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Research Article

Stochastic analysis of longevity risk in dependent multiple life annuities

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ABSTRACT

The aim of this paper is to examine the net single premiums of multiple life annuities using stochastic rates of return and dynamic life table under the assumption of dependency of spouses' future lifetimes. In order to calculate the present value of the annuity or the net single premium, two parameters are needed: survival rate and the rate of return. For the survival rates, we used a life table with a time dimension for Turkey, in which mortality rates follow a declining pattern, a major indicator of longevity. For the rate of return, two portfolios were created, low and high risk portfolios that include assets with different ratios and AR(1) process was used to model the rates of return based on both portfolios. To assess the dependency, future lifetimes of spouses were assumed to follow Frank's copula model. The effects of longevity, stochastic rates of return and dependency of future lifetimes on these net single premiums were analyzed.

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INTRODUCTION

Series of payments based on the living status of more than one individual are called multiple life annuities. As other annuity products, it is aimed to provide periodic income over lifetime for the individuals.

Insurance companies need two key variables: individuals' future lifetime which changes from day to day and the rate of return that defines the time value of money in order to calculate the net single premiums of annuities. Although, taking these variables into account in a deterministic way simplifies the calculations, it would lead to unrealistic results. There are several studies in the literature conducted using stochastic rate of return and life tables in order to determine the value of payments of a life annuity such as, Boyle [1], Waters [2], Panjer and Bellhouse [3], Frees [4], Beekman and Fuelling [5], Coppola et al. [6], Hoodemakers [7]. In this study we restricted ourselves that the rate of return, following a Gaussian AR(1) process that was previously used in a similar way by Bellhouse and Panjer [8], Marceau and Gaillardetz [9], Nolde and Parker [10] and Chen et al. [11]. For the survival rates, we used the dynamic life table proposed by Sucu et. al. [25] which is considered to be a means for understanding longevity.

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Lee and Carter [12] carried out the first and most frequently referred study in the literature that takes into account the time component of stochastic mortality models. Li et al. [13], Cairns et al. [14] and Currie et al. [15], among others, used mortality improvements over time in their studies. We used a nonparametric life table which differs greatly from the traditional life tables for analyzing mortality experience. The dynamic life table allows the observer to understand the shifts in mortality with the inclusion of time dimension apart from age and gender. Since the life annuity includes payments depending on the persons' life, it is important to make a prediction of expected future lifetime that the researches focusing on for few decades. Longevity is defined as an increase in the life expectancy or a decrease in mortality process of individuals. The increase in life expectancy leads to spending more time in the retirement period. The longer the future lifetime, the more payments to the individual. Thus, longevity becomes a risk for the insurance companies. Without longevity being taken into account, the calculation of the premiums using mortality rates can lead to significant losses for the insurance companies.

Traditionally, the joint survival rates in multiple life products are calculated under the assumption of the independence of couples' future lifetimes. Under this assumption, the probability of joint life is considered as a separate component of the individual's survival rates Which is actually far from reality. Among the leading studies on the assumption of the dependence of individuals' future lifetimes, "Copula" was used for the first time by Sklar [16]. Due to their ease of understanding and analyzing the relationships between variables, Copula dependency models have been widely used to explain the relationship between individuals in the actuarial literature. Parkes et al. [17] in his study on married couples showed that after one of the spouses dies, the probability of death of the other increases. Shemyakin and Youn [18] analyzed the net single premium for the joint life and survivorship annuities under the dependence of future lifetimes and compared the results of copula families. Carrie and Chan [19] showed the boundaries of the net single premiums of multiple life annuities to measure the effect of dependency. Frees et al. [20] calculated net single premiums of multiple life annuities using the copula function. They showed positive dependence between joint lives. In this study, we assumed spouses' future lifetimes follow Frank's copula model that frequently used in the literature

Different from previous studies in which stochastic rate of return and the assumption of dependent future lifetimes of spouses were used, we calculated the net single premiums of multiple life annuities by including a dynamic life table in the model. We aimed to find the stochasticity effect of these parameters on net single premiums and make more realistic calculations. It is essential to remark that these calculations were made using the Monte-Carlo simulation method. We restricted ourselves only to the two types of multiple life annuities, namely joint life and survivorship.

The rest of this study is organized as follows. Section 2 involves information about AR(1) model, dynamic life table and Frank's copula model along with the necessary assumptions to make calculation possible. Net single premiums under different conditions are calculated and compared in the Section 3. Finally, the conclusion is given in the Section 4.

MODEL AND ASSUMPTIONS

In this section, information about AR(1) model, dynamic life table for survival rates of spouses, and Frank's Copula dependency for spouses future lifetimes are presented.

STOCHASTIC RATE OF RETURN

The AR(1) model may not reflect the non-stationary situation of the real world. However, using long-term rate of return can be a reasonable approach in common actuarial valuation. In this study, it is assumed the rate of return is constant within each year. Let δ_k denote the annual rate of return between the year k-1 and k, a Gaussian AR(1) model is shown as;

$$\delta_k - \mu = \Phi(\delta_{k-1} - \mu) + \varepsilon_k \quad (1)$$

where μ is the long term mean of the process and Φ ($|\Phi| < 1$ for stationary) is the autoregressive coefficient which states returning speed to the mean. Furthermore, ε_k is the white noise process, which means it is a sequence of independent and normally distributed random variables with mean zero and finite variance σ^2. Also, covariance between $(\varepsilon_k, \varepsilon_{k+1})$ must be zero when $k \neq 0$ [10], [11].

These stochastic rates of return which are generated using simulation techniques are needed for the calculation of net single premiums of annuities. To calculate the present value of a payment, the AR(1) process will generate the rate of return for every valid policy year. The sum of the discounted value of these conditional payments in the discrete time space also gives the present value of the annuity.

FRANK'S COPULA MODEL OF DEPENDENCY

There are many traditional studies in which multiple life annuities have been determined using the assumption of independency between assured individuals. The insurance company which wants to design a multiple life annuity product must consider the impact of individuals' lives together on the future lifetimes. In this section, we describe the model of Frank's copula and it's implementation.

Nelsen [21] states that the basis of the copula models is to express the multiple joint distribution function as a function of marginal distributions. Copula contains all the information about the dependence structure of random variables. While correlation only examines a linear relationship between variables, copulas can also capture nonlinear dependence [22]. The bivariate Frank copula function with the $\propto$ dependency parameter, which is frequently used in the actuarial literature defined as [23],

$$C(u,v) = \ln\left(1 + \frac{(e^{\propto u}-1)(e^{\propto v}-1)}{e^{\propto}-1}\right)/\propto. \tag{2}$$

In equation 2, the $\propto$ parameter ($-\infty < \propto < \infty$) shows the strength of the dependency. If u and v are the marginal distribution functions, then Frank's copula becomes the joint distribution function for u and v. It can be proven that $\lim_{\propto \to 0} C(u,v) = uv$ and that means u and v are independent [11].

In survival analysis, marginal survival functions are used instead of marginal distribution functions. According to the Sklar survival copula with the marginal u and v is shown as,

$$\breve{C}(u,v) = u + v - 1 + C(1-u, 1-v). \tag{3}$$

For more detailed information, Sklar [11], Nelsen [21] and Elliot [24] can be examined.

In this study we consider two types of multiple life annuities:

a) joint life annuity that continues to be paid until the first death,
b) survivorship or last survivor annuity that continues to be paid until the last death.

In joint life annuity payments end when the first death occurs, in survivorship annuity, on the other hand, payments end when all individuals die. To be able to calculate the net single premium of annuity that includes more than one individual in a discrete-time space, we need the joint survival rates under the assumption of Frank's copula. With the help of the formula 4 joint survival rates of two individual's life status can be calculated as [24],

$${}_t p_{xy} = {}_t p_x + {}_t p_y - 1 + C({}_t q_x, {}_t q_y). \tag{4}$$

This equation shows the joint survival rate of both individuals for t years under the assumption of Frank's Copula dependency model. This probability is used for the calculations of the net single premiums of the joint life annuity. Furthermore, the formula below presents the rate of survival at least for one of the individuals for t years under the assumption of Frank's Copula dependency which is used to calculate the net single premium of survivorship annuity.

$${}_t p_{\overline{xy}} = 1 - C({}_t q_x, {}_t q_y) \tag{5}$$

Table 1. An illustration of a dynamic life table

		YEAR				
		2018	***2019***	***2020***	**…**	***2035***
AGE	*0*	$q_{0,2018}$	$q_{0,2019}$	$q_{0,2020}$	…	$q_{0,2035}$
	1	$q_{1,2018}$	$q_{1,2019}$	$q_{1,2020}$	…	$q_{1,2035}$
	2	$q_{2,2018}$	$q_{2,2019}$	$q_{2,2020}$	…	$q_{2,2035}$
	…	…	…	…	…	…
	119	$q_{119,2018}$	$q_{119,2019}$	$q_{119,2020}$	…	$q_{119,2035}$
	120	*1*	*1*	*1*	*1*	*1*

DYNAMIC LIFE TABLE FOR SURVIVAL RATES

In this section we chose to use a dynamic life table that fits to Turkish mortality data of annuitants. Dynamic life table refers to a life table that includes mortality rates varying year after year. This life table, which was particularly prepared for Turkey, provides information about longevity by showing the decline in mortality in the future years.

Table 1 illustrates a form of a dynamic life table based on Sucu et al.'s [25] study called "Creating the Turkish Insured and Annuitant Life Tables and Projections". Instead of using single column of a traditional life table, employing the effect of subsequent years would be more significant and convenient. For example, when determining the probability of survival for 2 years of an individual at age 0 in the beginning of the year 2018, ($1-q_{0,2018}$) and ($1-q_{1,2019}$) must be multiplied. So, the result is the survival rate of the 0 years old individual in 2018 for 2 years. If it is calculated with the static life tables as in traditional studies, ($1-q_{0,2018}$) and ($1-q_{1,2018}$) must be multiplied.

One can calculate the net single premium of a whole joint life annuity, where 1 unit of payment is made at the end of each period, with stochastic rate of return and joint survival rates obtained from the dynamic life table with the formula below. Let x and y be the individuals, expected present value of future payments can be calculated as:

$$\begin{aligned} a_{xy} &= {}_1p_{xy}(1+i_1)^{-1} + {}_1p_{xy}\,{}_1p_{x+1:y+1}(1+i_1)^{-1}(1+i_2)^{-1} + \ldots \\ &+ {}_1p_{xy}\,{}_1p_{x+1:y+1}\cdots{}_1p_{x+\Delta_\omega-1:y+\Delta_\omega-1}(1+i_1)^{-1}(1+i_2)^{-1} \\ &\ldots(1+i_{\Delta_\omega})^{-1} \end{aligned}$$

There is no need to show the net single premium of a whole survivorship annuity. Only difference is the notation of the survival rates. In this formula, i represents the rate of return that is modelled by using the AR(1) process, p_{xy} stands for the joint survival rate of individuals that calculated under the assumption of Frank's Copula and Δ_ω shows the lifetime. Therefore, Δ_ω stands for the minimum lifetime of the individuals in joint life annuity and maximum lifetime of the individuals in survivorship annuity.

NUMERICAL ILLUSTRATIONS

In this section, we present and discuss on the results of net single premiums of whole multiple life annuities for the 65-year-old spouses obtained from Monte Carlo simulations. In finding the dependent survival rates of spouses We used the alpha parameter estimated in the study carried out by Frees et al. [20] while. In order to understand the effect of the stochastic rate of return, calculations were made using the deterministic rate of returns which are in fact the long term means of portfolios obtained from the AR(1) process. To see the longevity effect, in addition to dynamic survival rates, premiums were calculated using the constant survival rates for every year in the future. Instead of using another life table, these survival rates for each age were obtained from the first column of the dynamic life table. The probabilities in the first column of the dynamic table will hereafter be called static survival rates. One assumption was that the dynamic life table includes predicted survival rates from the year 2018 to 2035, therefore, the survival rates for the years after 2035 were assumed to be equal to the rates of 2035. All of the premiums were calculated for the policies valid at the beginning of 2018. Finally, premiums were shown both under the assumption of independency of spouses' future lifetimes and Frank's copula model to observe the effect of dependency.

The procedure of the simulation we used in this study can be explained in the following steps:

1. Identify the constants and required parameters in order to begin the process before simulation starts,
2. Generate stochastic future rates of return using AR(1) process,
3. Calculate the joint survival rates for each status,
4. Generate random numbers from Uniform distribution,
5. Make the payment until the number generated becomes bigger than the joint survival rate.
6. Calculate the present value of these payments using stochastic rate of returns,
7. Repeat the simulation as many times as needed.

As a result of these steps, there will be present values as much as the number of simulations. The mean of these present values, which were calculated for different conditions, is the net single premium. Since this is a calculation of a present value using a simulation technique, standard deviations should essentially be given. Therefore, the standard deviations of all calculated net single premiums were shown in the related section. The simulations repeated 1 million times for each net single premium calculation.

PARAMETERS OF AR(1) PROCESS OF RATE OF RETURN

For a realistic approach, we created two portfolios such as high and low risky ones with a different ratio of three assets: annual inflation-adjusted rate of returns of government securities, Turkish lira-dollar parity and the index of Istanbul Stock Exchange. Annual data between 2005-2018 is used to establish an AR(1) process and to predict the future rates. The ratios of assets in the portfolios were determined on the basis of the final insurance report of Turkish Secretariat of Treasury. This report includes investment strategies of insurance companies in Turkey. The ratios of the assets are shown in Table 2.

Table 2. Ratios of assets in the portfolios

Portfolios	*Government Securities*	*Dollar*	*Stocks*
Low Risky	0.80	0.15	0.05
High Risky	0.40	0.20	0.40

Table 3. Parameters of low and high risky portfolios of AR(1) model for rate of returns

Parameters	*Low Risky Portfolio*	*High Risky Portfolio*
μ	0.040128	0.066541
Φ	−0.001455	−0.002988
σ	0.063793	0.151281

Low and high risk rate of return series were obtained by using the ratios of the assets given in the Table2. These series are modeled with AR (1) process and parameters of the models are found. With these parameters, the rate of returns of the following periods, obtained stochastically with the AR (1) process, were estimated for the premium calculation.

In order to generate future rates from the AR(1) process, an initial rate of return is needed to find out the ones in the following years. Thus, these initial rates are the last rates of return that are obtained using the ratios the portfolios include.

JOINT SURVIVAL RATES

Before presenting the net single premiums, it is useful to show how joint survival rates changing over time. Figure 1 allows us to understand the change of survival rates under different circumstances. Here the spouses are assumed to be alive at the beginning of 65 years old and the last age of the life table is 120.

The enhancing effect of longevity on survival rates can be clearly seen as static survival rates are lower than the dynamic survival rates in both the joint life and the survivorship situation. The dependency assumption affects two types of annuity differently. For the joint life status, the dependent survival rates are higher than the independent

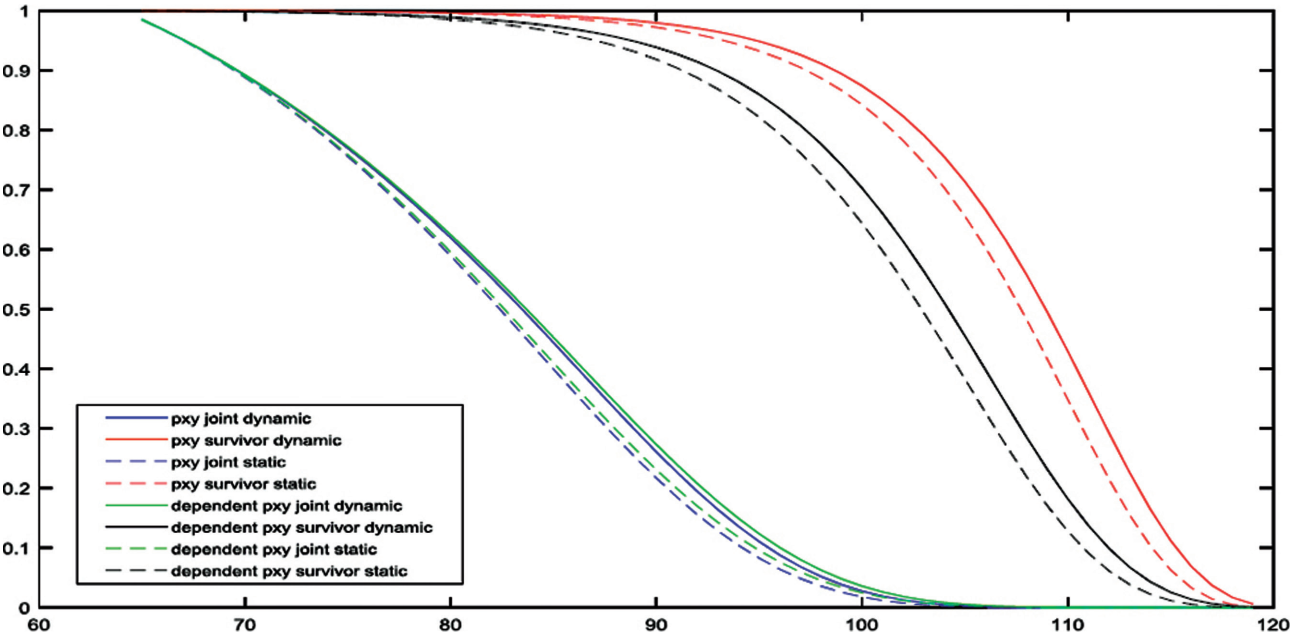

Figure 1. Survival Rates of 65 years old spouses over time.

Table 4. Net single premium of joint life whole annuity without the longevity risk

	Independent				*Dependent*			
	Stochastic		*Deterministic*		*Stochastic*		*Deterministic*	
	Low Risky	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**	**Low Risky**	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**
PV	12.1087	11.2588	11.6773	9.3387	12.2254	11.3646	11.7831	9.3980
Std. Dev.	5.4533	6.6202	4.7444	3.3371	5.5341	6.7368	4.8060	3.3650

survival rates. This is because, in the case of positive dependency, individuals increase each other's chances of living when they live together. On the other hand, in survivorship status, this yields the opposite result that is the dependent survival rates are lower than the independent survival rates. The reason for this is that in case of positive dependency, the death of one of the individuals, decreases the survival rate of the other and this results in a lower joint survival rate.

RESULTS AND DISCUSSION

In this section, we present the numerical results of the analysis based on the methodology developed so far. Table 4 and Table 5 show the net single premiums calculated with static survival rates, different rates of return and dependency assumptions. It is seen that the premiums calculated with the stochastic return rates labelled as low and high risk in the tables are higher in all cases than the ones calculated with the deterministic return rates indicated in the tables as the mean. As fluctuation of rate of return rises, the standard deviation increases. Also, it is obvious that when the rate of return increases, present value (PV) decreases. Under the dependency assumption, net single premiums are higher than the premiums calculated under the assumption of independency. This is because of the higher dependent joint survival rates of spouses. The distinct result is the higher premiums of survivorship annuity. Survivorship annuity continues to the last death thus there are more payments instead of the joint life annuity has. However, as mentioned before lower dependent survival rates in the survivorship status causes lower premiums than the ones calculated with independent survival rates.

Table 6 and Table 7 include the calculations made using the dynamic life table. There are similar impacts of stochastic rate of return and dependency assumption to

Table 5. Net single premium of survivorship whole annuity without the longevity risk

	Independent				*Dependent*			
	Stochastic		*Deterministic*		*Stochastic*		*Deterministic*	
	Low Risky	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**	**Low Risky**	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**
PV	21.1986	18.7558	19.9348	13.8911	19.9535	17.8103	18.8482	13.4400
Std. Dev.	4.6501	9.5561	1.6799	0.7636	4.6287	8.9152	2.2487	1.1324

Table 6. Net single premium of joint life whole annuity under the longevity risk

	Independent				*Dependent*			
	Stochastic		*Deterministic*		*Stochastic*		*Deterministic*	
	Low Risky	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**	**Low Risky**	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**
PV	12.4903	11.5924	12.0301	9.5489	12.6058	11.6898	12.1366	9.6098
Std. Dev.	5.6158	6.8775	4.8643	3.3931	5.6744	6.9434	4.9088	3.4081

Table 7. Net single premium of survivorship whole annuity under the longevity risk

	Independent				*Dependent*			
	Stochastic		*Deterministic*		*Stochastic*		*Deterministic*	
	Low Risky	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**	**Low Risky**	**High Risky**	**Low Mean Rate of Return**	**High Mean Rate of Return**
PV	21..4877	18.9656	20.1813	13.9829	20.3199	18.0912	19.1694	13.5765
Std. Dev.	4.6927	9.7207	1.5815	0.7052	4.6428	9.1130	2.1194	1.0479

Table 8. Effect of longevity risk on net single premiums

			Static		***Dynamic***		***Increase in Joint Life (%)***	***Increase in Survivorship (%)***
			Joint Life	***Survivorship***	***Joint Life***	***Survivorship***		
INDEPENDENT	*Stochastic*	*Low*	12.1087	21.1986	12.4903	21.4877	3.1515	1.3638
		High	11.2588	18.7558	11.5924	18.9656	2.9630	1.1186
	Deterministic	*Low*	11.6773	19.9348	12.0301	20.1813	3.0212	1.2365
		High	9.3387	13.8911	9.5489	13.9829	2.2508	0.6609
DEPENDENT	*Stochastic*	*Low*	12.2254	19.9535	12.6058	20.3199	3.1116	1.8363
		High	11.3646	17.8103	11.6898	18.0912	2.8615	1.5772
	Deterministic	*Low*	11.7831	18.8482	12.1366	19.1694	3.0001	1.7041
		High	9.398	13.44	9.6098	13.5765	2.2537	1.0156

all premiums calculated. One can see that premiums calculated with dynamic survival rates are higher than the ones calculated with static survival rates. It is obvious that while the probability of being alive rises the probability of payments of an annuity increases.

Table 8 contains the percentage of increase for both types of annuities to measure the effect of longevity. They show how much the risk of longevity increases the net single premiums of the annuity. The effects of the dependency assumptions and stochastic rate of returns on the net single

Table 9. Comparison of net single premiums

		Static-Deterministic-Independent	Dynamic-Stochastic-Dependent	Increase (%)
Joint Life	Low	11.6773	12.6058	7.9513
	High	9.3387	11.6898	25.1759
Survivorship	Low	19.9348	20.3199	1.9318
	High	13.8911	18.0912	30.2359

premium have been previously mentioned. In the survivorship whole life annuity, the effect of longevity is lower than the joint whole life annuity. The reason for this is that joint life whole annuity has a shorter period of payments compared to survivorship whole life annuity. As the payment period is extended, the effect of longevity is reduced.

Net single premiums are traditionally calculated using a fixed rate of return and constant survival rates with the assumption of independence between individuals' future lifetimes. The premiums calculated on the basis of the assumptions mentioned in this study, however, enables us to make a better comparison. In Table 9, premiums calculated using opposite rates and assumptions for both types of annuities, as well as changes in percentages are shown.

For the joint life annuity, it has been seen that the net single premium, calculated under the assumption of the dependency between the couples' future lifetimes using the dynamic life table and stochastic rate of return, is approximately 8 percent higher for the low rate of return and approximately 25 percent higher for the high rate of return, compared to the premium calculated with the traditional method. In the survivorship annuity, this rate is approximately 2 and 30 percent respectively. This calculation made only for one couple reveals a serious difference in both types of annuities.

CONCLUSION

This study examined the net single premiums of multiple life annuities in a stochastic environment. On the contrary of traditional studies, in order to see the longevity effect, we used dynamic life table and as a result of a declining mortality pattern, net single premiums were found higher than the ones calculated using a traditional life table. This study shows that when the volatility of the rate of return, the dependence of the individuals' lifetimes and the increase in the expected future lifetimes are not considered, insurance companies may face the risk of calculating the net single premiums of the multiple life annuities lower than expected.

The aim of this study was to calculate the net single premiums of multiple life annuities with a more realistic approach. Considering the dependency between lifetimes of individuals, the variability of the rate of return and the decrease in mortality rates in the future, we have shown that there is a significant difference in the calculations compared to the traditional method. It is believed that this study may be useful for insurance companies which deals with the multiple life annuity products.

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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