The flipped learning approach of instruction sees classroom lectures moved outside of classrooms through devices and technology. Homework is moved inside classrooms as learning activities. Due to the COVID-19 outbreak, cloud-based education platforms - the organization of the educational environment in the cloud as a tool to enable teaching and learning - have been widely deployed to support online instruction. In this study, flipped cloud learning, incorporating approach of advancing mathematical thinking, were conducted in Mathematical Analysis course to study students' mathematical critical thinking. A quantitative method, using a pre-test-post-test design, was employed with a group of 56 undergraduate mathematics students. The research instrument was a mathematical critical thinking ability test. The quantitative data were analyzed using the descriptive analysis, n-gain and t-test. The findings revealed that students had statistically significant mathematical critical thinking of higher than 60% of the full score at the .05 level. There was an increase in students' mathematical critical thinking abilities with a mean n-gain of .64, which was moderate. The findings also show that the students' mathematical critical thinking was significantly enhanced by this approach at .05 level. Thus, it could be summarized that integrating strategy for advancing mathematical thinking into flipped cloud learning positively affected students' mathematical critical thinking. The results also suggest that this reformed learning approach might be usefully employed as an instruction model in the new normal context.

Keywords: Cloud technology, flipped cloud learning, mathematical critical thinking, undergraduate students.

INTRODUCTION

Mathematics is a subject that involves reasoning, problem solving and thinking processes which is important for human development (Yang, 2017). Mathematics is usually used as a tool to express people's thoughts via the use of critical and systematic thinking (National Council of Teachers of Mathematics, 2011; Office of the Education Council, 2017). The nature of mathematics teaching is abstract with a focus on content. Traditional learning approaches often fail to create an appropriate learning environment and can cause students to misunderstand the content (Noor et al., 2020). According to Angraini and Wahyuni (2021), mathematical learning processes work well where teachers provide students with opportunities to practice their thinking processes while learning new content. Similarity, Ramananda and Srinivasan (2019) suggest that instructional approaches in teaching mathematics should encourage students to learn by thinking and...
problem solving on their own. In the twenty-first century, mathematics in higher education should not only focus on basic mathematical knowledge and skills, but also on developing mathematics graduates’ ability to express facts, solve problems and contribute to reasoning and advanced thinking (Alabdulaziz, 2022; Angraini et al., 2019). This accords with Kong (2015), who states that improving mathematics teaching methods will allow students to think and solve problems on their own by learning and exploring various media and technologies.

Mathematical critical thinking is considered to be higher order thinking; in other words, it goes beyond memorizing (Kraisuth & Panjakajornsak, 2017; Suardana et al., 2018). A critical thinker can distinguish relevant and unrelated information from mathematical problems (Silviariza et al., 2021). According to Wahyuni et al. (2019), critical thinking is essential for students to be able to find solutions to problems. Aizikovitsh-udi and Cheng (2015) suggest that critical thinking in learning supports learning regulation and the skills of students. Critical thinking in mathematics learning involves cognitive processes based on mathematical reasoning to obtain mathematical knowledge (Husnaeni, 2016). Students’ thinking, to explore, examine, use questions, make connections and evaluate all aspects of information related to mathematical problems, are requirements of mathematics learning processes. In some sense, it requires students to perform critical thinking (Angraini & Wahyuni, 2021).

The interesting strategy of organizing learning activities for advancing mathematical thinking proposed by Fraivillig (2001) focuses on students’ thinking processes. The approach consists of three components: (1) Eliciting. This is an important step for the development of the mathematical thinking of students, which challenges students’ mathematical thinking; for this phase, students are encouraged to express their thinking through speaking or writing; (2) Supporting. Students’ thinking and mathematical understanding is promoted in this phase; and (3) Extending. A learning process that expands students’ mathematical concepts by comparing solutions in different ways, extending to new problems.

Dynamic changes in the world and the advancement of digital technology have affected human learning (Shyshkina et al., 2017). The educational paradigm has shifted from a teacher-centered mode to a student-centered mode (Liu & Yan, 2021). Many educational institutions use information technology systems as part of their organization management, including in classrooms (Elena et al., 2019; Iji et al., 2017). The development of academic contexts with the use of more information technology corresponds to the behavior of most current students, who engage with the internet and social media (Techasukthavorn, 2019). Today’s students have different learning expectations; their dispositions differ from previous students (Cevikbas & Argun, 2017). In the twenty-first century, students want to be free agents; preferring to enjoy quick access to a variety of learning resources via digital technology channels to implement their own knowledge (Cevikbas & Kaiser, 2020; Engelbrecht et al., 2020). Therefore, instructors need to be involved in organizing content and activities using various media and digital technologies (Lai & Hwang, 2016). As a result, the use of technology as part of teaching and learning activities, to share knowledge and experiences with students, has gained more attention from and is desired by mathematics teachers (Goos et al., 2020; Hooks, 2015; Sen, 2022).

Cloud technology is a network-based resource sharing service that allows users to share resources as required and allows them to access resources and other applications, programs and services over a computer network (Yang, 2017). The storage space for the resources that users need is provided in the cloud. Users can access the data in the cloud from anywhere (Militsopoulos et al., 2016; Yimer, 2020). This ease of access means that anyone can search for knowledge using technology tools and store it on various mobile devices (Tiejun, 2019). The cloud technology has outstanding properties and features that facilitates connections between various data and prevents the loss of learning materials and information from teacher-student interactions (Cen & Cai, 2017; Etcuban & Pantinople, 2018). The extension of cloud technology into education has been developed in the form of teaching and learning interactive activities between teachers and students (Gonzalez-Martinez et al., 2015; Semenikhina et al., 2019). The organization of learning using cloud technology helps students to develop skills in learning performance (Kasiolas, 2017; Kraipiyaset et al., 2019). Furthermore, the exploration and self-learning of mathematical knowledge online in the cloud is likely to make learning more fun, which is a contributing factor to the increase in cloud mathematics learning (Jin & Ding, 2017; Ramananda & Srinivasan, 2019).
Flipped learning is one of the transformational approaches that has changed the digital reorganization of mathematics teaching, and contributed to the integration of technology into mathematics education and for engaging students in mathematics contexts (Cevikbas & Kaiser, 2020; He, 2020). It is an approach that has changed many elements of instruction, including pedagogical concepts, instructional purposes and more flexible teaching time; it is a mode of instruction that uses supported technologies (Jian, 2020; Kaya, 2021). The flipped model has also been identified as an approach that maximizes face-to-face sessions through lectures and has contributed to hands-on activities (DeLozier & Rhodes, 2017) in such a way that instructors teach students (including students exploring their own knowledge or learning by themselves) outside the classroom, and then the students do their homework or their tasks by themselves in class (He, 2020). The flipped learning approach requires students to have a high degree of responsibility for their learning outside the classroom and there must be sufficient technology and internet equipment for such teaching and learning to take place (Chen & Wen, 2019). A flipped approach is therefore likely to be more suitable for teaching at the higher education level and is one of the educational innovations that has been studied for enhancing learning experiences and competences (Kostaris et al., 2017; Sen, 2022).

Generally, regular classes are restricted to face-to-face interactions in classrooms. However, during the COVID-19 pandemic, many classroom sessions were replaced with online classes (Alabulaziz, 2022). For young modern students familiar with personal learning using social technology, online mathematics teaching has been widely applied in educational institutions (Elena et al., 2019). Recently, the idea of online learning has been increasingly used and applied as a means of enhancing mathematics learning for students (Anuor & Abah, 2014; Sen, 2022; Techasukthavorn, 2019). The process of online mathematics learning is allowing students to practice their divergent thinking processes, express their own opinions and self-assess according to their abilities (Huang & Su, 2016). A well-designed flipped mathematics class can improve students’ mathematical thinking and understanding (Cevikbas & Kaiser, 2020).

Although the flipped model is recognized as an effective teaching method, it does not mean there is only one type (Lencastre et al., 2020). For mathematics education, the integration of flipped learning with other techniques creates more active learning and enhances the purpose of mathematical performance, such as critical thinking (Joubert et al., 2020). Most research into flipped learning has primarily addressed the efficacy of using such an approach on student outcomes; it has not focused on classroom activities that develop mathematical thinking. Suardana et al. (2018) found that the teacher’s teaching methods affected students’ critical thinking, especially where the teachers used educational technological innovations in their learning activities and provided opportunities for learners to study and explore using various media and technologies (Yimer, 2020). In such contexts, students had higher critical thinking skills than those who were taught and studied using traditional methods. This follows Weinhandl et al. (2021), who suggest that using technology as a tool to learn mathematical algorithms, computation and problem solving to get a final solution, makes mathematics learning more interesting and improves students learning outcomes. Based on the theoretical background and the previously identified problem, online mathematics learning using a flipped cloud learning based approach to advancing mathematical thinking is deployed as the instructional pedagogy in this study. How learning, based on such an instructional model, influences mathematical critical thinking ability is also considered.

The purposes of this study were to examine the mathematical critical thinking abilities of students after they have undertaken flipped cloud learning incorporating advancing mathematical thinking compared to the 60% criteria, and to measure their mathematical critical thinking abilities both before and after implementing the proposed instructional pedagogy.

**LITERATURE REVIEW**

**Flipped Learning**

There are various names for flipped learning, such as; the inverted classroom (Lage et al., 2000), classroom flip (Baker, 2000), flipped classroom (Bergmann & Sams, 2012) and flipped learning (Ouda & Ahmed, 2016). The flipped learning approach focuses on student centered learning which uses the time previously spent on lectures (in class) for practical activities or assignments (Cevikbas & Kaiser, 2020; Kostaris et al., 2017).
The conventional events that used to take place in the classroom are shifted outside the classroom and vice versa (Voigt et al., 2020). It is one of the teaching reforms that uses supported technology to contribute to students’ learning (He, 2020). There are two aspects of this approach which may be referred to as ‘do the studying at home’ and ‘do the homework in school’. These are the two learning phases that distinguish it from other learning approaches (Khasanah & Anggoro, 2022). The lectures for basic knowledge and lesson content are available to do at home (out-of-class) using information technology tools for students to learn by themselves at home (Bergmann & Sams, 2012). It is one of the most innovative pedagogies that has led to the creation of active instruction environments, and has the potential to develop problem solving, thinking, communication and social interaction skills (Cevikbas & Argun 2017; Nayci, 2021). Flipped learning is an approach that provides an environment for students to actively construct their own knowledge (Cevikbas & Kaiser, 2020). Therefore, it is consistent with social constructivist theory (Ouda & Ahmed, 2016).

Flipped learning has been researched in a wide range of fields and at various levels of study (Sen, 2022) such as science, technology, engineering and mathematics (STEM) and information communication technology (ICT) (Jdaitawi, 2019; Kostaris et al., 2017), pronunciation instruction and speaking skills (Khasanah & Anggoro, 2022; Pratiwi et al., 2022) and English writing (Altas & Mede, 2021). The flipped learning model has been used in research studies to improve students’ academic learning outcomes (Kong, 2014; Love et al., 2014; Rahman et al., 2014; Saenboonsong, 2017; Sen, 2022). Bangpoophamorn and Wiriyanon (2019) deployed a flipped classroom with self-directed learning for undergraduate students, which enabled them to become higher critical thinkers. Studies have also been conducted to investigate the potential of the flipped model in mathematics classes. Lai and Hwang (2016) employed a self-regulated flipped classroom approach on a mathematics course and found that students achieved more. Bergmann and Sams (2012) note that the flipped pedagogy adapts mathematics classrooms and turns them into laboratories for mathematical inquiry and critical thinking. It is supposed that high school students can increase their performance in mathematics (Atwa et al., 2022; Bhagat et al., 2016; Wei et al., 2020), learning engagement (Kaya, 2021) and communication (Boubih et al., 2020) by studying based on the flipped model compared to learning traditionally. It has also been found that mathematics teaching and learning processes in high schools have been developed using a flipped classroom integrated with STEAM Education (Sutama et al., 2020). However, existing flipped model studies have solely addressed flipped mathematics classes in cloud environments for higher education. In this study, the analysis is based on flipped cloud learning for mathematics students at undergraduate level.

Cloud Learning in Mathematics

Cloud technology is a system in which the data processing and data storage are performed outside the mobile devices themselves (Sultana, 2020). As the computation and storage of data is done in the cloud, this allows applications to be distributed more widely to mobile users and not only to complex users (Xiong et al., 2019). Cloud technology services employed for mathematics learning consist of the three following models (Shyshkina et al., 2017): (1) Infrastructure as a Service (IaaS), which provides the development of infrastructure services, including processors, memory, storage system and networking. These resources are located on virtual systems that can be accessed via the internet; (2) Platform as a Service (PaaS), which is a system service for supporting application development, testing and the development of application management services; and (3) Software as a Service (SaaS), which is the provision of software or applications that are made available by processing to the service provider. Users can use it via the internet without installing the programs on the devices. The characteristics of the services of cloud technology have been developed to divide the resource infrastructure stored in phones, computers and other electronic devices (Chen & Huang, 2017). The educational cloud services include all websites in the cloud technology that can be used as tools to support and promote mathematics teaching and learning activities (Iji et al., 2017). In addition, there is ready access to the cloud service (Rimale et al., 2017). The cloud technology service allows various terminals to access it via a web browser. The cloud also serves as a mathematical learning website that provides software for teachers, students and anyone interested in using the programs for free without installing them on their computers (Saenboonsong, 2017). As cloud-based services have developed for educational applications, the tools now support learning in higher education mathematics (Iji et al., 2017). Online educational tools for mathematics promote students’ rational and problem-solving thinking skills.
the flipped approach uses technology and improvements in mathematics teaching to achieve these goals (Cevikbas & Kaiser, 2020). It is important for teachers to consider using technology strategically to enhance accessibility for all students (Attard et al., 2020; National Council of Teachers of Mathematics, 2011). For example, the features of cloud-based applications are suggested to enhance mathematics instruction for graduate students (Denton, 2012). Inquiry learning was used as the theoretical foundation via cloud technology in the study of Kanjug (2015) to enhance critical thinking and collaborative learning. Alternatively, Sukonwiriyakul (2017) employed mobile learning in the cloud with a 4E×2 model to develop the mathematical problem-solving abilities of seventh grade students. A study into technology enhanced mathematics teaching in secondary education (Weinhandl et al., 2021), suggests that it facilitates comprehension and exploration from various perspectives and assists performance in calculations making them quicker and more reliable. Furthermore, the study of Wahyuni et al. (2019) which deployed a pre-test post-test of edmodo-based blended learning with a group of students, found that their scientific critical thinking skills were enhanced.

The Teaching of Mathematics During the COVID-19 Pandemic

Distance Learning Mathematics

During the COVID-19 outbreak students and instructors were quarantined. For this period adjustments were made to instructional methods moving learning from on-site to online learning or digital learning (Giatman et al., 2020). This affected mathematics education and classes were primarily held using remote teaching during the pandemic (Lavidas et al., 2022).

Distance learning, also known as distance education, e-learning, and online learning became an important tool for teaching and learning during this period (Al-Naabi et al., 2022). In distance learning, there is a physical separation between instructors and students, and educational technologies are used to support communication between them (Karahisar & Unluer, 2022). This can be employed in both synchronous or asynchronous modes (Purbudak, Yilmaz, & Cakir, 2022). The learning process was carried out to meet the requirements of students using both synchronous and asynchronous blended teaching by various teachers at all levels of education (Lavidas et al., 2022). One positive outcome of the response to the COVID-19 pandemic has been the incorporation of digital technology and digital learning platforms and their infrastructures into the distance learning process (Lavidas et al., 2022; Mulenga & Marban, 2020).

The COVID-19 pandemic has changed the instruction process (Mulenga & Marban 2020). Alabdulaziz (2021) found that 98% of mathematics teachers said that the pandemic was a gateway for digital mathematics learning, and that even after the outbreak the use of online mathematics education has expanded. This explains why several types of online education tools for promoting online learning are now used. Alabdulaziz (2021) has elaborated on the digital technology frequently employed by teachers for facilitating teacher-student communication in mathematics learning during the pandemic, including mobile technologies, touchscreens and pen tablets. Other mathematical tools used for online teaching were the Massive Open Online Courses (MOOCs) and computer algebra systems (CAS). In terms of content communication, Zoom, Google Meet, email, and other communication facilitators were utilized (Goncalves et al., 2020). Riyanti and Nurhasana (2021), noted that those students who used Google Classroom during the pandemic for classroom interactions based on blended learning, have better logical thinking abilities. Additionally, the study by Lavidas et al. (2022) reveals that digital environments that have had an essential role for exchanging and communicating materials for mathematics distance learning include: WebEx, Zoom, e-class, Photodentro, Wordwall, Quiziiz, Google docs, Learning apps.

Distance learning has brought about a change in teaching techniques. Students can now have a range of learning experiences including, a discourse session, video lesson, lesson materials, group work, group discussion and presentation as well as other learning modes (Doz et al., 2022). For evaluation, several methods can be used, such as online tests, online work and presentations (Doz et al., 2022; Wahyuni et al. (2019). Regarding the usefulness of online learning, some studies found that students said it was more flexible in terms of time and location (Goncalves et al., 2020) and that it improved academic performance (Goncalves et al., 2020; Gonzalez et al., 2020), although it was possible to cheat during online exams (Nguyen et al., 2020). Doz et al. (2022) found that although students said they preferred in-class teaching, the grades
they obtained during remote mathematics teaching in the quarantine period increased, compared to their pre-epidemic grades. It is important to recognize the gains made when using technology for mathematics education (Radhy, 2019). This finding is consistent with those of Heyd-Metzuyanim et al. (2021) who found that educational success in mathematics was positively correlated with their success in mathematical media literacy tasks. Similarly, Riyanti and Nurhasana (2021) suggest that the ability to apply technology, as was done during the COVID-19 pandemic, is desirable as it enables students’ critical thinking competencies. Therefore, despite school and university closures during the pandemic, it can be seen that online learning in mathematics education has had a positive outcome (Alabdulaziz, 2021).

**Flipped Cloud Learning**

A cloud technology platform can be used to support distance learning and can also be adapted to the flipp learning model (Cen & Cai, 2017; Saenboonsong, 2017). From the pedagogical literature previously mentioned, there is limited scientific research regarding specialized cloud-based services that contribute to the development of thinking skills in mathematics undergraduate students. Flipped cloud learning as distance learning was deployed during this period. The flipped cloud learning for advancing mathematical thinking approach comprises principles that are supported in the model, namely, eliciting, supporting and extending, as is embedded in this research study.

The overview of this strategy framework is shown in Figure 1.

![Figure 1. The architecture of flipped cloud learning for mathematics with advancing mathematical thinking in this study](image)

**Mathematical Critical Thinking**

Mathematical thinking is the cognitive process of facing mathematical and reasoning problems to acquire knowledge and learn how to resolve them (Goos et al., 2020; Husnaeni, 2016). The important mathematics thinking skills are critical thinking, creative thinking, and problem solving (Suryati et al., 2019). Critical thinking is the ability to practice personal responsibility and self-control, obtained from observation, experience and prior knowledge to consider relevant problem information (Supriyatno et al., 2020). According to Ennis (2011), critical thinkers have basic classification skills, analytical ability, are able to make inferences, can clarify, make predictions and have integration skills. Various thinking skills are related to
critical thinking, such as the ability to determine the identification and credibility of information sources and observations, the consideration of consistency with prior knowledge and the ability to draw conclusions with principles and reasons (Jatmiko et al., 2018). A definition of critical thinking is rational reflection and thinking that allows for the formation of reasonable judgments.

Other indicators of critical thinking from researchers include that of Cottrell (2011), who describes the ability to think critically as being able to interpret, analyze and to develop a conceptual understanding and argument. Wang and Zheng (2016), say that critical thinking involves the ability to interpret, evaluate, observe, communicate, and acquire information. Wahyuni et al. (2019) mention five indicators for scientific critical thinking skills, as follows: fact analysis, submission of reason, conclusion, submission of arguments and presentation of implications. Critical thinking in mathematics as suggested by Angraini and Wahyuni (2021), has three indicators, the ability to identify relevance, the ability to determine problems into a mathematical model, and the ability to draw conclusions based on inference principles.

Therefore, it can be concluded that critical thinking is the careful consideration of a situation and/or problems that arise by gathering reliable information or evidence to support it or not, before deciding what action to take. Students will be able to learn mathematics critically when they use these indicators of critical thinking (Facione, 2020; Supratman et al., 2021). Table 1 illustrates the indicators and operational definitions of mathematical critical thinking that have been considered in this study.

**Table 1. Indicators and operational definitions of mathematical critical thinking abilities**

<table>
<thead>
<tr>
<th>No.</th>
<th>Indicators of mathematical critical thinking abilities</th>
<th>Operational definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fact analysis</td>
<td>Ability to identify questions, analyze key components and definitions of facts shown in the problem.</td>
</tr>
<tr>
<td>2</td>
<td>Selection of relevant information</td>
<td>Ability to select or write pertinent information such as the definition, concept, and theory to describe the given situation to be used to solve the problem.</td>
</tr>
<tr>
<td>3</td>
<td>Mathematical formulation</td>
<td>Ability to identify, interpret important information and evaluate assumptions into mathematical models. In other words, it is the ability to make assumptions in accordance with the mathematical problem.</td>
</tr>
<tr>
<td>4</td>
<td>Reasonable conclusions and references</td>
<td>Ability to accept the conclusion by using mathematical definitions, rules, principles, and theories for making valuable judgments including reliable presentation, explanation of implication and reference. References for problem solving and making reasonable.</td>
</tr>
</tbody>
</table>

*Source: Angraini & Wahyuni, 2021; Ennis, 2011; Wahyuni et al., 2019.*

From the study of the related research into critical thinking, it was found that most research studies have led to improvements in critical thinking ability by using teaching approaches that organize learning activities or by using training programs that the researcher has synthesized or developed. Some studies have been conducted regarding the implementation of flipped classrooms (Asmara et al., 2019; Bangoopamorn & Wiriyanon, 2019). Encouraging students’ thinking skills by employing learning strategies is associated with the practice of critical thinking. It is also expected to enhance higher mathematical critical thinking. In this regard, the relationship between advancing children’s thinking approaches (Fraivillig, 2001) and indicators of mathematical critical thinking are analyzed in Table 2.
Table 2. The analysis of the relevance of advancing children’s thinking approaches and mathematical critical thinking abilities in this study

<table>
<thead>
<tr>
<th>No.</th>
<th>Component</th>
<th>Strategy</th>
<th>Mathematical critical thinking abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Eliciting</td>
<td>The advanced problems based on the lessons are used as tool for students to express their thoughts. To achieve this step, the instructor must put emphasis on taking multiple answers from students for one problem.</td>
<td>Identification of questions/problems, analysis of the key components and definitions of facts shown in the problem.</td>
</tr>
<tr>
<td>2</td>
<td>Supporting</td>
<td>The instructor support students’ mathematical thinking.</td>
<td>The ability to explain their friends’ methods of solving problems in their own words while retaining the same meaning.</td>
</tr>
<tr>
<td>3</td>
<td>Extending</td>
<td>Providing analysis, comparing solutions in various ways by adjusting problems or changing new ones while also expanding students’ thinking by encouraging reflection of mathematical ideas and finding other methods and summarizing mathematical concepts.</td>
<td>The ability to comment, analyze, compare and summarize the mathematical lesson and problem formulations.</td>
</tr>
</tbody>
</table>

METHOD

Research Design

This study is pre-experimental research, involving one group, using a pre-test-post-test design (Edmonds & Kennedy, 2017). It has been carried out to determine any changes related to the treatment’s effect on the subjects, to identify any impact on mathematical critical thinking. It has been conducted without a comparison class because of the scheduled timetable provide by the university.

Participants

56 undergraduate mathematics students in their fourth year of university, enrolled on a Mathematical Analysis Course in The Faculty of Education located in Thailand, were involved in this study. The participants were selected using a purposive sampling technique. The study was conducted in the second semester of the 2021 academic year. The students were all aged between 21-22 years.

The information about the gender of the participants in this study is shown in Table 3.

Table 3. Demographic data

<table>
<thead>
<tr>
<th>Gender</th>
<th>N</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>12</td>
<td>21.42</td>
</tr>
<tr>
<td>Female</td>
<td>44</td>
<td>78.57</td>
</tr>
<tr>
<td>Total</td>
<td>56</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Table 3 shows the frequency distribution and percentage of subjects as classified by gender. There was a total of 56 subjects, of which 44 were female (78.57%), and 12 male (21.42%).

Procedure and Implementation

Procedure

The research procedure was initiated in the study on the related theoretical background and framework, and the development of the instructional approach, with reference to the strategy of advancing children’s mathematical thinking (eliciting, supporting and extending) of Fraivillig (2001) using the flipped model.
G Suite in Google Classroom was deployed to develop the cloud learning tools. Class preparation included the following: (a) the development of mathematics material in the form of videos and lessons. Assignments were given to the students for their out-of-class based study prior to their face-to-face session; (b) the design of the learning based on the cloud environment. The Google Meet cloud platform was employed for the mathematics online class session. Prior to the face-to-face online classes, the students were principally exploring the mathematics materials (video tutorials, lesson content) and self-assignments displayed in Google Classroom.

**Implementation**

The Mathematical Analysis Course was used to conduct the experimental research. The study was carried out during one semester which consisted of four stages:

Stage 1. The students were given an initial (pre-test) mathematical critical thinking ability test.

Stage 2. The instruction in Mathematical Analysis Course using the flipped cloud learning incorporating approach of advancing mathematical thinking (as shown in Table 3).

Stage 3. After completing the course, the students were given a second test (post-test) of their mathematical critical thinking abilities, the same as the first one (which contributed to their final grades). For this study, these abilities were measured using four indicators: fact analysis, selection of relevant information, mathematical formulation, and reasonable conclusions and references.

Stage 4. The scores were used for quantitative analysis to investigate the mathematical critical thinking abilities of the students.

The syntax and design of the cloud tools for the flipped cloud learning model with strategy for advancing mathematical thinking included the components shown in Table 4.

**Table 4.** The syntax and design of the learning activities and tools for education for each component in the cloud

<table>
<thead>
<tr>
<th>Learning Activities</th>
<th>Cloud based tools in Google Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Docs</td>
</tr>
<tr>
<td>Eliciting</td>
<td></td>
</tr>
<tr>
<td>(a) Instructor prepares problems to stimulate students’ thinking (in the cloud).</td>
<td>√</td>
</tr>
<tr>
<td>(b) Students express their thoughts and analyze methods to find answers by themselves through a cloud-based program where data can be collected.</td>
<td>√</td>
</tr>
<tr>
<td>Supporting</td>
<td></td>
</tr>
<tr>
<td>(a) Students are encouraged to think of similar problems and relevant concepts from the content that the instructor has posted in the cloud.</td>
<td>√</td>
</tr>
<tr>
<td>(b) Students review basic knowledge related to solving problems, including the problem-solving process by searching the cloud.</td>
<td>√</td>
</tr>
<tr>
<td>(c) Further explanation of the students’ solutions. Students and instructor together form conclusions.</td>
<td>√</td>
</tr>
<tr>
<td>Extending</td>
<td></td>
</tr>
<tr>
<td>(a) Interaction about the content that has been learned outside the class; students describe, give their opinions, analyze, compare and summarize mathematical knowledge.</td>
<td>√</td>
</tr>
<tr>
<td>(b) Instructor adjusts the conditions of the old problem or changes to new problems in the classroom and in the cloud.</td>
<td>√</td>
</tr>
<tr>
<td>(c) Preparation of electronic documents for students to challenge and engage them to find other methods and solutions.</td>
<td>√</td>
</tr>
</tbody>
</table>
Research Instruments

The instrument used in this study for data collection was a test of mathematical critical thinking abilities (see Appendix A). This test related to the content learned during the flipped cloud learning experience and operational definitions of mathematical critical thinking ability. The test was constructed by the authors of this study based on the concept of real numbers systems and limits as part of the course. It consisted of a six items essay test, where each item had four sub-items. The students had to do the same tests they completed at the pre-and post-test stages which refers to the four indicators of their mathematical critical thinking abilities. The content validity of the test was reviewed and approved by three experts selected for their academic expertise and qualifications in mathematics. The authors of this study made some improvements to the tests based on the suggestions given by the experts before it was distributed. The difficulty index was in the range 0.22 - 0.79, the discriminant index was in the range 0.45 - 0.80, the reliability using Cronbach’ alpha coefficient was 0.65, a medium reliability, and the McDonald’s omega reliability coefficient was 0.84, which is considered appropriate.

Data Collection and Analysis

The data collected for this study consists of the scores from the mathematical critical thinking tests. The Statistical Package for Social Sciences 23 (SPSS 23) software was used to analyze the data. To check the significance of the findings, a preliminary test was performed first by testing the normality and the variance homogeneity of the scores with regard to the mathematical critical thinking obtained.

The normality test of mathematical critical thinking ability was performed using the Shapiro-Wilk test as presented in Table 5.

<table>
<thead>
<tr>
<th>Table 5. Normality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
</tr>
<tr>
<td>Statistic</td>
</tr>
<tr>
<td>Pre-test</td>
</tr>
<tr>
<td>Post-test</td>
</tr>
</tbody>
</table>

Based on the Shapiro-Wilk test, the significance value for the pre-test was 0.056 > 0.05, and the significance value for the post-test was 0.924 > 0.05. Thus, it was concluded that the data were normally distributed at a significance level of 0.05.

The equality of variance using the Levene’s test is shown in Table 6.

<table>
<thead>
<tr>
<th>Table 6. Homogeneity test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
</tr>
<tr>
<td>Pre-test</td>
</tr>
<tr>
<td>Post-test</td>
</tr>
</tbody>
</table>

The value of the Levene statistic for the pre-test was 0.139 with a significance level of 0.710 >.05. For the post-test, the Levene statistic was 0.019 with a significance value of 0.891 > 0.05. Therefore, the data meet the assumption for the equality of variances.

The mean, standard deviation (S.D.), t-test, and normalized gain (n-gain) were used to analyze and calculate the quantitative data. A one sample t-test was used to consider the mathematical critical thinking scores compared to the 60% criteria. The paired samples t-test was used to compare the difference between the pre-test and post-test values, determined by the level of statistical significance (α <0.05).
The average score from the pre-test and post-test were analyzed using the following n-gain formula (Meltzer, 2002): 
\[ n\text{-gain} = \frac{X_m - X_n}{100 - X_n}, \] 
where \( X_m \) is post-test score, \( X_n \) is pre-test score, and n-gain is normalized gain. The criteria for interpreting the n-gain level are given in Table 7.

**Table 7. Interpretation of the criteria for the normalized gain level (Meltzer, 2002)**

<table>
<thead>
<tr>
<th>n-gain score</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-gain ≥ 0.7</td>
<td>High</td>
</tr>
<tr>
<td>0.3 ≤ n-gain &lt; 0.7</td>
<td>Medium</td>
</tr>
<tr>
<td>n-gain &lt; 0.3</td>
<td>Low</td>
</tr>
</tbody>
</table>

**FINDINGS**

The findings regarding the impact of using an advancing mathematical thinking approach with a cloud based flipped model on an online Mathematical Analysis course on students’ mathematical critical thinking are shown in Table 8.

**Table 8. Analysis of students’ mean post-test scores regarding mathematical critical thinking compared with the required criteria**

<table>
<thead>
<tr>
<th>Indicator of mathematical critical thinking</th>
<th>N</th>
<th>60 percent of the full score</th>
<th>Mean</th>
<th>S.D.</th>
<th>St. Error Mean</th>
<th>t</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical critical thinking ability</td>
<td>56</td>
<td>18</td>
<td>19.36</td>
<td>3.26</td>
<td>0.43</td>
<td>3.109</td>
<td>55</td>
<td>.003*</td>
</tr>
</tbody>
</table>

*Note. *p<.05

From the findings shown in Table 8, it can be seen that the mean mathematical critical thinking abilities score for students, after engaging with flipped cloud learning on advancing mathematical thinking was 19.36. The students had mathematical critical thinking abilities significantly higher than 60% at the .05 level.

To explore the improvement of mathematical critical thinking abilities before and after learning with the instructional method, we analyzed the normalized gain according to each indicator of mathematical critical thinking. The results are presented in Table 9.

**Table 9. The average normalized gain score of mathematical critical thinking**

<table>
<thead>
<tr>
<th>Indicator of mathematical critical thinking</th>
<th>n-gain</th>
<th>S.D.</th>
<th>St. Error Mean</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fact analysis</td>
<td>0.42</td>
<td>0.29</td>
<td>0.038</td>
<td>Medium</td>
</tr>
<tr>
<td>Selection of relevant information</td>
<td>0.29</td>
<td>0.19</td>
<td>0.025</td>
<td>Low</td>
</tr>
<tr>
<td>Mathematical formulation</td>
<td>0.57</td>
<td>0.33</td>
<td>0.045</td>
<td>Medium</td>
</tr>
<tr>
<td>Reasonable conclusions and references</td>
<td>0.59</td>
<td>0.34</td>
<td>0.046</td>
<td>Medium</td>
</tr>
<tr>
<td>Average</td>
<td>0.46</td>
<td>0.28</td>
<td>0.038</td>
<td>Medium</td>
</tr>
</tbody>
</table>

The students gained the highest n-gain on the reasonable conclusions and references indicator, which was 0.59, at medium level. This was followed by the mathematical formulation indicator, which was 0.57, and the lowest score on the selection of relevant information indicator was 0.29. Overall, it was found that the average increase in students’ mathematical critical thinking abilities was 0.46, which is considered as medium. This implies that using the flipped cloud learning approach with advancing mathematical thinking affected the students’ mathematical critical thinking.
To investigate the significance of the conclusion above, the difference between pre-test and post-test mathematical critical thinking was determined by a t-test, which is shown in Table 10.

<table>
<thead>
<tr>
<th>Test</th>
<th>N</th>
<th>Mean</th>
<th>S.D.</th>
<th>St. Error Mean</th>
<th>t</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>56</td>
<td>16.08</td>
<td>4.11</td>
<td>0.54</td>
<td>5.205</td>
<td>55</td>
<td>.000*</td>
</tr>
<tr>
<td>Post-test</td>
<td>56</td>
<td>19.36</td>
<td>3.26</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. *p* < .05

It can be seen from Table 10, that the mean score for mathematical critical thinking for the pre-test was 16.08, and for the post-test, it was 19.36. There is a statistically significant difference between the pre-test and post-test scores examined by a dependent sample t-test, which found that the post-test score was better than the pre-test score. An effect size based on Cohen’s d of 0.88 was a big difference. This verified that the treatment method was significant for eliciting students’ critical thinking abilities.

**DISCUSSION**

The integration of cloud technology and advancing mathematical thinking strategy in flipped classrooms has been considered as an instructional approach for studying students’ mathematical critical thinking abilities. After experimenting with this instructional approach, the mathematical critical thinking abilities of the students were higher than 60 percent, significantly. This may be due to the flipped learning model that provided students with opportunities to exchange ideas and collaborate, allowed them to solve problems independently and self-learn, which are ways of strengthening the ability to think critically (Kanjug, 2015; Sutama et al., 2020; Weinhandl et al., 2021). A cloud-based learning environment containing the eliciting, expanding and sharing of mathematical ideas and strategies encourages students to exchange knowledge, and to learn from and listen to suggestions from their classmates so they can fill in any gaps in their knowledge (Kraipiyaset et al., 2019). The findings of this study indicate that students had an average n-gain in mathematical critical thinking of 0.46, which was medium. This could be because learning performance was influenced by the instructional approach students experienced in the classroom (Goos et al., 2020). The application of cloud technology to advancing mathematical thinking in the flipped model improves the pedagogical process (Denton, 2012; Elena et al., 2019). When considering the top two highest-scoring indicators of mathematical critical thinking, mathematical formulation and reasonable conclusions and references, there is a medium average n-gain. This is consistent with the results of Wahyuni et al. (2019), who suggest that the flipped classroom process encourages students to explore their thoughts as a first step in practicing, which tends to result in better critical thinking. The students obtained the lowest score for the indicator of selection of relevant information with a low n-gain of 0.29. This may be because students were not yet able to consider the connections, relations and related concepts that must be used to solve a particular mathematical problem. Although the online learning environment contains activities involving interactions between students and instructors, some students are not in front of their screens all of the time. They may not understand some issues, and others may be unclear. However, thinking ability is a process that takes time to practice.

As the students had mathematical critical thinking abilities higher than before the experiment, at a statistically significant level of 0.05, this means students had better mathematical critical thinking abilities, which corresponds with the studies of Lai and Hwang (2016), Asmara et al. (2019), and Bangpoophamorn and Wiriyanon (2019). Further, this is in accordance with the findings of Saenboonsong (2017), who finds that undergraduate students’ academic achievements, after learning with the flipped classroom model using cloud technology are higher than before. The learning outside of the classroom mode, listening to video discourses, taking notes or exploring other online learning resources, are all activities that can be done independently and flexibly. The students practice their analysis of knowledge by integrating their prior knowledge with unfamiliar content and forming conclusions by themselves (Sukonwiriyakul, 2017).
cloud provides a space to exchange knowledge between students and between students and teachers, thereby giving students flexibility in reviewing and exploring contents (Boyinbode, 2018). In accordance with the study of Asmara et al. (2019), students’ exploration and extension of thought will eventually develop their critical thinking. The findings are also in accordance with the previous studies of Asmara et al. (2019), Atwa et al. (2022), Kanjug (2015) and Kostaris et al. (2017). The cloud learning resources make difficult points of mathematical knowledge visible (Cen & Cai, 2017). In addition, the cloud technology comprises tools that support learning activities for accessing essential initial knowledge such as data retrieving and sharing and provides students with the facility to select pertinent information to solve problems (Denton, 2012; Wahyuni et al., 2019).

Some previous studies found that students were more motivated to participate in class activities than in remote mathematics learning; but found that they felt more relaxed when studying at home (Goncalves et al., 2020; Gonzalez et al., 2020; Lassoued et al., 2020; Surani & Hamidah, 2020). Even though it is now convenient for students to attend classroom lectures, teaching and learning mathematics can be adjusted according to epidemic prevention policies with the use of distance learning models (Mulenga & Marban, 2020). In terms of the learning environment, the findings correspond with other research (Giatman et al., 2020; Goncalves et al., 2020; Gonzalez et al., 2020; Riyanti & Nurhasana, 2021). Instructors are recommended to retain the quality of online learning materials and to prepare for any infectious outbreaks or similar situations, but especially for students’ mathematical media literacy and for practicing critical thinking (Heyd-Metzuyanim et al., 2021).

The COVID-19 pandemic helped to facilitate an online instructional process that can be used for students’ learning mathematics. The flipped cloud model is a teaching and learning management system in the digital world that uses technology to expand the learning experience and provides active, engaged learning for students (Iji et al., 2017; Kostaris et al., 2017). Because of the epidemic with students not in the classrooms, teachers have gained experience in distance learning by integrating technology with the teaching of mathematics based on the flipped approach. Any mistakes made during this learning process will have been identified and feedback provided based on the communication and interaction that has taken place.

It is generally recognized that both students and instructors have been affected by the spread of the COVID-19 virus. As an instructor, it is imperative to determine teaching approaches that allow students to participate in lessons as much as possible. Flipped cloud learning is one of the methods that can be used for mathematics instruction and is a tool for communication of all course activities. This method can provide instruction for any students or teachers infected or in quarantine due to COVID-19 because they can still attend or hold classes in the form of distance learning. Therefore, flipped cloud learning is an appropriate way to support students.

CONCLUSION

The use of technological tools to help learners to visualize mathematical points while allowing them to practice their thinking skills are a significant aid that every instructor must think about using when traditional teaching inside the classroom is not possible. Online or distance learning has been used during the COVID-19 pandemic as well as during periods when socially distancing was necessary. Understanding the application of technology is important to efficiently develop a mathematics teaching and learning model and critical thinking skills. The flipped cloud distance learning for this study was carried out online using G Suite platforms.

This study has examined the flipped cloud learning approach based on advancing mathematical thinking strategy, implemented on a Mathematical Analysis course for undergraduate students. The study aimed to analyze students’ mathematical critical thinking abilities through learning with the treatment method. The evaluation was done before and after learning. In terms of statistical display, significant changes were indicated between pre-test and post-tests. The quantitative results revealed that the students increased their mathematical critical thinking more than the specified threshold. The findings of the study imply that a flipped model with cloud-based design for online mathematics learning enhances mathematical critical thinking. The results may support interested instructors and educators to apply the components in the model, contributing to the creation of a learning environment that emphasizes the thinking process, as
appropriate in their domain context. In addition, they can utilize the approach to enhance the performance of students’ mathematical critical thinking abilities.

Although the flipped cloud learning model alone may not be the best teaching and learning tool overall, it is the integration of technology with mathematics education that has allowed the teaching and learning process to continue and achieve success during the COVID-19 situation.

However, the study design was limited to one group. There was no control class to compare these results with, which would have provided more clarity. Therefore, further study is necessary that should include additional control groups as well as other strategies for teaching mathematics in the instructional model and which is supported by modern devices.

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**REFERENCES**


APPENDIX A

A test of mathematical critical thinking abilities used as a data collection tool for this study.

1. Consider the text “The sum of a rational number and an irrational number is an irrational number.”
   1.1 Clarify the keywords and describe the definition of keywords that appear in the aforementioned text. [Fact analysis].
   1.2 Write the relevant definitions, concepts, or theories needed to solve problems. [Selection of relevant information].
   1.3 Interpret the given text into a mathematical symbol and give the meaning of each symbol. [Mathematical formulation].
   1.4 Make conclusions from the given text (true or false) by deductive reasoning. [Reasonable conclusions and references].

2. Consider the text “The addition inverse of $\frac{1-\sqrt{5}}{2}$ is $-\frac{1+\sqrt{5}}{2}$”
   2.1 Clarify the keywords and describe the definition of keywords that appear in the aforementioned text. [Fact analysis].
   2.2 Write the relevant definitions, concepts, or theories needed to solve problems. [Selection of relevant information].
   2.3 Interpret the given text into a mathematical sentence and give the meaning of each symbol. [Mathematical formulation].
   2.4 Make conclusions from the given text (true or false) by deductive reasoning [Reasonable conclusions and references].

3. Consider the text “If $a \in \mathbb{R}$ and $a \neq 0$ then $a^2 > 0$”
   3.1 Clarify the keywords that appear in the aforementioned text. [Fact analysis].
   3.2 Describe the relevant definitions, concepts, or theories needed to solve problems. [Selection of relevant information].
   3.3 Interpret the given mathematical text into a text sentence. [Mathematical formulation].
   3.4 Make conclusions from the given text (true or false) by deductive reasoning. [Reasonable conclusions and references].

4. Consider the text “If $x$ is upper bound of $A$ and $y \in \mathbb{R}^+$ then $x + y$ is the upper bound of”
   4.1 Clarify the keywords and describe the definition of keywords that appear in the aforementioned text. [Fact analysis].
   4.2 Describe the relevant definitions, concepts, or theories needed to solve problems. [Selection of relevant information].
   4.3 Interpret the given text into mathematical model. [Mathematical formulation].
   4.4 Make conclusions from the given text (true or false) by deductive reasoning [Reasonable conclusions and references].

5. Consider the text “$\lim_{n \to \infty} \frac{-1}{\sqrt{n} + 5}$”
   5.1 Clarify the keywords and describe the definition of keywords that appear in the aforementioned text. [Fact analysis].
   5.2 Describe the relevant definitions, concepts, or theories needed to solve problems. [Selection of relevant information].
   5.3 Interpret the given text into mathematical hypothesis. [Mathematical formulation].
   5.4 Make conclusions from the given text by deductive reasoning, summarize the conclusion with references. [Reasonable conclusions and references].

6. Consider whether the sequence $\{(-1)^n\}$ is convergent sequence.
   6.1 Clarify the keywords and describe the definition of keywords that appear in the aforementioned text. [Fact analysis].
   6.2 Describe the relevant definitions, concepts, or theories needed to solve problems. [Selection of relevant information].
   6.3 Interpret the given text and symbol into mathematical hypothesis. [Mathematical formulation].
   6.4 Make conclusions from the given text by deductive reasoning, summarize the conclusion with references. [Reasonable conclusions and references].