Konuralp Journal of Mathematics, 11 (1) (2023) 8-14

Konuralp Journal of Mathematics

Research Paper

Journal Homepage: www.dergipark.gov.tr/konuralpjournalmath

e-ISSN: 2147-625X

A Note On The Matrix Operators Of Absolute Nörlund Series Space

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Abstract

On the recent study, the series space $\left|N_p^\theta\right|(q)$ has been introduced as the set of all series summable by the absolute Nörlund summability method, which includes the Maddox's space $l(q)$, the absolute Cesàro space $|C_\alpha|_k$ and the absolute Nörlund space $\left|N_p^\theta\right|_k$, and studied in terms of some topological and algebraic properties and matrix transformations by Gökçe and Sarıgöl. In this paper, some characterizations of matrix operators between the absolute Nörlund series space $\left|N_p^\theta\right|(q)$ and the classical sequence spaces c, c_0, l_∞ are given. Also, it is shown that the matrix operators are bounded operators. Finally, certain results are obtained as a special case.

Keywords: *Nörlund means; matrix transformation; bounded linear operators; Maddox's space.*

2010 Mathematics Subject Classification: *40C05, 40D25, 40F05, 46A45*

1. Introduction

The summability theory has very important role in mathematics, which has various applications in analysis, applied mathematics, engineering sciences, specially quantum mechanics, probability theory, Fourier analysis, approximation theory and fixed point theory, etc. It deals with the generalization of the concept of convergence of sequences and series, and aims to assign a limit for non-convergent sequences and series using an operator described by an infinite matrix. The reason why matrices are used for a general linear operator is that a linear operator from a sequence space to another can generally be given with an infinite matrix. This reveals the importance of sequence spaces and matrix operators in summability theory. In recent times, the literature has grown up concerned with characterizing all matrices operators which transform one given sequence space into another (see [1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 19]). In the study, we investigate the necessary and sufficient conditions for $A \in (\left|N_p^\theta\right|(q), \zeta)$ where $\zeta = \{c, c_0, l_\infty\}$, and then we present some results.
A sequence space is a vector subspace of ω which is the set of all complex valued sequences. Some of the most important sequence spaces in analysis are the set of all convergent, null and bounded sequences and the set of all convergent and bounded series spaces which are represented by $c, c_0, l_\infty, c_s, b_s$, respectively. Let $U = (u_{nv})$ be any infinite matrix with complex components and Λ, Γ be arbitrary sequence spaces. It is denoted the U-transform of the sequence $\lambda = (\lambda_v)$ by $U(\lambda) = (U_n(\lambda))$, if the series

$$U_n(\lambda) = \sum_{j=0}^{\infty} u_{nj}\lambda_j,$$

is convergent for each $n \in \mathbb{N}$. If $U(\lambda) \in \Gamma$, whenever $\lambda \in \Lambda$, then it is said that U describes a matrix transformation from the space Λ into another space Γ, and the class of all infinite matrices U such that $U : \Lambda \to \Gamma$ is represented by matrix class (Λ, Γ). The concept of matrix domain of an infinite matrix U in a sequence space Λ is described by the set

$$\Lambda_U = \{\lambda = (\lambda_n) \in \omega : U(\lambda) \in \Lambda\} \tag{1.1}$$

which is also a sequence space.
The Maddox's space which is defined by

$$l(q) = \left\{\lambda = (\lambda_n) : \sum_{n=0}^{\infty} |\lambda_n|^{q_n} < \infty\right\},$$

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is an $FK-$space which is a complete metrizable locally convex space with continuous coordinates $r_n : \Lambda \to \mathbb{C}$ defined by $r_n(\lambda) = \lambda_n$ for all $n \in \mathbb{N}$, according to its natural paranorm given by

$$g(\lambda) = \left(\sum_{n=0}^{\infty} |\lambda_n|^{q_n} \right)^{1/H},$$

where $H = \max\{1, \sup_n q_n\}$. Also, with together this paranorm, the Maddox's space $l(q)$ has AK property i.e., every sequence $\lambda \in l(q)$ has a unique representation $\lambda = \sum_{v=0}^{\infty} \lambda_v e^{(v)}$ where $e^{(v)}$ is the sequence whose only non-zero term is 1 in vth place for each $v \in \mathbb{N}$. On the other hand, if $q_v \geq 1$ for all v, also it is even a $BK-$space, normed FK-spaces, with respect to following norm

$$\|\lambda\| = \inf\left\{ \eta > 0 : \sum_{v=0}^{\infty} |\lambda_v/\eta|^{q_v} \leq 1 \right\},$$

([14], [15], [16]).
Let take an infinite series $\sum \lambda_v$ with the sequence of its partial sum (s_n). Also, assume that (θ_n) is a sequence of positive numbers and $q = (q_n)$ is a bounded sequence of non-negative terms. If

$$\sum_{n=0}^{\infty} \theta_n^{q_n-1} |U_n(s) - U_{n-1}(s)|^{q_n} < \infty$$

then, the series $\sum \lambda_v$ is said to be summable $|U, \theta_n|(q)$ (see[9]).
Considering the Nörlund matrix given by

$$a_{vj} = \begin{cases} p_{v-j}/P_v, & 0 \leq j \leq v \\ 0, & j > v \end{cases}$$

where (p_v) is a sequence of complex numbers with $P_v = p_0 + p_1 + ... + p_v \neq 0$, and the concept of absolute summability, we immediately get the absolute Nörlund summability $|N, p_n, \theta_n|(q)$. Then, with a few calculation, the space consisting all series summable by the absolute Nörlund summability method can be expressed as

$$\left|N_p^\theta\right|(q) = \left\{ \lambda \in \omega : \sum_{n=0}^{\infty} \theta_n^{q_n-1} \left| \sum_{j=0}^{n} \left(\frac{P_{n-j}}{P_n} - \frac{P_{n-j-1}}{P_{n-1}} \right) \lambda_j \right|^{q_n} < \infty \right\}.$$

Note that according to the notation (1.1), the space is redefined by means of the domain of a matrix in the Maddox's space $l(q)$ as follows: if we determine the infinite matrices $L^{(q)}(\theta)$ and $T(p)$ as

$$t_{nj}(p) = \begin{cases} \frac{P_{n-j}}{P_n}, & 0 \leq j \leq n \\ 0, & j > n, \end{cases} \tag{1.2}$$

and

$$l_{nj}^{(q)}(\theta) = \begin{cases} \theta_n^{1/q_n^*}, & j = n \\ -\theta_n^{1/q_n^*}, & j = n-1 \\ 0, & j \neq n, n-1, \end{cases} \tag{1.3}$$

then, it is written that $\left|N_p^\theta\right|(q) = (l(q))_{L^{(q)}(\theta) \circ T(p)}$.
Moreover, it is known that every triangle matrix has a unique inverse which is also a triangle, so the matrices $L^{(q)}(\theta)$ and $T(p)$ have inverse matrices $L^{(q)}(\theta)^{-1} = (l_{nj}^{(q)}(\theta)^{-1})$ and $T(p)^{-1} = (t_{nj}(p)^{-1})$ given by

$$l_{nj}^{(q)}(\theta)^{-1} = \begin{cases} \theta_j^{-1/q_j^*}, & 0 \leq j \leq n \\ 0, & j > n, \end{cases} \tag{1.4}$$

$$t_{nj}(p)^{-1} = \begin{cases} C_{n-j}P_j, & 0 \leq j \leq n \\ 0, & j > n. \end{cases} \tag{1.5}$$

Here, p_0 is a non-zero number and (C_n) is a sequence such that

$$\sum_{j=0}^{n} P_{n-j}C_j = \begin{cases} 1, & n = 0 \\ 0, & n > 0. \end{cases}$$

Throughout the whole paper, we suppose that $q = (q_j)$ is any bounded sequence of positive real numbers with $0 < inf q_j < \infty$ and q_j^* is the conjugate of q_j such that $1/q_j + 1/q_j^* = 1$ for $q_j > 0$, $1/q_j^* = 0$ for $q_j = 1$.
Before stating the main theorems, we remind some lemmas which have significant place in the remaining part of the study.

Lemma 1.1. *[11] Let $q=(q_j)$ be arbitrary bounded sequence of strictly positive numbers.*
(a) If $q_j \leq 1$, for all j, then,

$$U \in (l(q),c) \Leftrightarrow (i) \lim_{n\to\infty} u_{nj} \text{ exists for each } j, (ii) \sup_{n,j} |u_{nj}|^{q_j} < \infty$$

$$U \in (l(q),c_0)(iii) \Leftrightarrow \lim_{n\to\infty} u_{nj} = 0 \text{ for each } j, (ii) \text{ holds,}$$

and

$$U \in (l(q),l_\infty) \Leftrightarrow (ii) \text{ holds.}$$

(b) If $q_j > 1$ for all j, then,

$$U \in (l(q),c) \Leftrightarrow (i') \lim_{n\to\infty} u_{nj} \text{ exists for each } j, (ii') \text{ there is a number } K > 1 \text{ such that}$$

$$\sup_n \sum_{j=0}^{\infty} |K^{-1} u_{nj}|^{q_j^*} < \infty,$$

$$U \in (l(q),c_0) \Leftrightarrow (iii') \lim_{n\to\infty} u_{nj} = 0 \text{ for each } j, (ii') \text{ holds,}$$

and

$$U \in (l(q),l_\infty) \Leftrightarrow (ii') \text{ holds.}$$

Lemma 1.2. *[17] Let Λ be an FK space with AK, Γ be any subset of ω, R be triangle and S be its inverse. Then, $U \in (\Lambda_R, \Gamma)$ if and only if $\tilde{U} \in (\Lambda, \Gamma)$ and $V^{(n)} \in (\Lambda, c)$ for all n, where*

$$\tilde{u}_{nj} = \sum_{i=j}^{\infty} u_{ni} s_{ij};\ n,j = 0,1,...,$$

and

$$v_{mj}^{(n)} = \begin{cases} \sum_{i=j}^{m} u_{ni} s_{ij}, & 0 \leq j \leq m \\ 0, & j > m. \end{cases}$$

Lemma 1.3. *[18] Let R be a triangle. Then, for $\Lambda, \Gamma \in \omega, U \in (\Lambda, \Gamma_R)$ if and only if $B = RU \in (\Lambda, \Gamma)$.*

Lemma 1.4. *[20] Matrix transformations between FK-spaces are continuous.*

2. Main Theorems

Theorem 2.1. *[8] Assume that $q=(q_j)$ is a bounded sequence of non-negative numbers and $\theta=(\theta_n)$ is a sequence of positive numbers. Then, the set $|N_p^\theta|(q)$ is a vector space with the coordinate-wise addition and scalar multiplication. Besides, it is an FK-space with respect to the paranorm*

$$\|\lambda\|_{|N_p^\theta|(q)} = \left(\sum_{n=0}^{\infty} \left| \left(L^{(q)}(\theta) \circ T(p)(\lambda) \right)_n (\lambda) \right|^{q_n} \right)^{1/H},$$

where $H = \max\{1, \sup q_n\}$. Also, there is a linearly isomorphism between the spaces $|N_p^\theta|(q)$ and $l(q)$, that is, $|N_p^\theta|(q) \cong l(q)$.

After this point, let take

$$\sigma_{nj} = \sum_{v=j}^{n} C_{n-v} P_v$$

for ease of notation where p_0 is non-zero number.
Now, we continue with the theorem characterizing the matrix class $(|N_p^\theta|(q), \zeta)$ where $\zeta = \{c, c_0, l_\infty\}$:

Theorem 2.2. *Let $U=(u_{nj})$ be an infinite matrix of complex components, (θ_n) be any sequence of positive numbers, (q_j) be any bounded sequence of non-negative numbers with $q_j \leq 1$ for all $j \in \mathbb{N}$ and p_0 be non-zero number.*
(i) For $U \in (|N_p^\theta|(q), c)$ the necessary and sufficient conditions are, for $n=0,1,...,$

$$\sum_{i=j}^{\infty} \theta_j^{-1/q_j^*} u_{ni} \sigma_{ij} \text{ converges for each } j, \tag{2.1}$$

$$\sup_{m,j}\left|\theta_j^{-1/q_j^*}\sum_{i=j}^{m}u_{ni}\sigma_{ij}\right|^{q_j}<\infty \tag{2.2}$$

$$\sup_{n,j}\left|\theta_j^{-1/\mu_j^*}\sum_{i=j}^{\infty}u_{ni}\sigma_{ij}\right|^{q_j}<\infty \tag{2.3}$$

and

$$\lim_{n\to\infty}\theta_j^{-1/q_j^*}\sum_{i=j}^{\infty}u_{ni}\sigma_{ij} \text{ exists for all } j. \tag{2.4}$$

(ii) $U\in\left(\left|N_p^{\theta}\right|(q),c_0\right)$ *if and only if* (2.1), (2.2), (2.3) *and*

$$\theta_j^{-1/q_j^*}\sum_{i=j}^{\infty}u_{ni}\sigma_{ij}=0 \ \text{ for each } \ j, \tag{2.5}$$

(iii) $U\in\left(\left|N_p^{\theta}\right|(q),l_\infty\right)$ *if and only if the conditions* (2.1), (2.2) *and* (2.3) *hold.*

Proof. Let $q_j\le 1$ for all $j\in\mathbb{N}$. Note that $\left|N_p^{\theta}\right|(q)=(l(q))_{L^{(q)}(\theta)\circ T(p)}$. So, considering by Lemma 1.2, it is easy to see that $U\in\left(\left|N_p^{\theta}\right|(q),c\right)$ is equal to $\tilde{U}\in(l(q),c)$ and $V^{(n)}\in(l(q),c)$. Here the matrices $\tilde{U}$ and $V^{(n)}$ are defined by

$$\tilde{u}_{nj}=\theta_j^{-1/q_j^*}\sum_{i=j}^{\infty}u_{ni}\sigma_{ij}.$$

$$v_{mj}^{(n)}=\begin{cases}\theta_j^{-1/q_j^*}\sum\limits_{i=j}^{m}u_{ni}\sigma_{ij}, & 0\le j\le m\\ 0, & j>m.\end{cases}$$

It can be obtained easily by Lemma 1.1 that $V^{(n)}\in(l(q),c)$ iff the conditions (2.1) and (2.2) hold. Again, to applying Lemma 1.1 to $\tilde{U}$, we get that $\tilde{U}\in(l(q),c)$ if and only if the conditions (2.3) and (2.4) hold, which completes the first part of proof.
Because the remaining part of the proof can be proved similar way, it left to reader. □

Theorem 2.3. *Suppose that* $U=(u_{nj})$ *is an infinite matrix of complex components and* (θ_n) *is a sequence of positive numbers. If* p_0 *is non-zero number and* (q_j) *is arbitrary bounded sequence of non-negative numbers such that* $q_j>1$ *for all* $j\in\mathbb{N}$, *then*
(i) $U\in\left(\left|N_p^{\theta}\right|(q),c\right)$ *if and only if there exists an integer* $K>1$ *such that, for* $n=0,1,...,$

$$\sum_{i=j}^{\infty}\theta_j^{-1/q_j^*}u_{ni}\sigma_{ij} \ \text{ converges for each } \ j, \tag{2.6}$$

$$\sup_{m}\sum_{j=0}^{\infty}\left|\theta_j^{-1/q_j^*}\sum_{i=j}^{m}u_{ni}\sigma_{ij}K^{-1}\right|^{q_j^*}<\infty \tag{2.7}$$

$$\sup_{n}\sum_{j=0}^{\infty}\left|\theta_j^{-1/q_j^*}\sum_{i=j}^{\infty}u_{ni}\sigma_{ij}K^{-1}\right|^{q_j^*}<\infty \tag{2.8}$$

and (2.4) *hold.*
(ii) $U\in\left(\left|N_p^{\theta}\right|(q),c_0\right)$ *if and only if* (2.5),(2.6),(2.7) *and* (2.8) *hold.*
(iii) $U\in\left(\left|N_p^{\theta}\right|(q),l_\infty\right)$ *if and only if* (2.6),(2.7) *and* (2.8) *hold.*

Proof. Let $q_n>1$ for all $n\in\mathbb{N}$. From Lemma 1.2, it is obtained immediately that $U\in\left(\left|N_p^{\theta}\right|(q),l_\infty\right)$ if and only if $\tilde{U}\in(l(q),l_\infty)$ and $V^{(n)}\in(l(q),c)$. Applying Lemma 1.1 to the matrices $V^{(n)}$ and $\tilde{U}$ which are given in above theorem, the conditions (2.6), (2.7) and the condition (2.8) holds, respectively.
The remaining part of the theorem can be proved by the similar method. □

Theorem 2.4. *Let* $U=(u_{nj})$ *be an infinite matrix of complex components,* $\theta=(\theta_n)$ *be any sequence of positive real numbers,* $q=(q_j)$ *be any bounded sequence of positive numbers and* $\zeta=\{c,c_0,l_\infty\}$. *If* $U\in(\left|N_p^{\theta}\right|(q),\zeta)$, *then* U *defines a bounded linear operator.*

Proof. Since the spaces c,c_0,l_∞ are BK- spaces, normed FK-spaces, using Lemma 1.4 and Theorem 2.1, the proof of Theorem can be obtained immediately. □

Since $b_s = (l_\infty)_Z$ and $c_s = (c)_Z$ where the matrix $Z = (z_{nj})$ is given by

$$z_{nj} = \begin{cases} 1, & 0 \le j \le n \\ 0, & j > n, \end{cases}$$

the matrix classes $\left(\left|N_p^\theta\right|(q), b_s\right)$ and $\left(\left|N_p^\theta\right|(q), c_s\right)$ can be characterized as follows with Lemma 1.3:

Corollary 2.5. *Take $u(n,k) = \sum_{j=0}^{n} u_{jk}$ instead of u_{nk} in the Theorem 2.2 and Theorem 2.3. Then*

(i) if $q_v > 1$ for all v,

$$U \in \left(\left|N_p^\theta\right|(q), c_s\right) \Leftrightarrow (2.4), (2.6), (2.7) \text{ and } (2.8) \text{ hold.}$$

$$U \in \left(\left|N_p^\theta\right|(q), b_s\right) \Leftrightarrow (2.6), (2.7) \text{ and } (2.8) \text{ hold.}$$

(ii) if $q_v \le 1$ for all v,

$$U \in \left(\left|N_p^\theta\right|(q), c_s\right) \Leftrightarrow (2.1), (2.2), (2.3) \text{ and } (2.4) \text{ hold.}$$

$$U \in \left(\left|N_p^\theta\right|(q), b_s\right) \Leftrightarrow (2.1), (2.2) \text{ and } (2.3) \text{ hold.}$$

Note that, with the special selections, the absolute Nörlund series space $\left|N_p^\theta\right|(q)$ is reduced to some well-known spaces. For instance, if we take $q_n = k$ for all n, the space is reduced to $\left|N_p^\theta\right|_k$ studied by Hazar and Sarıgöl [12] and also it is reduced to the space $|C_\alpha|_k$ [19] for $q_n = k$, $\theta_n = n$ and $P_n = E_n^\alpha$ for all n, where

$$E_n^\alpha = \frac{(\alpha+1)(\alpha+2)...(\alpha+n)}{n!},$$

$$E_0^\alpha = 1, \; E_{-n}^\alpha = 0, \; n \ge 1$$

[2, 19]. With this information, the following results are immediately obtained from Theorem 2.2 and Theorem 2.3.

Corollary 2.6. *Let $U = (u_{nv})$ be an infinite matrix of complex components and (θ_n) be a sequence of positive numbers. If p_0 is non-zero number, then,*

(i) $U \in \left(\left|N_p^\theta\right|, c\right)$ if and only if for $n = 0, 1, ...,$

$$\sum_{j=v}^{\infty} u_{nj}\sigma_{jv} \text{ converges for all } v, \tag{2.9}$$

$$\sup_{m,v} \left| \sum_{j=v}^{m} u_{nj}\sigma_{jv} \right| < \infty \tag{2.10}$$

$$\sup_{n,v} \left| \sum_{j=v}^{\infty} u_{nj}\sigma_{jv} \right| < \infty \tag{2.11}$$

$$\lim_{n\to\infty} \sum_{j=v}^{\infty} u_{nj}\sigma_{jv} \text{ exists for all } v. \tag{2.12}$$

(ii) $U \in \left(\left|N_p^\theta\right|, c_0\right)$ if and only if (2.9), (2.10), (2.11) and

$$\sum_{j=v}^{\infty} u_{nj}\sigma_{jv} = 0 \text{ for each } v.$$

(iii) $U \in \left(\left|N_p^\theta\right|, l_\infty\right)$ if and only if the conditions (2.9), (2.10) and (2.11) hold.

Corollary 2.7. *Assume that $U = (u_{nv})$ is an infinite matrix of complex components and (θ_n) is a sequence of positive numbers. If p_0 is non-zero number and $k > 1$, then*

(i) $U \in \left(\left|N_p^\theta\right|_k, c\right)$ if and only if for $n = 0, 1, ...,$

$$\sum_{j=v}^{\infty} \theta_v^{-1/k^*} u_{nj}\sigma_{jv} \text{ converges for each } v, \tag{2.13}$$

$$\sup_m \sum_{v=0}^{\infty} \left| \theta_v^{-1/k^*} \sum_{j=v}^{m} u_{nj}\sigma_{jv} \right|^{k^*} < \infty \tag{2.14}$$

$$\sup_{n}\sum_{v=0}^{\infty}\left|\theta_v^{-1/k^*}\sum_{j=v}^{\infty}u_{nj}\sigma_{jv}\right|^{k^*}<\infty \tag{2.15}$$

and (2.12) *hold.*

(ii) $U\in\left(\left|N_p^{\theta}\right|_k,c_0\right)$ *if and only if* (2.13),(2.14), (2.15) *and*

$$\sum_{j=v}^{\infty}\theta_v^{-1/k^*}u_{nj}\sigma_{jv}=0 \ \text{ for each } v$$

hold.

(iii) $U\in\left(\left|N_p^{\theta}\right|_k,l_\infty\right)$ *if and only if* (2.13),(2.14) *and* (2.15) *hold.*

Corollary 2.8. *Let λ be non-negative integers, $U=(u_{nv})$ be an infinite matrix of complex numbers. Then $U\in(|C_\alpha|,c)$ if and only if*

$$\sum_{j=v}^{\infty}\frac{E_{j-v}^{-\alpha-1}}{j}u_{nj} \text{ converges for all } v\geq 1 \tag{2.16}$$

$$\sup_{m,v\geq 1}\left|vE_v^{\alpha}\sum_{j=v}^{m}\frac{E_{j-v}^{-\alpha-1}}{j}u_{nj}\right|<\infty \tag{2.17}$$

$$\sup_{n,v\geq 1}\left|vE_v^{\alpha}\sum_{j=v}^{\infty}\frac{E_{j-v}^{-\alpha-1}}{j}u_{nj}\right|<\infty \tag{2.18}$$

$$\lim_{n\to\infty}\sum_{j=v}^{\infty}\frac{vE_v^{\alpha}E_{j-v}^{-\alpha-1}}{j}u_{nj} \text{ exists for each } v\geq 1. \tag{2.19}$$

(ii) $U\in(|C_\alpha|,c_0)$ *if and only if* (2.16), (2.17),(2.19) *and*

$$\lim_{n\to\infty}\sum_{j=v}^{\infty}\frac{vE_v^{\alpha}E_{j-v}^{-\alpha-1}}{j}u_{nj}=0 \text{ for each } v\geq 1.$$

hold.

(iii) $U\in(|C_\alpha|,l_\infty)$ *if and only if (* (2.16), (2.17),(2.19) *hold.*

Corollary 2.9. *Let α be non-negative integers, $U=(u_{nv})$ be an infinite matrix of complex numbers. Then $U\in(|C_\alpha|_k,c)$ if and only if*

$$\sum_{j=v}^{\infty}\frac{E_{j-v}^{-\alpha-1}}{j}u_{nj} \text{ converges for all } v\geq 1 \tag{2.20}$$

$$\sup_{m}\sum_{v=1}^{m}\left|v^{1/k}E_v^{\alpha}\sum_{j=v}^{m}\frac{E_{j-v}^{-\alpha-1}}{j}u_{nj}\right|^{k^*}<\infty \tag{2.21}$$

$$\sup_{n\geq 1}\sum_{v=1}^{\infty}\left|v^{1/k}E_v^{\alpha}\sum_{j=v}^{\infty}\frac{E_{j-v}^{-\alpha-1}}{j}u_{nj}\right|^{k^*}<\infty \tag{2.22}$$

$$\lim_{n\to\infty}\sum_{j=v}^{\infty}\frac{v^{1/k}E_v^{\alpha}E_{j-v}^{-\alpha-1}}{j}u_{nj} \text{ exists for each } v\geq 1. \tag{2.23}$$

(ii) $U\in(|C_\alpha|_k,c_0)$ *if and only if* (2.20), (2.21), (2.22) *and*

$$\lim_{n\to\infty}\sum_{j=v}^{\infty}\frac{v^{1/k}E_v^{\alpha}E_{j-v}^{-\alpha-1}}{j}u_{nj}=0 \text{ for each } v\geq 1$$

hold.

(iii) $U\in(|C_\alpha|_k,l_\infty)$ *if and only if* (2.20), (2.21), (2.22) *hold.*

Article Information

Acknowledgements: The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Author's contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflict of Interest Disclosure: No potential conflict of interest was declared by the author.

Supporting/Supporting Organizations: No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent: It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism detected.

Availability of data and materials: Not applicable.

References

[1] P. Z. Alp, M. İlkhan, On the difference sequence space $l_p(\hat{T}^q)$, Math. Sci. Appl. E-Notes Vol: 7, No. 2 (2019), 161-173.
[2] T. M. Flett, On an extension of absolute summability and some theorems of Littlewood and Paley, Proc. London Math. Soc. Vol: 7 (1957), 113-141.
[3] F. Gökçe, Compact and Matrix Operators on the Space $\left|\overline{N}_p^{\phi}\right|_k$. FUJMA. Vol: 4 No. 2 (2021), 124-133.
[4] F. Gökçe, M.A. Sarıgöl, Some matrix and compact operators of the absolute Fibonacci series spaces. Kragujevac J. Math. Vol: 44, No.2, (2020), 273–286.
[5] F. Gökçe, M.A. Sarıgöl, Series spaces derived from absolute Fibonacci summability and matrix transformations. Boll. Unione Mat. Ital. Vol: 13, (2020), 29-38.
[6] F. Gökçe, M.A. Sarıgöl, On absolute Euler spaces and related matrix operators, Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci. Vol: 90 No:5 (2020), 769-775.
[7] F. Gökçe, M.A. Sarıgöl, Generalization of the absolute Cesàro space and some matrix transformations, Num. Func. Anal. Opt. Vol: 40, (2019), 1039-1052.
[8] F. Gökçe, M.A. Sarıgöl, Extension of Maddox's space $l(\mu)$ with Nörlund means, Asian-Eur. J. Math. Vol: 12 No.6 (2019), 1-12.
[9] F. Gökçe, M.A. Sarıgöl, A new series space $\left|\overline{N}_p^{\theta}\right|(\mu)$ and matrix transformations with applications, Kuwait J. Sci. Vol:45 No:4 (2018), 1-8.
[10] F. Gökçe, M.A. Sarıgöl, Generalization of the space $l(p)$ derived by absolute Euler summability and matrix operators. J. Ineq. Appl. Vol: 2018 (2018), 133
[11] K.G. Grosse-Erdmann, Matrix transformations between the sequence spaces of Maddox, J. Mathe.Anal, Appl., Vol: (1993), 223–238.
[12] G. C. Hazar, M. A. Sarıgöl, On Absolute Nörlund Spaces and Matrix Operators, Acta Math. Sinica, English Series Vol: 34 (2018), 812-826.
[13] M. Ilkhan, Matrix Domain of a Regular Matrix Derived by Euler Totient Function in the Spaces c_0 and c, Mediterr. J. Math.,Vol: 17, No.1 (2020), 1-21.
[14] I. J. Maddox, Some properties of paranormed sequence spaces, J. London Math. Soc. Vol: 1, (1969), 316-322.
[15] I. J. Maddox, Paranormed sequence spaces generated by infinite matrices, Math. Proc. Cambridge Philos. Soc. Vol: 64, (1968), 335-340.
[16] I. J. Maddox, Spaces of strongly summable sequences, The Quart. J. Math. Vol: 18, (1967), 345-355.
[17] E. Malkowsky, V. Rakocevic, On matrix domains of triangles. Appl.Math. Comp. Vol:189, No. 2, (2007), 1146-1163.
[18] E. Malkowsky, V. Rakocevic, An introduction into the theory of sequence space and measures of noncompactness, Zb. Rad.(Beogr), Vol:9, No.17, (2000), 143-234.
[19] M.A. Sarıgöl, Spaces of series summable by absolute Cesàro and matrix operators, Comm. Math. Appl. Vol:7 (2016), 11-22.
[20] A. Wilansky, *Summability Through Functional Analysis*, Math. Studies, 85. North Holland , Amsterdam, (1984).