Fen, Matematik, Girişimcilik ve Teknoloji Eğitimi Dergisi
Journal of Science, Mathematics, Entrepreneurship and Technology Education
https://dergipark.org.tr/tr/pub/fmgted
© ISSN: 2667-5323

Investigating the Effect of Origami Use in Geometry Teaching on Pre-service Teachers' Concept Images*

Aliye Beyza VERGİSİZ[1], Atilla ÖZDEMİR[2]

[1]Süleyman Demirel Üniversitesi, Eğitim Fakültesi, İlköğretim Matematik Öğretmenliği, ORCID ID: 0000-0001-7750-5013, bvergisiz@gmail.com
[2]Dr. Öğr. Üyesi, Süleyman Demirel Üniversitesi, Eğitim Fakültesi, İlköğretim Matematik Öğretmenliği, ORCID ID: 0000-0003-4775-4435

**This study was presented as an oral presentation at the International Pegem Conference on Education (IPCEDU-2021).*

ABSTRACT
The paper folding method for teaching mathematics is used as a concrete material in primary and secondary schools to explore concepts, primarily geometry, and understand the relationship between these concepts. The overarching definitions expressing the relations of the quadrilaterals are ignored when teaching plane geometry. This research is differentiated from other studies in creating original content by approaching origami and quadrilaterals conceptually. In this study, pre-service teachers will be provided to rethink the conceptual relationship between quadrilaterals with origami. The primary purpose is to determine the effect of paper folding activities on pre-service teachers' concept images of quadrilaterals. A quantitative research method, a single group pre-test-post-test design, will be used to achieve this aim. The study data were collected with "Concept Image Questions" and "Geometry Attitude Scale." As a result of the data analysis, it was seen that while the pre-test was directed to the exclusionary definitions, the concept images of the inclusive definitions intensified in the post-test. In addition, no significant change was observed in the geometry attitudes of pre-service teachers. At the end of the study, it was concluded that the application of origami activities would enable pre-service teachers to express comprehensive definitions of quadrilaterals and to deal with the relations of quadrilaterals with each other in a conceptual dimension. Thus, students' awareness of plane geometry subjects taught in schools will increase indirectly.

ARTICLE TYPE
Research

ARTICLE INFORMATION
Received:
01.06.2022
Accepted:
05.09.2022

KEYWORDS:
Origami, geometry, quadrilaterals, concept image.

Geometri Öğretiminde Origami Kullanımının Öğretmen Adaylarının Kavram İmajlarına Etkisinin İncelenmesi

ÖZET
Matematik öğretimi için kağıt katlama yöntemi ilkokul ve ortaokulda geometri öncelikli olmak üzere kavramların keşfedilmesi ve bu kavramlar arasındaki ilişkinin anlaşılması için somut bir materyal olarak kullanılmaktadır. Dörtgenlerin birbiriyle olan ilişkilerini ifade eden kapsayıcı tanımları ise düzlem geometrisi öğretilirken göz ardı edilmektedir. Bu araştırma origami ile dörtgenler konusuna kavramsal olarak yaklaşarak özgün içerikler oluşturulması açısından diğer çalışmalardan farklılaştırılmıştır. Bu çalışmada öğretmen adaylarının dörtgenler arasındaki kavramsal ilişkiyi origami ile yeniden düşünmeleri sağlanacaktır. Burada temel amaç kağıt katlama etkinliklerinin öğretmen

MAKALE TÜRÜ
Araştırma

MAKALE BİLGİLERİ
Gönderilme Tarihi:
01.06.2022
Kabul Edilme Tarihi:
05.09.2022

adaylarının dörtgenlere ilişkin kavram imajlarına olan etkisini belirlemektir. Bu amaca ulaşmak için nicel araştırma yöntemi, tek grup ön test – son test deseni kullanılacaktır. Çalışmanın verileri ''Kavram İmajı Soruları'' ve ''Geometri Tutum Ölçeği'' ile toplanmıştır. Veri analizinin sonucunda ön testte daha çok hariç tutan tanımlara yönlenirken son testte kapsayıcı tanımlara ilişkin kavram imajlarının yoğunlaştığı görülmüştür. Bunun yanında öğretmen adaylarının geometri tutumlarında anlamlı bir değişme görülmemiştir. Çalışma sonunda origami etkinliklerinin uygulanmasının öğretmen adaylarının dörtgenlere ilişkin kapsayıcı tanımları ifade edebilme ve dörtgenlerin birbirleriyle olan ilişkilerini kavramsal boyutta ele alabilmelerini sağlayacağı sonucuna ulaşılmıştır. Böylece okullarda öğretilen düzlem geometri konularında öğrencilerde bu farkındalıkları dolaylı olarak artırılacaktır.

ANAHTAR KELİMELER: Origami, geometri, dörtgenler, kavram imajı.

Introduction

Since geometry teaching contributes to the development of skills such as critical thinking and problem-solving, geometry occupies an essential place in primary education mathematics, but it is an area that requires attention in teaching processes because it is built on abstract concepts and relationships (Bayraktar-Kurt, 2012; Toptaş, 2008). The National Council of Teachers of Mathematics (2000) defined geometry as a field where students have the opportunity to develop their reasoning and reasoning skills. Küçük-Demir (2020) stated that geometry teaching aims to realize mathematical strategies such as abstraction, expression, generalization, proof, and visualization. The individual's recognition of abstract concepts, specifying their features, and associating them with each other can be achieved by developing geometric thinking levels. Dina van Hiele-Geldof and Pierre van Hiele's approaches to developing geometric thinking are mentioned. Van Hiele's geometric thinking levels consist of five stages. These stages are visual, descriptive, informal inference, formal inference, and systematic thinking.

1. Visual Level: An individual at this level evaluates geometric shapes only according to appearance and is not interested in their features (Duatepe-Paksu, 2016). For example, a student at this level may look at a door and state that it is a rectangle but cannot grasp that a square can also be a rectangle.

2. Descriptive Level: An individual at this level can analyze the features of shapes but cannot see the hierarchical relationship between shapes (Duatepe-Paksu, 2016). Students at this level can talk about all rectangles instead of a specific rectangle. They can express the properties of a rectangle (four sides, four right angles, mutually parallel sides, etc.) (Van De Walle, Karp, & Bay-Williams, 2019).

3. Level of Informal Inference: An individual at this level can express the hierarchical relationship of figures but cannot prove it with a deductive inference (Duatepe-Paksu, 2016). The student can express shapes with necessary and sufficient features at this level.

4. Formal Inference Level: While seeing the relationship between shapes, an individual at this level can express this through reasoning and proof (Baki, 2018). For example, a student at this level can prove that when opposite sides of a quadrilateral are parallel, its opposite angles are congruent. At this level, students gain their first experience with the deductive structure of geometry (Yavuz, 2020).

5. Systematic Thinking Level: An individual at this level can prove the correctness of propositions in Euclidean geometry; Can interpret axioms, theorems, and definitions in Euclidean geometry in other geometries (Baki, 2018).

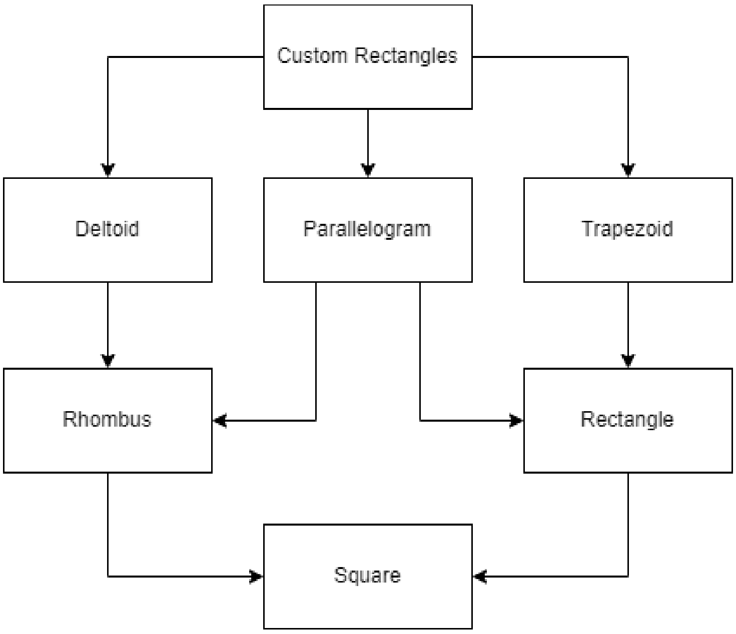
As mentioned in the Van Hiele approach, establishing the relationships between geometric shapes and comprehending the hierarchical relationship of shapes is an integral part of the field of geometry. It is stated that understanding the hierarchical relations between geometric shapes improves deductive reasoning and reasoning ability (Fujita & Jones, 2007). It is also clear that geometric reasoning often significantly influences mental shape images (Fujita & Jones, 2007). The image formed in the mind of the concept may belong to a prototype definition that expresses that shape. Prototype shapes are the first examples of the concept (Hershkowitz, 1990). Due to the dominance of the visual character in some examples of the concepts, prototype images may occur, and these examples may affect the formation of the concept (Zeybek-Şimşek, 2019). For example, a prototype image comes to mind when the concept of a rectangle is considered since it is emphasized that only the opposite side lengths of the rectangle are equal to each other. However, a square with all sides equal is also a rectangle. The first use of prototype examples while learning a concept may create limited visual perceptions and limit the reasoning ability in classifying concepts (Hershkowitz, 1990; Monaghan; 2000). If the concepts are classified with the help of prototype examples, it is possible for the concept images formed in the students' minds to be prototypes. Tall and Vinner (1981) defined the term "Concept Image" as defined by Tall and Vinner (1981). Accordingly, concept image is expressed as all the visuals and cognitive structures related to a concept in the individual's mind. The image of a concept can be a prototype structure of the defined shape. In this case, the formation of concept images can be associated with how the concept is defined and the person's experiences of that concept. According to Vinner (1983), concept images are generally used when the defined concepts are expressed later. Geometric reasoning and reasoning skills are also provided by concept images formed by the interactions between concepts and shapes (Fischbein, 1993; Tall & Vinner, 1981, Vinner, 1983; Vinner & Hershkowitz, 1980).

In the first stage of this study, the concept images of the pre-service teachers of special quadrilaterals were determined. Gülkılık (2008) stated that determining the concept images of teacher candidates will help eliminate the misconceptions and deficiencies of concepts. When the national and international exams are examined, it is seen that students' achievement in geometry, in general, and quadrilaterals in particular, is not at a sufficient level (Dağdelen, 2012). When the literature is examined, it is seen that students and teacher candidates have difficulties in expressing the hierarchical relationship between quadrilaterals (Erşen & Karakuş, 2013; Fujita, 2012; Fujita & Jones, 2006, 2007; Özdemir&Çekirdekci, 2022; Türnüklü, 2014; Ubuz, 2017; Zeybek-Şimşek, 2019). It is thought that it would be beneficial if the subject was well understood by the teachers so that the difficulties experienced by the students about quadrilaterals could be understood and eliminated.

If custom quadrilaterals are defined with their container relationships, they are classified hierarchically. According to Öztoprakçı and Çakıroğlu (2013), a mathematical definition should specify the necessary and sufficient properties of a concept. Usiskin, Griffin, and Willmore (2008) considered quadrilaterals with a different approach at the definition point. While the definitions made by considering the relationship between the quadrilaterals are expressed as inclusive, the definitions used only to specify the properties of a quadrilateral are expressed as excluding definitions. Overarching definitions consist of excellent and necessary properties to express a quadrilateral. According to the Van Hiele approach, defining a shape by specifying sufficient and necessary features is at the level of informal inference. Fujita and Jones (2007) stated that a feature belonging to a concept within the hierarchical classification would be correct for other concepts covered by that concept. It will not need to be re-examined. For example, since a parallelogram is "a quadrilateral with two pairs of opposite sides of equal length," the same expression can be used for a square because the parallelogram includes the square. The hierarchical structure of special quadrilaterals is given in Figure 1.

Figure 1

Hierarchical Structure of Special Quadrilaterals



According to the Van Hiele geometric thinking levels, grasping special quadrilaterals in a hierarchical structure and making their comprehensive definitions take place at the informal inference level, proving these definitions through reasoning is at the formal inference level. For example, An individual at the level of formal inference can show the correctness of the statement that when opposite sides of a quadrilateral are parallel, its opposite angles will also be congruent.

The use of inclusive definitions helps express the special quadrilaterals that have a conceptual relationship between them more practically by using a meta-concept and increasing the quality of cognitive thinking by providing convenience during problem-solving (Çalık-Uzun, 2020; De Villers, 1994). Erşen and Karakuş (2013), on the other hand, stated that pre-service teachers had difficulties expressing the hierarchical structure of quadrilaterals and that it would be more beneficial to make connections within the features instead of memorizing the features of special quadrilaterals. Thanks to concrete objects and models, generalizations can be made by establishing a connection between the elements and features that make geometric shapes (Toptaş, 2008). According to this expression, the teaching of the properties of special quadrilaterals can be achieved through various activities. According to Koç et al. (2010), activity is the process of explaining a concept more easily through concrete materials. One of these activities is origami, known as the art of paper folding.

Origami is a Japanese word meaning "folding paper," which combines the words Ori and Kami. Although there is no definite information about its origin, it is said that it was discovered in China approximately 1200 years ago, then reached Japan thanks to the Buddhist monks, and developed here as well (İnce, Mertol, & Kaya, 2021). Tuğrul and Kavici (2002) stated that origami in mathematics teaching is an activity-based method associated with contemporary learning methods such as creative, active, project and brain-based learning. In addition, Boyacıoğlu et al. (2013) stated that students would get rid of their negative thoughts with the activities implemented in the classroom. Negative thoughts about a subject can affect the attitude towards that subject. Tarım and Artut (2016) stated that the attitude can or can be changed and that an individual's success in a subject can lead to a positive attitude.

Origami is frequently preferred in geometry teaching, especially in recognizing the properties and relationships of geometric shapes. Krier (2007) stated that origami might be necessary for terms of mathematical comprehension, and Pearl (2008) stated that geometric shapes could be grasped better thanks to origami (cited in Dağdelen, 2012). Accordingly, it is thought that encountering many geometric shapes while creating origami will benefit learning the properties of these shapes by making connections. Demir-Küçük (2020), on the other hand, stated that the use of origami and the visualization of theoretical geometry information with constructed origami shapes could be beneficial during the transition from experimental thinking to abstract thinking.

When the studies on origami are examined, the studies generally include classroom activities for middle school and higher age groups (Aksu, 2020; Boakes, 2009; Boz-Yaman, 2015; Boz-Yaman & Bulut, 2019; Boz-Yaman & Duatepe-Paksu, 2019; Dağdelen, 2012; Duatepe-Paksu & Boz-Yaman, 2021; Polat, 2013; Doğan & Bayraktar-Kurt, 2021; Hacısalihoğlu-Karadeniz, 2020; Masal, Ergene, Takunyacı & Masal, 2018). Another purpose of studying teacher candidates is the expectation that these pieces of training to be given to teacher candidates will primarily create positive views and attitudes towards the profession (Senemoğlu & Özçelik, 1989). For this reason, in this study, an applied education process was designed with primary school mathematics teacher candidates, and their development and thoughts in this process were discussed. In the study, while expressing the inclusive definitions of special quadrilaterals, origami activities were prepared in which shapes suitable for these definitions were created. In the second stage of the research, origami activities were applied to prospective primary school mathematics teachers for five weeks to help them understand the relationship between special quadrilaterals at a conceptual level.

The Problem of Research

This study is aimed to determine the effect of origami activities prepared for inclusive definitions of quadrilaterals on the concept images of teacher candidates. For this purpose, the research problem and sub-problem questions are given below:

What is the effect of origami activities on the geometry attitude of elementary mathematics teacher candidates and their concept images regarding the definition of quadrilaterals?

Sub Problems

1. How did the geometry attitude scores of primary school mathematics teacher candidates change with origami activities?
2. What is the effect of origami activities on the change in the concept images of elementary mathematics teacher candidates regarding the definition of quadrilaterals?

Method

In this study, one group pre-test-post-test design, one of the weak experimental designs, was used to examine the change in the concept images of the pre-service teachers about quadrilaterals by using origami in geometry teaching. The effect of the activities performed in this design is tested with a study on a single group.

Study Group

The research study group was chosen as the appropriate case study group from the purposeful study groups. Thus, it is aimed to determine the people and groups that are suitable for the research and to be easily researched (Creswell, 2012; Sönmez & Alacapınar, 2016). The focus of the research is the use of quadrilaterals and origami. For this reason, the research study group was composed of pre-service mathematics teachers, who will be the most important human factor in delivering mathematics to society. The research study group consists of pre-service teachers in the primary education mathematics teaching program of two state universities in Turkey in the 2020-2021 academic year. There are ten primary school mathematics teacher candidates in the study group, including two female teacher candidates who have completed the 1st grade and eight female teacher candidates who have completed the 2nd grade. The entire study group participated in the study voluntarily.

Data Collection Tools

Concept-Image Questions

In order to determine the concept images of teacher candidates regarding special quadrilaterals, six questions were prepared by the researchers based on the literature, and their final form was given in line with the opinions of three faculty members who are experts in the field of mathematics education (Appendix-1). The questions were applied as pre-test and post-test. It was aimed to determine the images related to the concept with the answers given by the participants to the questions. Thus, it aims to determine the effect of origami activities on concept images.

Geometry Attitude Scale

The geometry attitude scale, which was used to determine the effect of origami activities on pre-service teachers' geometry attitudes, was developed by Aktaş and Aktaş (2013). The scale consists of 4 factors and 23 items. The variances explained by the factors were 14.69%, 12.48%, 12.31%, and 7.54%, respectively; The total explained variance is 47.03%. The high Cronbach Alpha coefficients for the sub-dimensions of the scale (seeing oneself as self-sufficient and enjoying geometry = .810; associating geometric concepts sub-dimension = .717; geometry real-life relationship = .780; place of geometry course in the curriculum = .614). shows that the items in the dimensions are consistent with each other.

Data Collection Process

Before the application, the pre-service teachers who participated in the study answered the concept image questions and the geometry attitude scale. The data collected here constitute the pre-tests of the study. The implementation process lasted five weeks. The lessons were held for at least one hour each week on a free online live lesson platform accessible to all participants (Appendix 2 for a sample activity). The order of events is as follows:

Week 1: Activity 1) The Relationship Between Deltoid, Rhombus, and Square

Week 2: Activity 2) Relationship between Trapezoid, rectangle, and square

Week 3: Activity 3) The Relationship Between Parallelogram, Rectangle, and Square

Week 4: Activity 4) The Relationship Between Parallelogram, Rhombus, and Square

Week 5: At the end of the activities, which lasted for four weeks, the participants were tasked with making five origami shapes as homework. The QR codes of the construction videos of the shapes were shared with the participants. You can reach video 1 by scanning the QR code attached.

Video 1

Sonobe Cube

After the application, the participants were asked to answer concept image questions and geometry attitude scales. The data collected here constitute the final test of the study.

Data Analysis

The proper filling of the data collection tools was checked by the researcher one by one. The researchers transferred the information about the scale data and concept images obtained from the pre-service teachers to the computer environment. All the data were checked one by one to prevent any erroneous coding that may occur in this process. There is no missing data in the generated data file. In order to analyze the prospective teachers' data obtained from the geometric attitude scale, which is the first sub-problem of the study, with the appropriate statistical technique, the condition of meeting the normality assumptions was examined (Mertler & Vannatta, 2005; Tabachnick & Fidell, 2014; Thode, 2002). Although the assumption of continuity of data is met, the assumption of the number of data (N=10) is not met. As a next step, the values obtained by dividing the skewness and kurtosis coefficients of the data by the standard errors were examined (Howitt & Cramer, 2011; Tabachnick & Fidell, 2014). As a result of this analysis, it was seen that the skewness value was outside the ± 2 limits. In addition, since the total number of participants was less than 50, the Shapiro-Wilk test was applied, and it was seen that it did not meet the normal distribution condition ($p<.05$) according to the results. Similarly, Q-Q plots support this finding. As a result, it was decided that the data were not normally distributed, and the Wilcoxon Signed Ranks test, one of the non-parametric tests, was used to analyze the data obtained from the geometry attitude scale.

The second problem case of the study was aimed to determine the change in the concept images of the pre-service teachers. Concept images of pre-service teachers include expressions related to the concept or not (Vinner & Hershkowitz, 1980). Among these statements, the first and last ones that come to mind of the pre-service teachers are important in helping us to determine the strong and weak ones of the classes that the concept images contain. Therefore, pre-service teachers were asked to write five answers, starting with their first thought of them about the concepts of square, rhombus, trapezoid, rectangle, and parallelogram. Two researchers analyzed the data and gathered it under six themes (Table 2).

Findings

Pre-service Teachers' Attitudes to Geometry

The data collected from 10 pre-service teachers before and after the study were analyzed using the Wilcoxon signed-rank test, one of the non-parametric tests. The data obtained are as in Table 1.

Table 1

Wilcoxon Signed Ranks Test Results of Geometry Attitude Scores Before and After the Experiment

Geometry Attitude Scores	N	Rank Average	Rank Total	z	p
Negative Rank	3	7.50	22.50	-0.511	0.609
Positive Sequence	7	4.64	32.50		
Equal	0				

When Table 1 is examined, it is seen that the mean rank of negative (7.50) is higher than the mean positive rank (4.64). This finding shows that the geometry attitude scores before the experiment are higher than those obtained after the experiment. Analysis results show that this difference before and after the activity is not statistically significant ($z=-0.511$, $p > 0.05$).

Concept Images of Teacher Candidates

The answers given by the teacher candidates to the applied concept image questions in the pre-test and post-tests were examined in 6 themes. It includes answers expressing definitions that exclude themes 1 and 3. The sixth theme includes answers expressing inclusive definitions.

The themes examined and their corresponding explanations are given in Table 2.

Table 2

Themes Related to Quadrilaterals

Codes	Themes	Explanation
1*	Elements and properties of a quadrilateral	This group includes the elements that make up the quadrilateral and their properties.
2	Daily life	Some expressions are associated with the shape or other features of the rectangle in the context of daily life.
3*	Definition – theorem	There are definitions and theorems about quadrilaterals.
4	Other shapes	There are other geometric shapes outside the relevant quadrilateral.
5	Subject areas	There are subject areas that the participants identify with the quadrilateral.
6	Hierarchical relationship/overarching definition	Some features express the hierarchical relationship of the quadrilaterals.

**1 and 3 include answers expressing the exclusionary definitions.*

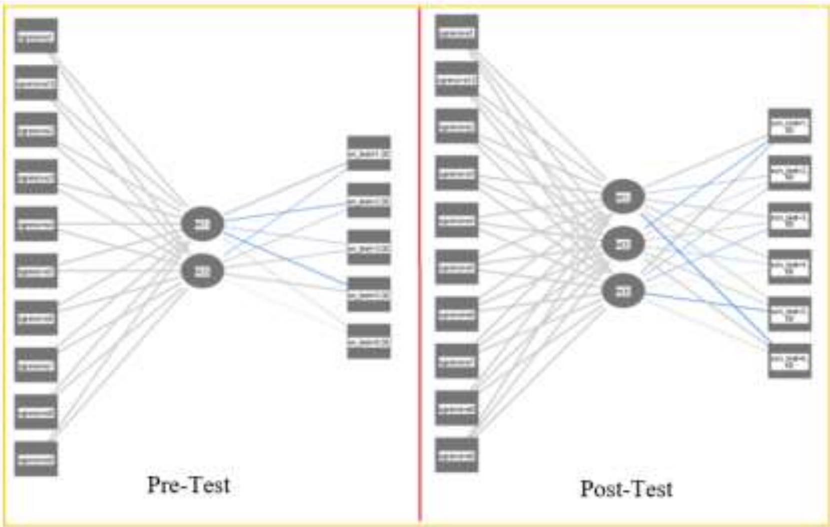
Accordingly, the information on each data was analyzed separately.

i. Square

Two different figures were obtained regarding the change of concept images before and after the training about the square. Accordingly, while the paths formed by the students in the pre-test and post-test were determined in Figure 2, the connections between the data obtained from the pre-test and post-test were examined separately in Figure 3.

Figure 2

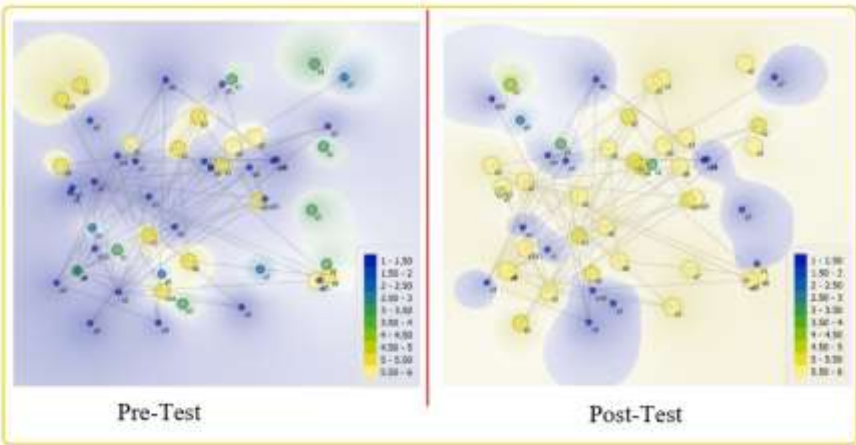
Pre-Test and Post-Test Neural Links of Concept Images of Square



When Figure 2 is examined, it is seen that there are clear connections between two hidden layers and incredibly exclusionary definitions in the pre-test. There are clear connections regarding three hidden layers and inclusive definitions in the post-test. These connection points are detailed in Figure 2.

Figure 3

Pre-Test and Post-Test Neural Point Links of Concept Images of Square



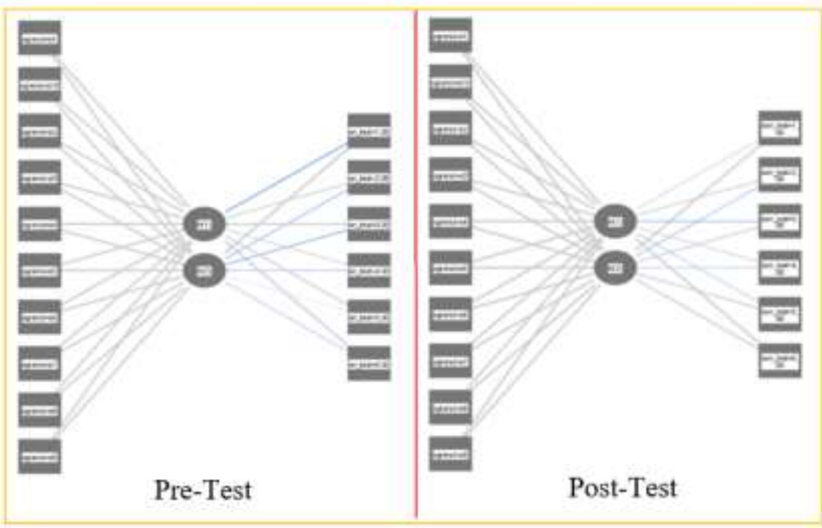
When Figure 3 is examined, it can be stated that while the pre-test tends to more exclusionary definitions, concept images of inclusive definitions are concentrated in the post-test.

ii. Equilateral quadrangle

Two different figures were obtained regarding the change of concept images before and after the education about the rhombus. Accordingly, while the paths formed by the students in the pre-test and post-test were determined in Figure 4, the connections between the data obtained from the pre-test and post-test were examined separately in Figure 5.

Figure 4

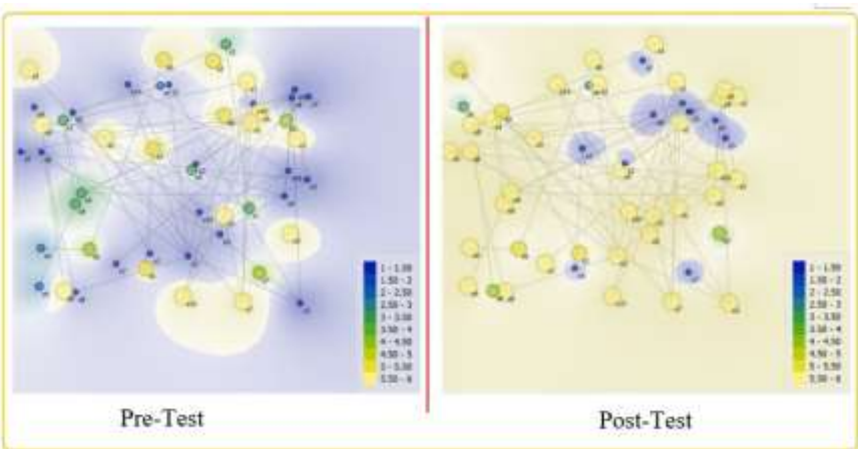
Pre-test and Posttest Neural Links of Rhombus Concept Images



When Figure 4 is examined, two hidden layers in the pre-test and undeniable connections of the exclusion definitions are seen. Similarly, there is a trend towards two hidden layers and inclusive connections in the post-test. However, these connection points are detailed in Figure 4 to clarify this orientation.

Figure 5

Pre-Test and Post-Test Neural Point Links of Concept Images of Rhombus



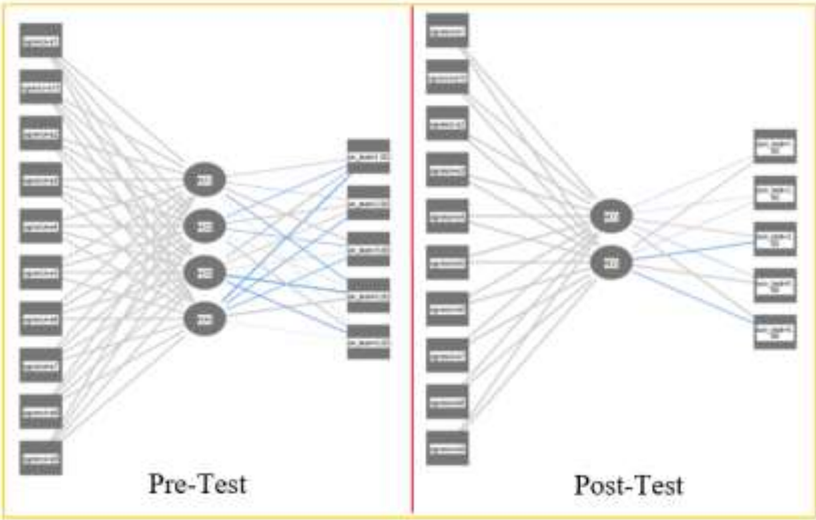
When Figure 5 is examined, it can be stated that while the pre-test tends to more exclusionary definitions, concept images of inclusive definitions are concentrated in the post-test.

iii. Deltoid

Two different figures were obtained regarding the change of concept images before and after the training about the deltoid. Accordingly, in Figure 6, the paths formed by the students in the pre-test and post-test were determined, while in Figure 7, the connections between the data obtained from the pre-test and post-test were examined separately.

Figure 6

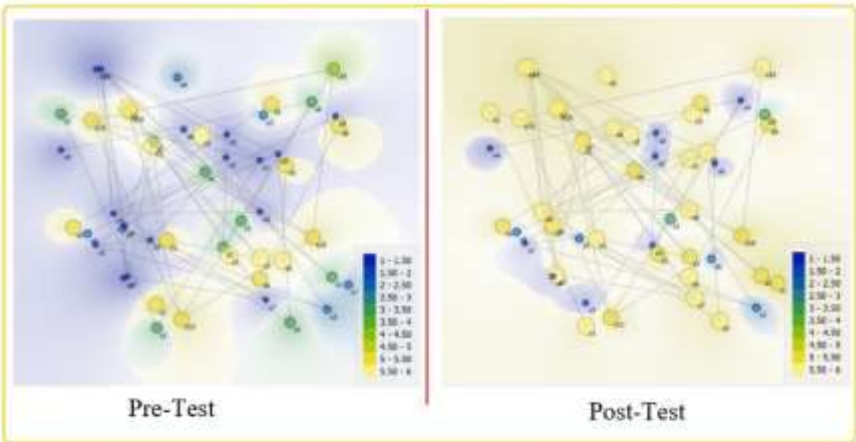
Pre-test and Posttest Neural Links of Deltoid Concept Images



When Figure 6 is examined, the four hidden layers in the pre-test and especially the apparent connections of the exclusion definitions are seen. In the post-test, it is seen that the number of layers is two, and the connections related to the inclusive definition are intensified. These connection points are detailed in Figure 6 to clarify the current situation.

Figure 7

Pre-Test and Post-Test Neural Punctuation of Concept Images of Deltoid



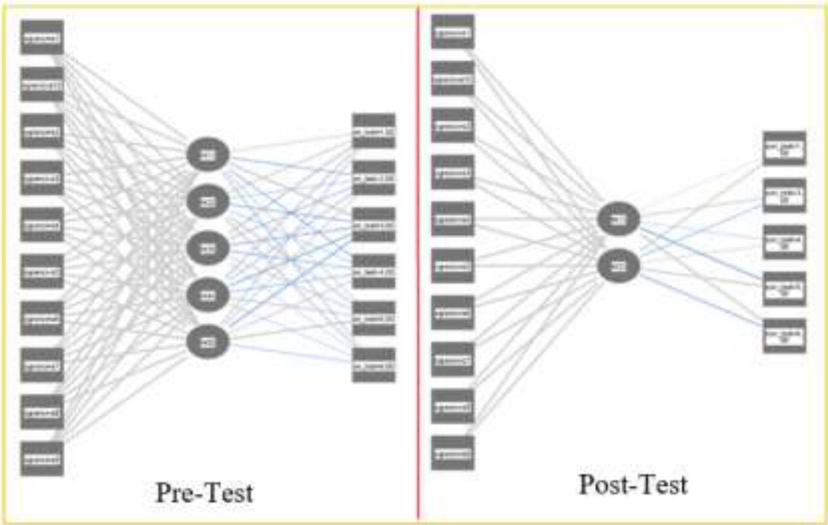
When Figure 7 is examined, it can be stated that while the pre-test tends to more exclusionary definitions, concept images of inclusive definitions are concentrated in the post-test.

iv. Trapezoid

Two different figures were obtained regarding the change of concept images before and after the trapezoid training. Accordingly, while the paths formed by the students in the pre-test and post-test were determined in Figure 8, the connections between the data obtained from the pre-test and post-test were examined separately in Figure 9.

Figure 8

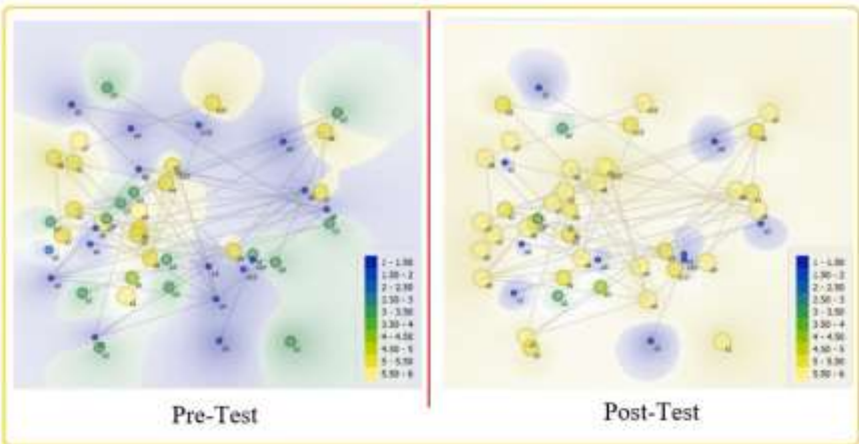
Pre-Test and Post-Test Neural Links of Concept Images of Trapezoid



When Figure 8 is examined, it is seen that there are five hidden layers in the pre-test and the obvious connections of the exclusionary definitions. In the final test, while the number of layers is two, it is seen that the connections that hold the container are concentrated. These connection points are detailed in Figure 8.

Figure 9

Pre-test and Posttest Neural Point Links of Concept Images of Trapezoid



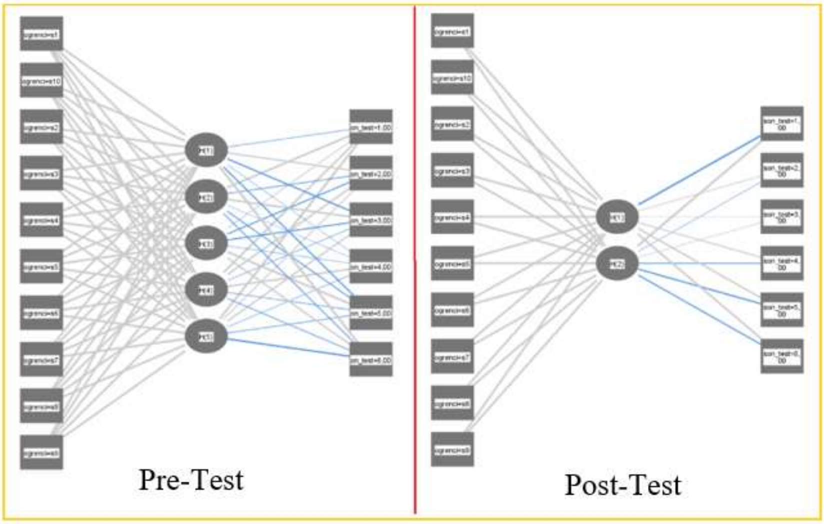
When Figure 9 is examined, it can be stated that while the pre-test tends to more exclusionary definitions, concept images of inclusive definitions are concentrated in the post-test.

v. Rectangle

Two different figures were obtained regarding the change of the concept images before and after the education about the rectangle. Accordingly, while the paths formed by the students in the pre-test and post-test were determined in Figure 10, the connections between the data obtained from the pre-test and post-test were examined separately in Figure 11.

Figure 10

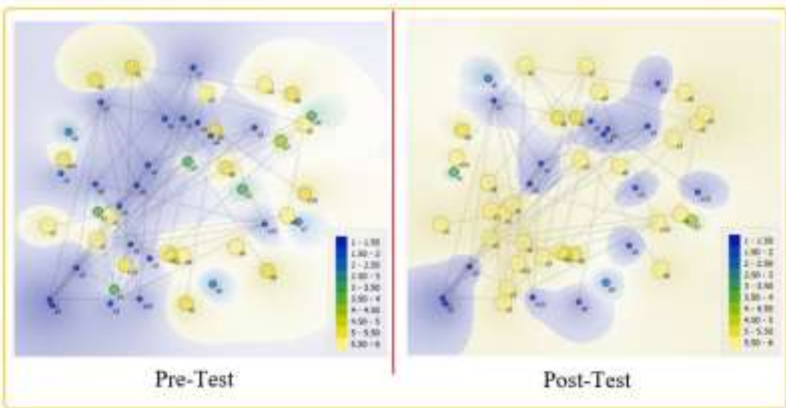
Pre-test and Posttest Neural Links of Rectangle Concept Images



When Figure 10 is examined, it is seen that there are five hidden layers in the pre-test and the obvious connections of the exclusionary definitions. In the final test, while the number of layers is two, it is seen that the connections that hold the container are concentrated. These connection points are detailed in Figure 10.

Figure 11

Pre-Test and Post-Test Neural Point Links of Rectangle Concept Images



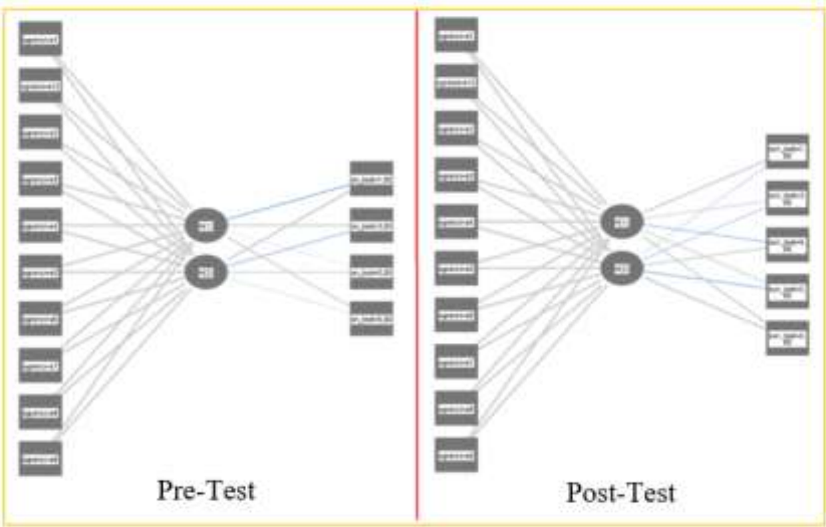
When Figure 11 is examined, it can be stated that while the pre-test tends to more exclusionary definitions, concept images of inclusive definitions are concentrated in the post-test.

vi. Parallelogram

Two different figures were obtained regarding the change of concept images before and after the training related to the parallelogram. Accordingly, while the paths formed by the students in the pre-test and post-test were determined in Figure 12, the connections between the data obtained from the pre-test and post-test were examined separately in Figure 13.

Figure 12

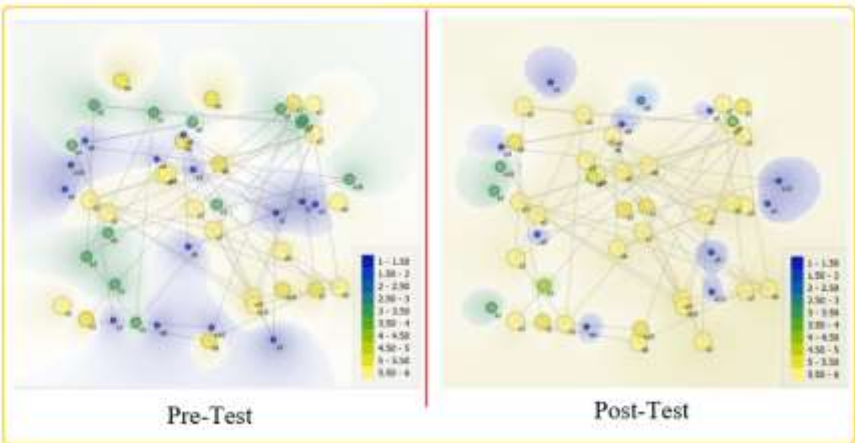
Pre-Test and Post-Test Neural Links of Concept Images of Parallelograms



When Figure 12 is examined, it is seen that there are two hidden layers in the pre-test and the obvious connections of the exclusionary definitions. In the post-test, it is seen that while the number of layers does not change, the connections that hold the container are concentrated. These connection points are detailed in Figure 12.

Figure 13

Pre-Test and Post-Test Neural Point Links of Concept Images of Parallelograms



When Figure 13 is examined, it can be stated that while the pre-test tends to more exclusionary definitions, concept images of inclusive definitions are concentrated in the post-test.

Discussion, Conclusion, and Recommendations

According to the pre-test results of the concept image questions before the application, it was observed that the pre-service teachers had difficulties in expressing the inclusive definitions of special quadrilaterals. The findings are consistent with the studies in the literature (Bütüner & Filiz, 2016; Erşen & Karakuş, 2013; Fujita & Jones, 2007; Fujita, 2008; Horzum, 2018; Kazak & Duatepe-Paksu, 2019; Türnüklü et al. 2013; Zeybek-Şimşek, 2019). When the findings obtained from the post-tests after the activities are examined, it is seen that the pre-service teachers' concept images of special quadrilaterals have changed at the conceptual level. Bingölbali and Monaghan (2008) stated that concept images have a dynamic structure and can change with individual experiences. In this context, origami activities experienced by pre-service teachers were effective in changing their concept images. In this direction, it can be said that using origami effectively teaches the hierarchical relationship between special quadrilaterals. One of the reasons for this is that the quadrilaterals built with the applied paper folding activities according to the traditional teaching approach have become more understandable for the participants, and they have the opportunity to examine the shapes. While making origami, students have the opportunity to create and examine geometric shapes. It is essential to recognize the shapes and their features when folding the paper in the stage of creating a product. Şimşek (2012) concluded in his study that origami activities effectively increase student success compared to the lessons taught with traditional methods. Kazak and Duatepe-Paksu (2019) stated that due to their study in which quadrilaterals were examined from the point of view of symmetry through paper folding, pre-service teachers reached the level of informal inference. In this study, it was seen that the pre-service teachers showed improvement in expressing the quadrilaterals hierarchically. These statements are consistent with the result that origami activities are an effective method for understanding the hierarchical relationship between quadrilaterals.

One source that affects our thoughts or behaviors towards a particular object or event is attitude (Tarım & Artut, 2016). A geometry attitude can be expressed as a person's positive or negative feelings and behaviors about geometry. When the results of the geometry attitude scale pre-test and the post-test were examined at the end of the application, it was seen that the geometry attitudes of the pre-service teachers did not change. Attitudes learned at an early age do not change quickly, but they may undergo some changes thanks to new experiences and new learnings (Kağıtçıbaşı & Cemalcılar, 2014). In this respect, it is thought that the limited duration of the origami activities applied in the study to 5 weeks is not enough to change the attitudes.

Although the study was conducted with a limited number of primary school mathematics teaching students, as a result of the application, it was concluded that origami activities were effective in expressing special quadrilaterals with inclusive definitions. Considering those classroom teachers will apply geometry for the first time in primary education, it may be beneficial to apply the study to prospective teachers studying in the classroom teaching program. Considering that undergraduate students have difficulty expressing the hierarchical relationship of quadrilaterals, it is recommended that this practice be done early. The activities can be applied at the secondary school level by making them suitable. In addition, the 5-week process can be extended and implemented by supporting it with different activities or dynamic geometry software.

References

Aksu, H. H. (2020). The effect of teaching the triangular issue thorugh origami-based learning approach on the success of 9th grade students. *Turkish Studies-Educational Sciences*, *15*(4), 2343-2354.

Baki, A. (2018). *Matematiği öğretme bilgisi [Knowledge of teaching mathematics]* (1st edition). Ankara: Pegem Akademi.

Bayraktar-Kurt, E. (2012). *In elementary geometry education the passing of geometric shapes and objects by the help of origami.* [Unpublished master’s thesis]. On Dokuz Mayıs University.

Bingölbali, E. & Monaghan, J. (2008). Concept image revisited. *Educational Studies in Mathematics*, *68*(1), 19-35.

Boakes, N. J. (2009). Origami instruction in the middle school mathematics classroom: Its impact on spatial visualization and geometry knowledge of students. *RMLE Online*, *32*(7), 1-12.

Boyacıoğlu, H., Köroğlu, H., & Alkan, H. (2003). İlköğretimin ilk beş sınıfında matematik etkinlikleri *[Mathematics activities in the first five grades of primary education]*. Matematikçiler Derneği Bilim Köşesi.

Boz-Yaman, B. (2015). A journey from two-dimensional papers to three-dimensional origamic cube. *Journal of Inquiry Based Activities*, *5*(1), 20-33.

Boz-Yaman, B., & Duatepe-Paksu, A. (2019). Triggering geometrical habits of mind by origami activity: Folding a dodecahedron. *Australian Mathematics Education Journal*, *1*(4), 4-10.

Duatepe-Paksu, A., & Boz-Yaman, B. (2021). Short activity: Examining a platonic octahedron through origami. *Australian Mathematics Education Journal*, *3*(2), 4-7.

Bütüner, S. Ö. & Filiz, M. (2016). Examining prospective mathematics teachers' understanding of the classification of quadrilaterals. *Journal of Subject Teaching Research*, *2* (2), 43-56.

Cansız-Aktaş, M. & Aktaş, D. Y. (2013). The development of a current attitude scale towards geometry. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *7* (2), 225-247.

Creswell, J. W. (2012). Educational research: planning. *Conducting, and Evaluating*, *260*, 375-382.

Çalık-Uzun, S. (2020). Dörtgen kavramı ve öğretimi [Quadrilateral concept and teaching]. Ertekin, E., Ünlü, M. (Ed.) *Geometri ve ölçme öğretimi: tanımlar, kavramlar ve etkinlikler [Geometry and measurement teaching: definitions, concepts and activities].* Ankara: Pegem Akademi.

Dağdelen, M. G. (2012). *Effect of origami in comprehending special quadrilaterals in primary 5th grade geometry education*. [Unpublished master's thesis.] On Dokuz Mayıs University.

De Villers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics*, *14*(1), 11-18.

Doğan, M., & Bayraktar-Kurt, E. (2021). Origami in mathematics education: the research trends of the postgraduate theses. *Journal of International Social Research*, *14*(77).

Duatepe-Paksu, A. (2016). Van Hiele geometrik düşünme düzeyleri [Van Hiele geometric thinking levels] . Bingölbali, E., Arslan, S. ve Zemzat, İÖ. (Ed.), *Matematik eğitiminde teoriler [Theories in mathematics education]* (ss. 265-275). Ankara: Pegem Akademi.

Erşen, Z.&Karakuş, F. (2013). Evaluation of pre-service elementary teachers' concept ımages for quadrilaterals. *Turkish Journal of Computer and Mathematics Education, 4*(2), 124-146.

Fischbein, E. (1993). The theory of figural concept. *Educational Studies in Mathematics*, *24*(2), 139-162.

Fujita, T. (2008). Learners' understanding of the hierarchical classification of quadrilaterals. Joubert. *Proceedings of the British Society for Research into Learning Mathematics, 28*(2).

Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon. *The Journal of Mathematical Behavior*, 31, 60–72.

Fujita, T., & Jones, K. (2006). Primary trainee teachers' understanding of basic geometrical figures in Scotland. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), Proceedings of 30th Conference of the International Group for the Psychology of Mathematics Education (Vol.3, pp. 129–136). Prague: PME.

Fujita, T.&Jones, K., (2007). Learners' understanding of the definitions and hierarchical classification of quadriılaterals: towards a theoretical framing. *Research in Mathematics Education*, *9*(1), 3-20.

Gülkılık, H. (2008). *A phenomenographic study on the concept images and image development of prospective secondary mathematics teachers related with some geometric concepts*. [Unpublished master's thesis] Gazi University, Ankara.

Hacısalihoğlu-Karadeniz, M. (2020). Mathematics teaching via paper folding method, *Elementary Education Online*, *16*(2): 663-692.

Hershkowitz, R. (1990). Psychological aspects of learning geometry, in P. Nesher and J. Kilpatrick (Ed.) Mathematics and cognition. Cambridge: Cambridge University Press.

Horzum, T. (2018). The investigation of pre-service mathematics teachers' knowledge about quadrilaterals through concept maps. *Turkish Journal of Computer and Mathematics Education*, *9* (1), 1-30.

Howitt, D. & Cramer, D. (2011). *Introduction to SPSS statistics in psychology: For version 19 and earlier (5th Ed.).* Pearson Education Limited.

İnce, B., Mertol, H. & Kaya, N. (2021). Historical development and benefits of origami. *XI. Umteb International Congress On Vocational & Technical Sciences*, Ankara.

Kağıtçıbaşı, Ç. & Cemalcılar, Z. (2014). Dünden bugüne insan ve insanlar [People and people from yesterday to today]. İstanbul: Evrim Publishing.

Kazak, S., Duatepe-Paksu, A. (2019). The Impact of Exploring Quadrilaterals from Symmetry Perspective on Geometric Thinking: Cases of Definitions of Rectangle and Rhombus. *Gazi University Journal of Gazi Education Faculty, 39*(3), 1421 - 1451.

Koç, H., Aksoy, B., Sönmez, Ö.F., & Yeşiltaş, E. (2010). The activities making students active during teaching process and geography teaching depending on activities. *The Journal of Social Sciences Research*. Volume 5, Issue 2, 181 - 196

Küçük-Demir, B. (2020). Geometri ve geometrik düşünce [Geometry and geometric thinking]. T. Ağırman, B. Küçük-Demir (Ed.), *Geometri ve öğretimi [Geometry and its teaching]*(pp. 1-10). Ankara: Pegem Akademi.

Masal, M., Ergene, Ö., Takunyacı, M., & Masal, E. (2018). Prospective teachers' views about using origami in mathematics lessons. *International Journal of Educational Studies in Mathematics*, *5*(2), 56-65.

Mertler, C. A. & Vannatta, R. A. (2005). *Advanced and multivariate statistical methods: Practical application and interpretation (3rd Ed.).* Pyrczak Publishing.

Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics, 42*(2),179-196.

NCTM. (2000). Curriculum and Evaluation Standards for School Mathematics, Retrieved from http://standards.nctm.org/document/chapter7/geom.htm

Özdemir, A. & Çekirdekci, S. (2022). Geometric Habits of Mind: The Meaning of Quadrilaterals for Elementary School Student Teachers . *International Journal of Educational Studies in Mathematics, 9* (1), 49-66.

Öztoprakçı, S. & Çakıroğlu, E. (2013). Dörtgenler [Quadrangles]. Zembat, İ. Ö., Özmantar, M. F., Bingölbali, E., Şandır, H. ve Delice, A. (Ed.), *Tanımları ve tarihsel gelişimleriyle matematiksel kavramlar [Mathematical concepts with their definitions and historical development]* (2nd edition, pp. 249-272). Ankara: Pegem Akademi.

Polat, S. (2013). Teaching mathematic with origami. *Mustafa Kemal University Journal of Social Sciences Institute, 10* (21), 15-27.

Senemoğlu, N., & Özçelik, D. A. (1989). Öğretmen adaylarına "öğretmenlik bilgisi" kazandırma bakımından fen-edebiyat ve eğitim fakültelerinin etkililiği [The effectiveness of science-literature and education faculties in terms of gaining "teaching knowledge" to teacher candidates]. *Çağdaş Eğitim Dergisi*, *142*, 18-21.

Sönmez, V., & Alacapınar, F. G. (2016). *Örneklendirilmiş araştırma yöntemleri [Illustrated Research Methods]*, Ankara: Anı Publishing.

Tabachnick, B. G. & Fidel, L. S. (2014). *Using multivariate statistics (6th Ed.).* Pearson Education Limited.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151–169.

Tarım, K.&Dinç-Artut, P. (2016). Attitude and math achievement. Bingölbali, E., Arslan, S. ve Zemzat, İÖ. (Ed.), *Theories in mathematics education* (pp. 767-784). Ankara: Pegem Academy.

Thode, H. C. (2002). *Testing for normality*. Marcel Dekker, Inc.

Toptas, V. (2008). Analysis of the teaching – learning process by the activities applied during geometry teaching. *Elementary Education Online*, Volume 7, Issue 1, 91.

Tuğrul, B. & Kavici, M. (2002). The art of paper folding origami and learning. *Pamukkale University Journal of Education*, *11*(11), 1 – 17.

Türnüklü, E., Alaylı, F. G., & Akkaş, E. N. (2013). Investigation of prospective primary mathematics teachers' perceptions and ımages for quadrilaterals. *Educational Sciences: Theory & Practice. 3*(2), 1213-1232.

Türnüklü, E. (2014). Construction of inclusion relations of quadrilaterals: Analysis of pre-service elementary mathematics teachers' lesson plans. *Education and Science, 39*(173), 198–208.

Ubuz, B. (2017). Relations among quadrilaterals: 7th grade students' concept ımages. *Education for Life.* 31(1), 55-68.

Usiskin, Z., Griffin, J., Witonsky, D. & Willmore, E. (2008). *The classification of quadrilaterals: a study in definition.* Charlotte, NC: Information Age Publishing.

Van De Walle, J. A., Karp, K. S.&Bay-Williams, J. M. (2019). *İlkokul ve ortaokul matematiği gelişimsel yaklaşımla öğretim [Elementary and middle school mathematics: teaching developmentally].* (S. Durmuş, Translation edit.) Ankara: Nobel Academy.

Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, *14*, 293– 305.

Vinner, S., & Hershkowitz, R. (1980). Concept images and common cognitive paths in developing some simple geometrical concepts. In R. Karplus (Ed.), *Proceedings of the Fourth International Conference for the Psychology of Mathematics Education:* Vol.1 (pp. 177–184). Berkeley: University of California, Lawrence Hall of Science.

Yaman, B. B., & Bulut, S. Stake holders experıences of usıng 5e learnıng cycle approach ın teachıng of Euler's formula. *Bolu Abant Izzet Baysal University Journal of Faculty of Education*, *19*(3), 836-852.

Yavuz, E. (2020). *An investigation into concept image of primary school mathematics pre-service teacher is on the hyperbola.* [Unpublished master's thesis] Dokuz Eylül University.

Zeybek-Şimşek, Z. (2019). Investigating pre-service teachers' ability to recognize and classify geometric concepts hierarchically. *Turkish Journal of Computer and Mathematics Education, 10* (3), 680-710.

Appendices

Appendix-1: Concept Image Questions

Question 1: What are the five things that come to mind when you think of the square concept?
Question 2: What are the five things that come to mind when you think of the concept of a rhombus?
Question 3: What are the five things that come to mind when you think of the concept of deltoid?
Question 4: What are the five things that come to mind when you think of the trapezoidal concept?
Question 5: What are the five things that come to mind when you think of the rectangle concept?
Question 6: What are the five things that come to mind when you think of the parallelogram concept?

***Appendix-2: A Sample** Activity:* Relationship Between Deltoid and Rhombus

Participants: Teacher candidates
Event duration: 1 hour
Materials needed: Origami paper, scissors, colored pencils.
Goals:
- Determine the overarching properties of deltoid and rhombus
- Understanding the hierarchical structure of deltoid, rhombus, and square;
- The diagonals of a rhombus are deltoids that bisect each other.
- A rhombus is a deltoid with two pairs of opposite angles congruent.
- A square is a rhombus with an angle of 90 degrees or diagonals of equal length.
- A square is a deltoid with diagonals of equal length or three angles of 90 degrees.

Application Stage:
Deltoid

"How can we create a deltoid?"

1) The two adjacent sides of the square paper are folded on the diagonal to form a deltoid.

"What is the reason for choosing two adjacent sides to fold?"

2) It is folded from its first diagonal (line of symmetry).

“How can we interpret the edge properties of the deltoid?”

A deltoid is a quadrilateral with two pairs of adjacent sides.

3) Angle properties of the deltoid folded from the symmetry line are also determined.

4) Its second diagonal is also folded.

5) It is shown that the second diagonal is not symmetrical.

A deltoid is a symmetrical quadrilateral for at least one of its diagonals.

6) It is shown that the second diagonal is the line segment where the bases of two isosceles triangles coincide.

A deltoid is a quadrilateral formed by coining the bases of two isosceles triangles with equal bases.

7) Express the states of the diagonals relative to each other.

"We said that the first diagonal divides the deltoid into two parts symmetrically. So we can say that the first diagonal averages the second diagonal."

"The fact that the second diagonal does not center the first diagonal is related to the fact that the diagonal is also not an axis of symmetry."

"And at what angle does the first diagonal center the second diagonal? We know that the second diagonal is a line segment that forms an angle of 180 degrees to the plane.

Then the other line segment (first diagonal), which divides this line into two symmetrically, will also split the angle. '"

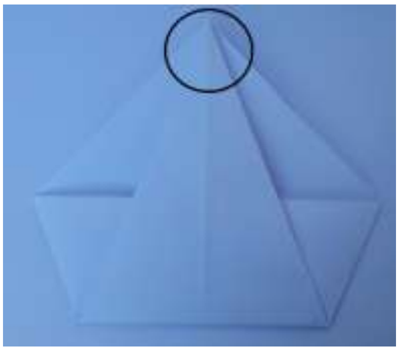
A deltoid is a quadrilateral that centers one diagonal perpendicular to the other.

Equılateral Quadrangle

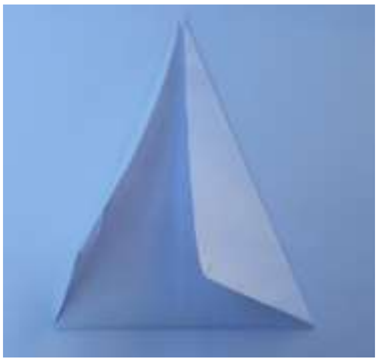
"Can we construct a deltoid with two congruent angles?"

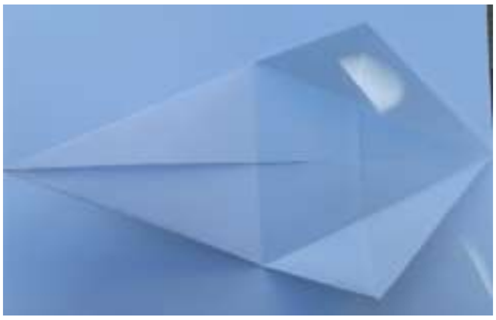
"We have shown the congruent angles of the deltoid; let us make the other angles equal to each other."

1) Angles are superimposed as in the figure.



2) To equalize the measure of one angle to the other, fold traces are determined as in the figure, and the shape is opened.





3) Fold from the determined fold marks.

"We have folded the other adjacent edges, which we have shown the deltoid is congruent."

4) The figure is folded from one diagonal.

"How can we interpret the shape by folding this diagonal of the shape?"

A quadrilateral whose adjacent side lengths are equal.

5) It is folded from the other diagonal, and the side lengths are expressed.

"We have obtained a shape whose side lengths are equal to each other, and at the same time, its diagonals form the axis of symmetry, so we can say that one diagonal is centered on the other. Furthermore, at what angle does it center?"

"We can apply the method of finding the angle at which the diagonals intersect, which we use in the deltoid to examine the relative positions of the diagonals. We can also do this over 360 degrees for a different method."

6) The quadrilateral is folded into four over its diagonals.

“As the four angles overlap, it is 360/4 = 90. The diagonals center each other perpendicularly.

"Then, since the shape meets all the properties of the deltoid, we have created a deltoid whose opposite angles are equal to each other."

"What can we say about opposite sides of a quadrilateral with all sides equal and opposite angles equal?"

7) By naming the angles of the figure, the state of the opposite sides is expressed.

(180 – 2b) + 2b = 180
A rhombus is a quadrilateral with a pair of adjacent sides of equal length and opposite sides parallel, that is, all sides of equal length.

8) It is shown that a quadrilateral whose side lengths are not equal to each other cannot be parallel to opposite sides by slowly opening one side of the rhombus.

"We first turned the square paper shape into a deltoid and a rhombus. Let us review how we did it."

"Can we say that two pairs of adjacent sides of a rhombus are also equal?"

"Is a rhombus centering one diagonal perpendicular to the other and symmetrical for at least one of its diagonals?"

According to them, a rhombus is a deltoid whose diagonals center each other perpendicularly, or a rhombus is a deltoid with two double angles congruent.

Ethics Committee Approval

Ethical permission was obtained from the Süleyman Demirel University Social and Human Sciences Ethics Committee for this research (Date of Decision: 28.06.2021; Number of Decisions: E- 87432956-050.99-73702). All the rules in the "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed in this research. None of the actions in "Ethics Actions Against Scientific Research and Publication " were carried out.

Statement of Contribution of Researchers to the Article:

1st author contribution rate: %40
2nd author contribution rate: %60

Conflict of Interest Statement:

There is no conflict of interest

Statement of Financial Support or Acknowledgment:

This study was supported as the project numbered SLP-2021-8362 of Süleyman Demirel University BAP Coordination unit.