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Research Article

Why Do Pre-Service Teachers Who Believe In The Necessity Of Proof Have Low-Level Self-Efficacy With The Proof?

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Abstract – This research aimed to examine the opinions of pre-service mathematics teachers towards proof. The students received low scores in the items containing the self-evaluation of the proof in the quantitative dimension, a qualitative form was applied to obtain more comprehensive information. Therefore, quantitative and qualitative research approaches were used together along with mixed methods. Although all students expressed their opinions on the necessity of proof, it was concluded that they had difficulties in the proof process due to the difficulty level of proof and their lack of mathematical knowledge in their self-evaluation. And also, a great majority of pre-service teachers stated that they intended to use alternative learning approaches instead of proof in their future careers. Although proof is seen as a necessary process, pre-service teachers' proof self-efficacy levels are low and their thoughts on using proof in their professional life are insufficient. Therefore, it can be recommended to include proof in pre-graduate education and to eliminate the students' lack of basic knowledge.

Key words: proof, mathematics education, proof self- efficacy.

Introduction

The crucial factor determining the degree of difficulty in mathematics is “precision”. Mathematics, unlike other fields of science, requires absolute precision (King, 1992). It can be argued that the situation that gives this certainty to mathematics is proof because mathematical objects gain certainty as they are proven. Showing the accuracy of mathematical knowledge is an essential activity for mathematics (Almeida, 2000). Bell (1976) sees proof as a process that takes place as a result of sequential steps and adds that there should be categories of verification (showing the truth of a proposition), explanation (showing why the proposition is true), and systematization (organizing propositions and theorems in an inductive system). Similarly, Baki (2008) saw mathematical proof as a process and defined this process as verification, explanation, and abstraction.

The source of the proof is Euclid's book “Elements” which was published in the 4th century BC. According to Euclid, the process of proving consists of four stages; verifying the result, convincing others, finding a result, and placing the results in a deductive system (Almeida, 2003). There are two purposes in mathematical proof. The first of these is to show that the hypothesis is brought to an end with logical steps, and the second is to understand why and how to go from assumptions. Therefore, proof provides an understanding not only that a theorem is true, but also why it is true (Doruk & Kaplan, 2013). Considered in this way, it can be said that the proof is both convincing and exploratory, and can contribute to the systematization of mathematics by reconciling the results obtained with it (Hanna & Barbeau, 2009).

In a mathematical proof, there should be definitions, theorems, and operational processes, and these structures should reveal the mathematical-logical predictions hidden in the expressions. Therefore, the purpose of mathematical proof can be expressed as proving the truth or falsity of what is claimed in every situation (Baki, 2008) as well as showing the relationship between verifications (Lee, 2002). Thus, it can be ensured that students predict facts that are not seen in a real-life situation or event, and that they can reach positive results by testing their accuracy and falsity (Zaslovsky & Peled, 1996). Therefore, it is possible to say that mathematical proof is important and necessary for relational, permanent, and meaningful learning. For this important process to come to the fore in mathematics teaching steps, it is necessary to know the opinions and self-evaluations of both teachers and pre-service teachers in terms of an effective roadmap.

There are various studies regarding the notion of mathematical proof. In one of these studies, for instance, Moralı et al. (2006), who investigated the opinions of pre-service teachers on proof, stated that the opinions of the participants on proof were not fully formed. According to them, this stems from the fact that pre-service teachers do not know the importance of proof in teaching mathematics. In another study, Saeed (1996) investigated the relationship between students' understanding of some concepts related to mathematical proof and their attitudes towards mathematics. As a result of the research, he discovered that many students at the university level had difficulties in learning and teaching proof. Another study by Kotelawala (2007) aimed to reveal teachers' attitudes and beliefs towards proof in mathematics classrooms.

Kotelawala found that teachers' past concerns and difficulties with proof affected their use of proof in their classrooms. In another study, Nordström (2002) reported that, although students exhibited positive attitudes towards proof, they had great difficulties in proving many intermediate-level statements.

Considering the studies carried out, teachers and pre-service teachers should understand the importance of proving and having proof skills and include proof processes in their professional lives. As stated by Weber (2004), one of the aims of mathematics is to make students acquire the ability to prove. It is, therefore, thought that the attitudes and self-evaluations of mathematics teachers and prospective teachers towards proof are also likely to affect their use of proof in their lessons and their teaching methods. In this respect, it is important for people who will take part in the education task to focus on proof in mathematics education starting from the primary education level in terms of the development of students' mathematical thinking. And thus, it is effective in developing positive attitudes towards mathematics in students (Moralı et al., 2006). For this reason, knowing the opinions and self-evaluations of people who will undertake the education task will be beneficial in interpreting the mathematics education processes of the students transferable.

For this reason, within the scope of this research, first of all, pre-service teachers' views on proof, their awareness of the importance of proof, and their self-evaluations on proof skills were discussed. In the data obtained, a difference was determined in the levels of evaluating the necessity of proof, and the self-evaluation levels of the pre-service teachers, and it was aimed to determine the pre-service teachers' definitions of proof, the stages they had difficulty in, and their preferences to include proof in mathematics lessons in their future careers. The subject of proof was analyzed not only as an attitude towards proof but also as a self-evaluation of proof skill and related factors.

It is thought that pre-service teachers' views on proof are an important determinant of their ability to benefit from proof in their classrooms, their mathematics education, and their future careers because knowing pre-service teachers' views on proof and self-evaluation can provide a roadmap for proof skills. When the existing literature is examined, students' attitudes and beliefs towards proof were determined in several studies (Kotelawala, 2007; Mingus & Grassl, 1999; Saeed, 1996; Üzel & Özdemir, 2009). In addition, there are also various studies on proof self-efficacy in the literature. For example, in one study, Regier and Savic (2020) explored how fostering mathematical creativity may impact student self-efficacy for proof. In another study, the relationship between persistence and self-efficacy in proof construction was investigated (Selden & Selden, 2014). In another study by Shonge and Mudaly (2021), the aim was to develop and investigate preliminary validity evidence for a new instrument for measuring self-efficacy for mathematical proof in high school students. In another study, Viholainen et al. (2019) examined university students' motivation and self-efficacy beliefs about proof and proving. Although there are similar studies involving different variables related to self-efficacy with the proof, there is no study that deals with the relationship between students' opinions and self-evaluations and related factors. It is thought that this study will contribute to the literature.

Method

The research was designed only in a quantitative dimension at the first stage, and the scope of the research was expanded based on the difference between the opinions of the pre-service teachers on the necessity of proof and the self-efficacy levels of proof skill, according to the data obtained at the first stage. The in-depth analysis was carried out in a mixed research model in which quantitative and qualitative approaches were used together with open-ended scale application.

In this study, a non-random convenient sampling method was adopted in the selection of the study group. The study group consists of 34 pre-service mathematics teachers studying in a state university's primary education mathematics teaching program. Although the number of participants is small, the sample can be considered quite representative. The spring semester was waited for the application of the scales in the research because the pre-service teachers had taken the courses of Abstract Mathematics, General Mathematics, Analysis-I, and Analysis-II, in which they knew of the existence and use of the proof process.

The quantitative dimension of the research was carried out with a descriptive survey. In the qualitative dimension, the document analysis of the answer codes given to the opinion, self-evaluation, and design questions for proof was made and the mixed method was used. The 5-point Likert-type "Opinion Scale for Proof" developed by Moralı et al. (2006) was used in the research. The reliability of the scale was determined as 0.80 and factor analysis was performed to determine its construct validity. The scale has seven factors; personal proof proficiency, the importance of proving, the effect of proof on understanding the theorem, self-perceptions about proving, general views on proving, examples, theorem perspectives, and views on the relationship between problem-solving and mathematical proof.

In the evaluation of the quantitative scale, the arithmetic mean of the scale items was used and it was tried to determine the direction of the score distribution (high, medium, low). The attitudes of the sample were also examined in terms of the factors that emerged as a result of the factor analysis performed with the help of the SPSS 25.0 program, and it was observed that opposite average values were obtained especially between some factors in the scale. Contrary to the importance of proof, the effect of proof on understanding the theorem, and the fact that their general views on proving reflect a high mean score, the pre-service teachers have a low mean score, especially regarding their proof proficiency and self-perceptions about proof. To understand the reason for these contradictory results obtained for proof and to obtain a detailed result, a qualitative form that contained 3 open-ended questions was applied as a second step after the quantitative scale was applied. As a result, data were collected with a questionnaire containing both the statements and open questions.

Findings and Discussions

Within the scope of the research, firstly, the "Opinion Scale for Proof" was applied to obtain the opinions of the pre-service teachers about proof, and then a qualitative scale application consisting of open-ended questions was applied for in-depth analysis due to the contrast between the average scores obtained among the sub-factors of the scale. Table 1 presents the descriptive statistical values of the answers given by the pre-service teachers to the "Opinion Scale for Proof".

Table 1
Descriptive Statistics Values of Pre-service Teachers' Opinions on Proof

	Min.	Max.	Mean	Std. Deviation
1. A mathematical proof is about facts and explaining facts.	3.00	5.00	4.23	0.65
2. When a mathematical result is proven, I believe it to be true.	4.00	5.00	4.64	0.48
3. Seeing a result illustrated by an example does not always help me understand why that conclusion is true.	3.00	4.00	3.55	0.50
4. Proof is indispensable for theoretical mathematics.	3.00	5.00	4.38	0.65
5. In mathematics, we can only understand whether something is true with the help of examples.	1.00	3.00	2.32	0.68
6. I do not understand why we need to do the proofs. All the results we see in the	1.00	5.00	2.38	1.15

lesson were proven before without a doubt by famous mathematicians.				
7. Proofs sometimes involve strategies that are not very clear.	2.00	5.00	3.52	0.78
8. In mathematics, if a result is clearly true, there is no point in proving it.	1.00	5.00	2.17	0.99
9. I like doing mathematical proofs.	1.00	5.00	2.20	0.91
10. I am confident in my ability to prove myself.	1.00	5.00	1.85	0.98
11. Working through the stages of proof helps me understand why something is true.	2.00	5.00	3.64	0.81
12. Seeing different proofs of a theorem helps me understand it better.	2.00	5.00	4.00	0.85
13. A mathematical proof also depends on other mathematical results.	2.00	5.00	3.94	0.81
14. I usually have a hard time understanding the proofs.	1.00	5.00	3.97	0.99
15. Proving is, in a sense, problem-solving.	1.00	5.00	3.55	0.99
16. Only professional mathematicians can do mathematical proofs.	1.00	3.00	1.94	0.81
17. I think knowing the theorem (or proposition) is more important than proving it.	1.00	4.00	2.58	0.74
18. Dealing with proofs is very boring.	1.00	5.00	3.94	0.95
19. Although I generally understand what a theorem means, I find it difficult to understand its proof.	1.00	5.00	4.02	0.71
20. I can only understand proof when the teacher does it in the classroom.	1.00	5.00	3.97	0.99

In this scale, which consists of 7 factors, it was determined that there is an opposite correlation between the mean score values of some factors that are directly or indirectly related to each other. The first factor is the students' personal proof proficiency (items 14, 18, 19, and 20 of the scale), the second factor is the students' views on the importance of proving (items 6, 7, 8, and 17), the third factor is the students' views on the effect of proof on understanding the theorem (11, 12, 13 and 16), the fourth factor is students' self-perceptions towards proving (items 9 and 10), the fifth factor is students' general views on proving (items 1, 2 and 4), the sixth factor is students' perspectives on examples and theorems (items 3 and 5) and the seventh factor is students' views on the relationship between problem-solving and mathematical proof (item 15). Considering these sub-factors ("personal proof proficiency" and "self-perceptions towards proving"), the mean scores of these items (9, 10, 14, 18, 19, 20) of the preservice teachers showed that they had low self-efficacy in proof.

Since the self-evaluation of the personal proof proficiency was low, the reasons for this were investigated more comprehensively with open-ended questions. In the qualitative scale applied, the students were asked what it means to prove, the parts that they have difficulty in proving, and their preferences for proving in their future careers.

Table 2 shows the answers given to the question "What does mathematical proof mean to you? Please explain."

Table 2
Statements of Pre-Service Teachers about Mathematical Proof

Expression categories	f	Some sample expressions from participant opinions
Meaningful Learning	18	*"Explaining with proof provides more meaningful learning, so it has an important place for mathematics."* *"It makes sense of mathematics. It makes me understand that the formulas I learned came from somewhere, not just a written formula."*
Permanent learning	11	*"Rather than memorizing the subject, I think one of the basic steps of permanent learning is proof."* *"Proof is important both for better understanding of a subject and for keeping the formulas in mind as we can recreate the formulas in moments of forgetting."*
Causality Relationship	5	*"The human brain has an inquiring nature and seeks reasons in mathematics, not memorization. To present these reasons, the proof is important."* *"Proof takes us to the basics of knowledge. It makes us understand where the information comes from, it makes us think."*

In Table 2, the answers of the pre-service teachers on what proof means are grouped under three categories. The vast majority of pre-service teachers stated that proof is a process that provides meaningful learning (f=19). As can be seen in the sample expressions in the table, the answers of all pre-service teachers about the concept of proof point to the necessity and importance of proof. The question was asked to determine whether students' positive opinions about the functionality and importance of proving on a quantitative scale would be repeated on a qualitative scale, and it was observed that the data derived from both qualitative and quantitative scales were consistent.

Table 3 contains data on student answers to the open-ended question "Are there any stages that you have difficulty in proving, and if so, what are they?"

Table 3
Statements of Pre-Service Teachers about Situations in which They Have Difficulty in Proving

Expression categories	f	Some sample expressions from participant opinions
Structure of the proof	15	*"I generally have no difficulty in proving, but the content of some proofs can be difficult.* *"In some proofs, I cannot relate some steps to each other, so it can be difficult."* *"Proofs that are complex and require some acceptances are difficult for me."*
A Mixed Process	10	*"Since proof involves some complicated steps, constant proofing in class can scare the student. It would be better if the proofs were chosen for some specific topics."* *"In some subjects, making a demonstration can be easier and more effective than proof."*
Mathematical Knowledge Level	9	*"Whether I am successful or unsuccessful in proving is related to my mastery of the subjects."* *"I think being able to prove is very high level and requires knowing a lot about different subjects related to mathematics. So, even though it is very important, I cannot always succeed.*

According to the data in Table 3, the situations that pre-service teachers had difficulty in proving were grouped under three categories. The point that students frequently expressed was that some proofs included more difficult steps due to their nature (f=15). This situation is named under the category of content/structure of the proof. The second category was the expression of proof as a complex process in general (f=10). In the answers under this category, it was stated that it is generally difficult to prove and situations such as showing examples, making demonstrations, and solving problems provide an easier learning process than proof. Another category is related to basic mathematical knowledge level. The pre-service teachers who responded under this category associated their inadequacy in proving with their lack of knowledge rather than the proof process and showed mastery of mathematics as a prerequisite for proving. In that case, it can be argued that pre-service teachers divide the self-evaluation of proof into internal factors (level of mathematical knowledge) and external factors (difficulty of proof, seeing proof as a high-level process).

Table 4 contains data on student responses to the open-ended question "When you become a teacher, would you consider using proof in your lessons, please explain".

Table 4

Pre-Service Teachers' Views on Using Proof in their Lessons

Expression categories	f	Some sample expressions from participant opinions
Conditional Yes Opinion (I Use But…)	27	*"I do not think of using it all the time, I want to make some subjects permanent by giving examples from daily life, not by proof.* *"Proving all the time can scare the student. Students may see the mathematics course as a lesson in which every subject must be proven."*
Unconditional Yes Opinion	7	*"I use proof because learning proof is important."* *"I am thinking of using proof because I think knowing where the theorems come from is more memorable than memorizing the theorems."* *"Yes, I would consider using it. Even if it is high-level for students, I try to think about where the information I give comes from and how I can show it."*

In Table 4, after the general opinions and self-evaluations of the pre-service teachers on proof and their ideas about their future career choices were examined, these ideas were gathered in two categories; "conditional yes" and "unconditional yes". Although the majority of pre-service teachers (f=27) stated that proof is important in mathematics education, they stated that they thought of using proof only under certain conditions in their professional life. As a justification for this, they put forward reasons such as the fact that it is difficult for students to use proof all the time, and demonstration by example is more understandable. Pre-service teachers who said "unconditional yes" (f=7) stated that they intended to use proof in their lessons because it is important and useful to prove without specifying any conditions.

In general, pre-service teachers expressed a positive opinion on the importance of proving on a quantitative scale and similarly repeated their positive views on the effectiveness of proof on a qualitative scale. However, when the self-assessments of the pre-service teachers about proving proficiency were examined, it was determined that the pre-service teachers found their proficiency levels to be insufficient. It was also determined that the inadequacy of the pre-service teachers in self-evaluation in making proofs is related to the difficulties arising from the structure of the proof and their lack of basic mathematical knowledge, and in connection with these difficulties, the pre-service teachers also stated that they would not always use proof in their professional life, but only under certain conditions.

Conclusions and Suggestions

When the scores of the students are examined, the average score of the answers given by the participants about the necessity of proof and the importance of proof indicates that the pre-service teachers have positive opinions about the importance of proof and its contribution to the learning process. The average scores of the items (9, 10, 14, 18, 19, 20) related to proof self-efficacy in the scale showed that the students had low proof self-efficacy. However, the average score of the items of the sub-factors in which proof is evaluated as a cognitive skill and students making self-assessments about proof proficiency generally shows that pre-service teachers find their proficiency levels lacking. This shows that although students consider proof important in the learning process, they have difficulty in performing it and they see themselves as inadequate in proving. With this result, to understand the underlying reason for this more clearly, after the quantitative scale, the students were applied a qualitative scale and were asked what it means to make a proof, the parts that they had difficulty in proving, and their preferences for proving in the future. Most of the pre-service teachers expressed their views on what proof meant, and said that proof is a process providing meaningful learning. They also mentioned the relationship of proof with permanent learning and causality. And also, the answers of all pre-service teachers in the study to the concept of proof pointed to the necessity and importance of proof. All pre-service teachers who participated in the research expressed positive views on the functionality and importance of proving on a quantitative scale, and they similarly repeated their positive views on the effectiveness of proof in the open-ended question in the qualitative scale, and thus consistent results were obtained in the quantitative-qualitative scale data. Similarly, the results of another study showed that the students were highly motivated to learn to understand and construct proofs, but they were more uncertain about their proving skills (Viholainen et al., 2019).

As the second question in the qualitative scale, the situations in which the pre-service teachers had difficulties in proving were examined, and the answers were gathered in three categories; the structure of the proof, the fact that it is a complex process, and the lack of basic mathematical knowledge. The most frequent opinion of the students was that some proofs included more difficult steps due to their nature. The second most frequently answered category was the expression of proof as a complex process in general. In the answers in this category, it was stated that it is generally difficult to prove, and alternative methods such as showing examples, demonstrations, and problem-solving offer more understandable learning systematic than proof. Another category was determined as the level of basic mathematical knowledge, and the pre-service teachers in this category associated their inadequacy in proving with the existing knowledge deficiencies rather than the proof itself and showed mastery of mathematics as a prerequisite for proving. When these results are evaluated, it is possible to argue that pre-service teachers divide the self-evaluation of proof into internal factors (level of mathematical knowledge) and external factors (difficulty of proof and seeing proof as a high-level process).

Although the importance of proof for mathematics was emphasized in different studies, it was determined that university students and mathematics teachers were unsuccessful in proof (Cusi & Malara, 2007; Doruk & Kaplan, 2015; Ko & Knuth, 2009; Weber, 2001). In Nordström's (2002) study, it was stated that although students exhibited positive attitudes towards proof and learning proof, they had great difficulties in proving many intermediate-level statements. Within the scope of the current research, pre-service teachers' self-evaluations about proof skills were examined and it was determined that they had insufficient self-efficacy beliefs in this regard.

After the general opinions and self-evaluations of the pre-service teachers about proof, their ideas about their future career choices were also examined. These opinions were gathered under the categories of "conditional yes" and "unconditional yes", and the pre-service teachers who answered "unconditional yes" stated that they thought of using proof in their lessons without specifying any conditions. Those who answered "conditionally yes" stated that they found it appropriate to use proof only if the subject was suitable and there were no alternative teaching practices and that it would be difficult for students to prove constantly. Considering that the vast majority of pre-service teachers gave the "conditional yes" answer, it can be thought that this situation is also related to their low self-efficacy beliefs about proof skills.

As a result, in the current study, the prominent situation is that although proof is considered important, having an incomplete self-efficacy belief, especially because the proof process is described as difficult, negatively affects the pre-service teachers' preferences for using proof in their professional lives. Mathematics is considered difficult because it is precise rather than abstract, and, unlike all other fields, requires absolute precision (King, 1992). It can be said that the concept that gives this certainty to mathematics is proof and that the pre-service teachers' self-insufficiency beliefs about proof skills are based on the precision aspect of mathematics. However, although mathematics is known as a science of precision, and thus, of proof, the role of proof is not generally reflected in curricula (Reiss, Heinze & Klieme, 2002), and it has become a priority to focus more on it in mathematics education (Schabel, 2005).

It is thought that, in addition to the attitudes towards proof, the proving skills and proof self-efficacy beliefs are also important determinants in the ability of pre-service teachers to benefit from proof in their mathematics learning processes and in their future professional life. And also, in connection with the intrinsic factors (insufficient mathematical knowledge level) in the self-evaluation of pre-service teachers, it can be suggested to determine the knowledge deficiencies with a test application that covers all the conceptual knowledge that pre-service teachers should have according to their grade level and to carry out a compensatory learning process that focuses on these deficiencies. More radical applications can be suggested for the external factors, namely the structure of proof and seeing it as a complex process, which is another issue in pre-service teachers' beliefs about their inadequacy of proof skills. As a result, it is thought that it would be beneficial to give more place to proof teaching at every grade, both in curricula and in classroom studies, to make students acquire the habit of making proof, therefore meaningful learning, self-efficacy belief in proof skill and the preference of making proof in their professional life.

İspatın Gerekliliğine İnanan Öğretmen Adayları İspatla İlgili Neden Düşük Düzeyde Özyeterliğe Sahiptir?

Özet:

Bu araştırma matematik öğretmen adaylarının ispata yönelik görüşlerini incelemeyi amaçlamaktadır. Araştırmanın nicel boyutunda, öğretmen adayları ispat öz değerlendirmesine yönelik maddelerden düşük puan aldığı için daha kapsamlı bilgi elde etmek amacıyla nitel bir ölçek uygulanmıştır. Bu nedenle nicel ve nitel araştırmanın birlikte uygulandığı karma yöntem yaklaşımı kullanılmıştır. Öğretmen adaylarının tamamı ispatın gerekliliği doğrultusunda görüş bildirmesine rağmen ispatın zorluk derecesi ve matematiksel bilgi eksikliklerinden dolayı ispat sürecinde zorlandıklarını belirtmişlerdir. Ayrıca öğretmen adaylarının büyük çoğunluğu mesleki hayatlarında ispat yerine alternatif öğrenme yaklaşımlarını kullanmayı amaçladıklarını belirtmişlerdir. İspat gerekli bir süreç olarak görülse de öğretmen adaylarının ispat öz-yeterlik düzeylerinin düşük ve meslek hayatlarında ispatı kullanma düşüncelerinin yetersiz olduğu sonucuna ulaşılmıştır..Bu nedenle lisans öncesi eğitimde de ispata yer verilmesi ve öğrencilerin temel bilgi eksikliklerinin giderilmesi önerilebilir.

Anahtar kelimeler: ispat, matematik eğitimi, ispat özyeterliliği.

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Statements Of Publication Ethics
This research was reviewed by the Izmir Demokrasi University Social and Humanities Ethics Committee and it was decided that the research was ethically appropriate. Date and ethical decision number: 09-05/2022- 2022/05/03

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