International Journal of Technological Sciences, 14(3), 153-162, 2022 DOI: 10.55974/utbd.1130198

Uluslararası Teknolojik Bilimler Dergisi

International Journal of Technological Sciences

UTBD
IJTS

Research Article/Orijinal Makale

Trajectory tracking performance comparison of kinematic bicycle model with LQR and Lyapunov-based controllers on a circular trajectory

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Keywords
Wheeled mobile robot
LQR
Lyapunov
Kinematic bicycle model
Trajectory tracking

Article history:
Received: 13.06.2022
Accepted: 31.12.2022

Abstract: This paper focuses on comparative results of two different controllers applied to kinematic bicycle model with rear wheel contact point to the ground as the reference point. The wide range of representation of different types of robots and vehicles of kinematic bicycle model is the main reason for this model selection. This paper has three main sections. The first section of the paper is mathematical modeling of the model. The second section is describing the utilized control techniques. The last section shares results of the simulations. The simulations have been carried out with pure feedback signals in absence of noise. The compared two controllers are an (Linear Quadratic Regulator)LQR controller and a Lyapunov based controller. The objective in the simulations is to track and complete a given constant radius trajectory. Last section includes comparison of results by analyzing statistical values of a defined error signal.

To Cite/Atıf için: Ceyhun HE. Gören A. Trajectory tracking performance comparison of kinematic bicycle model with LQR and Lyapunov-based controllers on a circular trajectory. International Journal of Technological Sciences, 14(3), 153-162, 2022.

LQR ve Lyapunov temelli iki kontrolcünün kinematik bisiklet tipi model için güzergah izleme performansının dairesel bir güzergahta karşılaştırması

Anahtar Kelimeler
Mobil robot
LQR
Lyapunov
Kinematik bisiklet modeli
Güzergah takibi

Makale geçmişi:
Geliş Tarihi: 13.06.2022
Kabul Tarihi: 31.12.2022

Öz: Bu makale farklı kontrol tekniklerinin arka tekerleğinin yere temas noktasını referans alan kinematik bisiklet tipi robot üzerine uygulamasının karşılaştırmalı sonuçlarına odaklanmıştır. Kinematik bisiklet tipi modelin seçilmesinin ardında yatan asıl sebep, bu modelin günümüzde kullanılan birçok robot modelini ve aracı temsil edebilmesidir. Çalışma üç ana bölümden oluşmaktadır. İlk bölüm robotun modellenmesini içermekte. İkinci bölüm uygulanan kontrol teknikleri ile ilgili çalışmaları barındırmakta. Son bölüm ise simülasyon sonuçlarını paylaşmaktadır. Kullanılan sinyaller gürültüsüz ortamda bir geribesleme sinyali için oluşturulmuştur. Karşılaştırılan kontrolcüler (Linear Quadratic Regulator)LQR ve bir Lyapunov temelli kontrolcülerdir. Simülasyonların ana amacı modelin verilen sabit yarıçaplı güzergahı takip ederek tamamlamasıdır. Son bölüm içerisinde tanımlanan hata sinyalinin istatistiksel olarak değerlendirmesini içerir.

1. Introduction

Nowadays, the achievements in mobile robotics allow advanced applications in real life. One of the popular tasks for autonomous and mobile robots is trajectory tracking. This task requires mathematical model of the mobile robot and a proper control approach to minimize the total deviation from a desired trajectory.

In this research, the main aspect is to investigate the control performances of two different controllers which have been designed for a kinematic bicycle type wheeled mobile robot (WMR). The rear tire contact point assumed as a reference to accomplish trajectory tracking task. Linear Quadratic Regulator (LQR) and Lyapunov based controllers are the applied control

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approaches in this paper. MATLAB/Simulink environment has been used to simulate the trajectory tracking performances of the WMR.
Robotic motion, kinematic models and well-known control techniques are presented in many references. While [1-3], [6] and [16] are covering most of these topics, other references such as [5] is concentrated on nonlinear control techniques and [4] is scoping Lyapunov stability and regulation problem.

We can see that, using a linear controller is a practical approach to the problem as stated in [7]. This paper uses a differential drive type robot. Then subjects it to a coordinate transformation by using reference frames and input transformation. PI tuning with LQR is realized in 3-dimensional state space system. In [8] a dynamic model of bicycle type robot is used together with its kinematic model. By this manner, a Lyapunov function is used for control. Stability through Barbalat condition is studied.

The kinematic model and linearization through input transformation is validated and a control scheme shown in [9]. This paper also uses a differential drive robot in 3-dimensional error system linearized around origin. Linear feedback control is proposed with pole placement approach to regulate the desired polynomials.

In [10] a 3-dimensional state space system is considered under Lyapunov conditions. Dynamic model is utilized for the related study. [11] follows a similar approach for a welding application combined with a wheeled mobile robot control.

In [12] a differential drive model is the subject of feedback linearizing control to accomplish “Follow the Carrot” algorithm by replacing system nonlinearities with desired dynamics. Position and orientation error form the error system. [17] and [18] are detailed and explanatory sources for optimal control theory.

An inclusive reference for path tracking methods of autonomously steering vehicles is [13]. Dynamic and kinematic models are shown together with geometric tracking algorithms such as Pure Pursuit and Stanley Control. Kinematic and optimal control including LQR form are also investigated deeply.

[14] considers center of wheelbase as the reference point of the kinematic model. Therefore, another 3-dimensional approach is made by LQR-LMI tuning for a Lyapunov based controller.

[15] Both forward and backward motions are considered for tractor trailer wheeled robot. Lyapunov based approach is stated for different types of trajectories. Slip condition is also considered.

Instead of taking statically balanced models [19] takes an e-scooter model and investigates a higher dimensional state space system. This new system also takes self-stabilization as an additional task other than motion control.

[20] takes even larger state space system with 12 degrees of freedom. Tracking problem simulations are done for PD and H_∞ controllers then tested for PD controller.

A time-varying LQR control is applied in [22] for tracking task. This paper uses a 4-dimensional state space system as in our study.

Contributions to WMR studies in “Automatic Control and Robotics Laboratory” of Dokuz Eylül University focus on linear and non-linear controller design, navigation, sensor fusion and implementations on experimental models. [23] realizes a path following task for autonomous vehicles by GPS measurement. [24] considers relative positioning of a robot together with trajectory tracking and obstacle avoidance task. Sensory measurement and indoor mapping by a LIDAR are researched deeply in order to ensure autonomous navigation in [25]. Alongside traditional approaches, a neural network-based control approach is applied to a non-holonomic model using multiple sensors in [26].

As another perspective, [27] takes the standard form of a PI controller and combine it with a nonlinear gain and control a second order system. Stability analysis is done for different criteria. Then all these considerations are subjected to a higher order system which is the active suspension system. Reference tracking performances are evaluated for different system parameter settings. On the other hand [28] suggests a hybrid control approach by including a neural network to the tuning part. Then tracking performances are evaluated and compared for regular PD, Anfis and a hybrid controller constructed by both.

1.1. Problem Definition

Our work, unlike many applications, takes a four-dimensional state space for kinematic bicycle model. Considered model uses reference as rear wheel center. Reference inputs are longitudinal velocity and rate of change in steering angle. This consideration allows us to model many of the real-life robots and vehicles. Therefore, this paper can be considered as a case study for a wide range autonomous wheeled mobile robots and a large class of vehicle family. Cars, busses, motorcycles, scooters can be given as examples of this model.

Moreover, the problem of tracking for this type of model is taken as the studied topic to test the tracking controller performances. As stated above, the model is a representative of frequently encountered vehicle families and therefore tracking control for these types of vehicles becomes vital for the futuristic autonomous driving scenarios. This paper searches the less deviating tracking control for that matter.

2. Mathematical Modeling

The rear wheel reference kinematic bicycle model is a very common model for a family of wheeled mobile robots. Equation 1 show the states for this type of model. First two states x and y are taken as planar position information. Next states are orientation angle of the robot/vehicle and steering angle. They are shown with θ and φ respectively. As a result, a 4-dimensional state vector is created.

$$p = \begin{pmatrix} x \\ y \\ \theta \\ \varphi \end{pmatrix} \quad (1)$$

The main principle is to apply a non-holonomic constraint for two wheels. The wheels are assumed not sliding sideways and motion is obtained only by rolling. This assumption results two constraints for two wheels.

In Figure 1, coordinates of the robot can be seen as a vector of rear wheel cartesian coordinates, front wheel cartesian coordinates, orientation angle of whole body and steering angle. This is a redundant representation of this robot. Because with a constant wheelbase "L" it is possible to rewrite one wheel's coordinates by using the other one as in Equation 2 and 3.

$$x_{front} = x_{rear} + L\cos(\theta) \quad (2)$$

$$y_{front} = y_{rear} + L\sin(\theta) \quad (3)$$

As can be seen in Equation 4 and 5, by derivation of these equations with respect to time, evolutions of states in time are obtained.

$$\dot{x}_{front} = \dot{x}_{rear} - \dot{\theta}L\sin(\theta) \quad (4)$$
$$\dot{y}_{front} = \dot{y}_{rear} + \dot{\theta}L\cos(\theta) \quad (5)$$

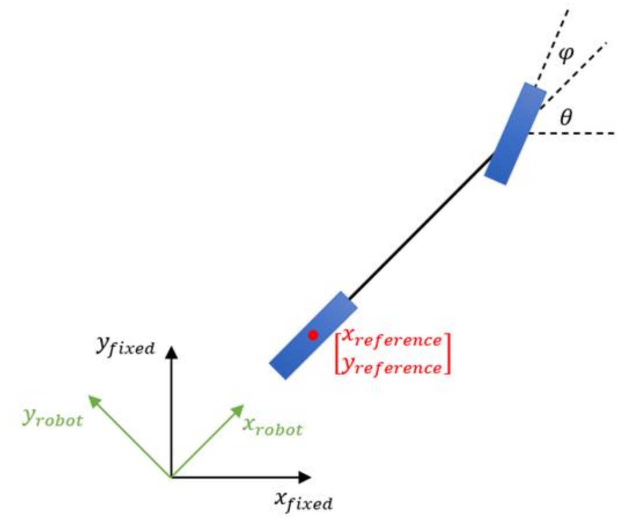

Figure 1. Kinematic Rear Wheel Reference Bicycle Model

If this new representation is used for writing the wheel constraints equation, the input vector fields can be obtained by Equation 6 and 7.

$$\dot{x}_{rear}\sin(\theta) - \dot{y}_{rear}\cos(\theta) = 0 \quad (6)$$
$$\dot{x}_{front}\sin(\theta+\varphi) - \dot{y}_{front}\cos(\theta+\varphi) = 0 \quad (7)$$

It is possible to rewrite this in matrix form. Integrability of the input vector fields will be guaranteed by this way. For simplicity of notation, rear subindices are dropped. Hence, coordinates are changed to a vector containing only one pair of cartesian coordinates of rear wheel and two angular coordinates with orientation angle of the body and steering angle. Matrix form of the constraints on the wheels can be seen in Equation 8.

$$\begin{bmatrix} \sin(\theta) & -\cos(\theta) & 0 & 0 \\ \sin(\theta+\varphi) & -\cos(\theta+\varphi) & -L\cos(\varphi) & 0 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{pmatrix} = 0 \quad (8)$$

The vector fields which satisfy these constraints are found and can be used for determination of input vector fields. Non-holonomy condition is satisfied for this model by a null space solution.

Obtained vectors can be seen below in mathematically defined rear wheel kinematic bicycle model. By this knowledge it is possible to write the rear wheel reference kinematic bicycle model in state space form as in Equation 9.

$$\dot{p} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} cos(\theta) \\ sin(\theta) \\ tan(\varphi)/L \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2 \quad (9)$$

Inputs are chosen to be linear velocity of body and angular velocity (10) as rate of change of steering angle (11) and can be written as:

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} \tag{10}$$
$$w = \dot{\varphi} \tag{11}$$

However, the resulting model is highly nonlinear and a direct linearization by derivation gives a set of equations which do not represent the robot motion. Therefore, a feedback linearization through coordinate transformation is done. Then control approaches are applied for this new system representation. To get this new system, a reference state vector must be defined as in Equation 12. Ref subindices represent reference.

$$p_{ref} = \begin{pmatrix} x_{ref} \\ y_{ref} \\ \theta_{ref} \\ \varphi_{ref} \end{pmatrix} \tag{12}$$

By the measured states, the error vector is obtained and represented in a rotated coordinate system attached to the rear wheel of the kinematic bicycle type model. Full state measurement is assumed to be available. Error system can be defined as in Equation 13.

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_{ref} - x \\ y_{ref} - y \\ \theta_{ref} - \theta \\ \varphi_{ref} - \varphi \end{pmatrix} \tag{13}$$

The transformation matrix in Equation 13 has the form of a planar rotation by the orientation angle of the model. This rotation is applied to x and y coordinates of model. However, angular states are expressed in their natural form. Followed procedure is allowing a definition of an error system which is described on longitudinal and lateral dynamics of the model. By rewriting the reference states like the model's kinematic equations, the error system is rewritten clearly. Equations 14, 15, 16 and 17 show time derivatives of the reference states.

$$\dot{x}_{ref} = v_{ref} \cos(\theta_{ref}) \tag{14}$$
$$\dot{y}_{ref} = v_{ref} sin(\theta_{ref}) \tag{15}$$
$$\dot{\theta}_{ref} = \frac{v_{ref} tan(\varphi_{ref})}{L} \tag{16}$$
$$\dot{\varphi}_{ref} = w_{ref} \tag{17}$$

Thanks to the differential flatness and constant parameters of the model, it is possible to obtain these reference states of the system. However, an additional calculation is necessary to express the curvature of desired trajectory. By this way it is possible to represent reference steering angle change rate and therefore rate of change of steering angle.

Reference orientation of the trajectory is calculated in Equation 18.

$$\theta_{ref} = \tan^{-1}\left(\frac{\dot{y}_{ref}}{\dot{x}_{ref}}\right) \tag{18}$$

Reference linear velocity of the trajectory is calculated in Equation 18.

$$v_{ref} = \sqrt{{\dot{x}_{ref}}^2 + {\dot{y}_{ref}}^2} \tag{19}$$

Reference curvature and instantaneous center of rotation of the trajectory are calculated in Equation 20 and 21.

$$\kappa_{ref} = \frac{\ddot{y}_{ref}\dot{x}_{ref} - \ddot{x}_{ref}\dot{y}_{ref}}{({\dot{x}_{ref}}^2 + {\dot{y}_{ref}}^2)^{3/2}} \tag{20}$$
$$R_{ref} = \frac{1}{\kappa_{ref}} \tag{21}$$

Reference steering angle and its rate of change can be directly found by derivation of Equation 22.

$$\varphi_{ref} = \tan^{-1}\left(\frac{L}{R_{ref}}\right) \tag{22}$$

Where, R_{ref} is the distance between instantaneous center of rotation and rear wheel. L is the wheelbase of the robot. By this way, the rotated error equations are derived for cartesian coordinates. Then, reference expressions are used. Note that angular states are kept the same for error system generation.

Evolution of error states with respect to time can be seen in Equation 23, 24, 25 and 26. This procedure is done by calculating Equation 13 with the terms calculated in Equations 14, 15, 16, 17, 18, 19, 20, 21 and 22.

$$\dot{e}_1 = v_{ref} cos(e_3) - v + \frac{v\tan(\varphi)}{L} e_2 \tag{23}$$
$$\dot{e}_2 = v_{ref} sin(e_3) - \frac{v\tan(\varphi)}{L} e_1 \tag{24}$$
$$\dot{e}_3 = \frac{v_{ref} tan(\varphi_{ref})}{L} - \frac{v\tan(\varphi)}{L} \tag{25}$$
$$\dot{e}_4 = \dot{\varphi}_{ref} - \dot{\varphi} \tag{26}$$

This form still has a highly nonlinear relation. Small angle assumption is made and artificial inputs are introduced by redefining original inputs. Redefined inputs are given in Equation 27, 28 and 29.

$$u_1 = v_{ref} cos(e_3) - v \quad (27)$$
$$u_2 = \dot{e}_3 \quad (28)$$
$$u_3 = \dot{e}_4 \quad (29)$$

New error system can be represented in linear time-invariant form if and only if reference linear speed and reference steering angle are constants. This is shown in Equation 30.

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\\ \dot{e}_4\end{bmatrix} = \begin{bmatrix} 0 & \frac{v_{ref} tan(\varphi_{ref})}{L} & 0 & 0 \\ -\frac{v_{ref} tan(\varphi_{ref})}{L} & 0 & v_{ref} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1\\ e_2\\ e_3\\ e_4\end{bmatrix} + \begin{bmatrix} 1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} \quad (30)$$

To avoid dynamic effects which can occur on tires and therefore on lateral motion, motion of the model has been investigated using relatively smaller velocities. In this work a constant radius circular path is considered and details are presented in next sections.

3. Control Methods

In this study two control approaches are applied and simulation results are compared. These are Linear Quadratic Regulator and a Lyapunov based controller.

3.1. Linear Quadratic Regulator (LQR)

In LQR design, linear quadratic cost function is used and shown in Equation 31.

$$J = \int (x^T Q x + u^T R u) dt \quad (31)$$

Where Q and R matrices are diagonal weights matrices for the states and inputs. In this case, states correspond to errors and inputs are the artificial ones shown in Equations 27, 28 and 29. The feedback is obtained in a static manner. Thus, standard form can be rewritten as in Equation 32.

$$J = \int (e^T Q e + u^T R u) dt \quad (32)$$

In this research, the inputs are not penalized but states are weighted differently. State and input weights are expressed in Equation 33 and 34. The selection of weights are done by inspecting the Equations 27, 28, 29 and 30. Since e_3 and e_4 represent the error on angular states in the original system and the rotation is done by the orientation angle itself, more weights are given to those states. Another supporting observation can be pointed out by the fact that e_1 and e_2 are highly dependent on both angular error states. Input weighting is not done and all virtual inputs are taken with the same weights. Therefore, Equations 32 and 33 would be a viable selection for weights matrices.

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix} \quad (33)$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (34)$$

Free system has two eigenvalues at origin and two complex eigenvalues at 0±0.6283i. LQR strategy places the eigenvalues to -31.6228, -31.6212, -2.9531, -0.7670. Corresponding gain matrix can be seen in Equation 35.

$$K = \begin{bmatrix} 3.5604 & -2.1689 & -0.2213 & 0 \\ -0.2213 & 1.6032 & 31.7809 & 0 \\ 0 & 0 & 0 & 31.6228 \end{bmatrix} \quad (35)$$

3.2. Lyapunov Based Controller

Lyapunov functions are widely used in many fields of nonlinear control application. This paper follows a control approach with a storage function which is semi-positive definite for any time while its derivative is negative semi-definite for any time instant. At the origin the storage function is equal to zero. Considered storage function is constructed by quadratic terms and a trigonometric term. Main reason behind this selection of structure is to stress out the property of boundaries of each function type. While quadratic functions have a lower bound at zero, trigonometric functions are defined in the range of ±1. Storage function and its derivative can be found in Equation 36 and 37.

$$V = \frac{1}{2}({e_1}^2 + {e_2}^2 + {e_4}^2) + \frac{1 - cos(e_3)}{k_2} \quad (36)$$

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_4 \dot{e}_4 + \frac{sin(e_3)}{k_2} \dot{e}_3 \quad (37)$$

For this approach, error system can be rewritten by releasing the small angle approximation and keeping the input transformation the same. A non-linear system is defined in Equation 38, 39, 40 and 41.

$$\dot{e}_1 = u_1 + \left(\frac{v_{ref} tan(\varphi_d)}{l} - u_2\right) e_2 \quad (38)$$

$$\dot{e}_2 = v_{ref} sin(e_3) - \left(\frac{v_{ref} tan(\varphi_d)}{l} - u_2\right) e_1 \quad (39)$$

$$\dot{e}_3 = u_2 \quad (40)$$
$$\dot{e}_4 = u_3 \quad (41)$$

Finally, the Lyapunov criteria are checked and the controller performance is simulated. In order to have a negative semi-definite derivative for storage function, following feedback terms in Equation 42, 43 and 44 are used.

$$u_1 = -k_1 e_1 \quad (42)$$
$$u_2 = -k_2 v_{ref} e_2 \quad (43)$$
$$u_3 = -k_3 e_4 \quad (44)$$

After adding the error system and feedback terms together, the resultant derivative function became negative semi-definite for all values of states. This operation is shown in Equation 45 and its simplified version in 46.

$$\dot{V} = -k_1 e_1^2 + e_2 v_{ref} sin(e_3) + \frac{sin(e_3)}{k_2}(-k_2 v_{ref} e_2) - k_3 e_4^2 \quad (45)$$
$$\dot{V} = -k_1 e_1^2 - k_3 e_4^2 \quad (46)$$

Controller gains are selected as follows in Equation 47, 48 and 49.

$$k_1 = 40 \quad (47)$$
$$k_2 = 40 \quad (48)$$
$$k_3 = 50 \quad (49)$$

Note that any positive value of Lyapunov gains will result a satisfactory result for Lyapunov Stability Theorem. Controller gains are selected by trial and error. Starting from a small positive value and slowly increasing each value resulted these gains. Greater values would result more aggressive results.

4. Simulation Results and Discussion

MATLAB/Simulink environment is utilized to simulate the trajectory and the motion of the WMR. The selected task is to follow a circular reference trajectory with a diameter of 10 meters in 10 seconds. Therefore, the reference linear and angular speed inputs are constant π m/s and 0.2π rad/s. Necessary references can be obtained directly. These references are expressed in Equation 50 and 51.

$$x(t) = 5\cos\left(\frac{\pi}{5}t\right) \quad (50)$$
$$y(t) = 5\sin\left(\frac{\pi}{5}t\right) \quad (51)$$

The single important parameter of bicycle is its wheelbase of 1.5 meters. This parameter selected by considering an average bicycle/scooter. Steering angle is saturated in ±1.07 radians.

Initial conditions are picked the same for both cases as shown in Equation 52. Which is the starting point on the trajectory. The goal is to follow the exact trajectory.

$$p_{initial} = \begin{bmatrix} 5 & 0 & \frac{\pi}{2} & 0 \end{bmatrix}^T \quad (52)$$

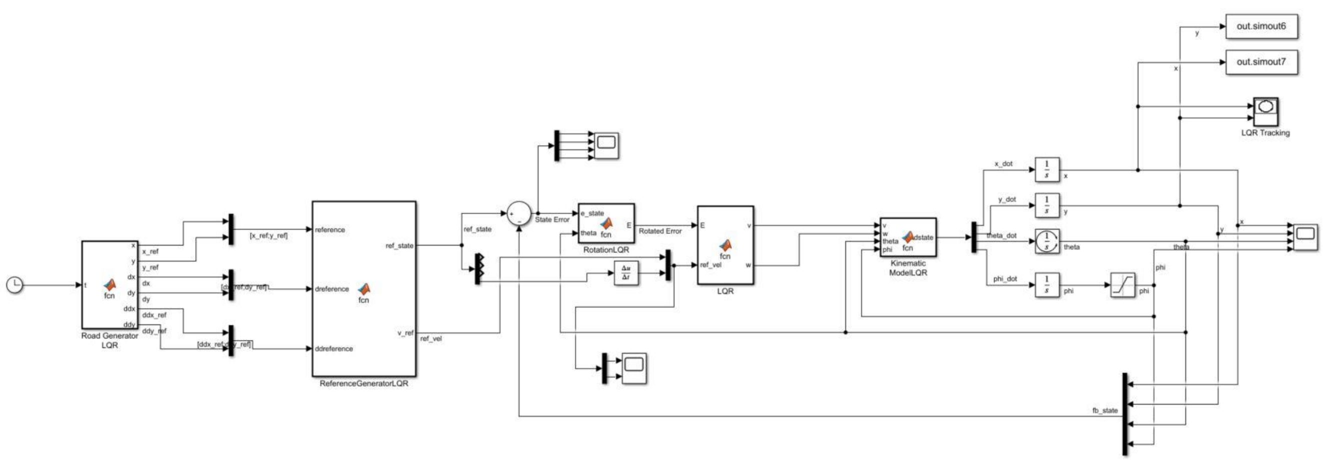

Figure 2. LQR Control Scheme

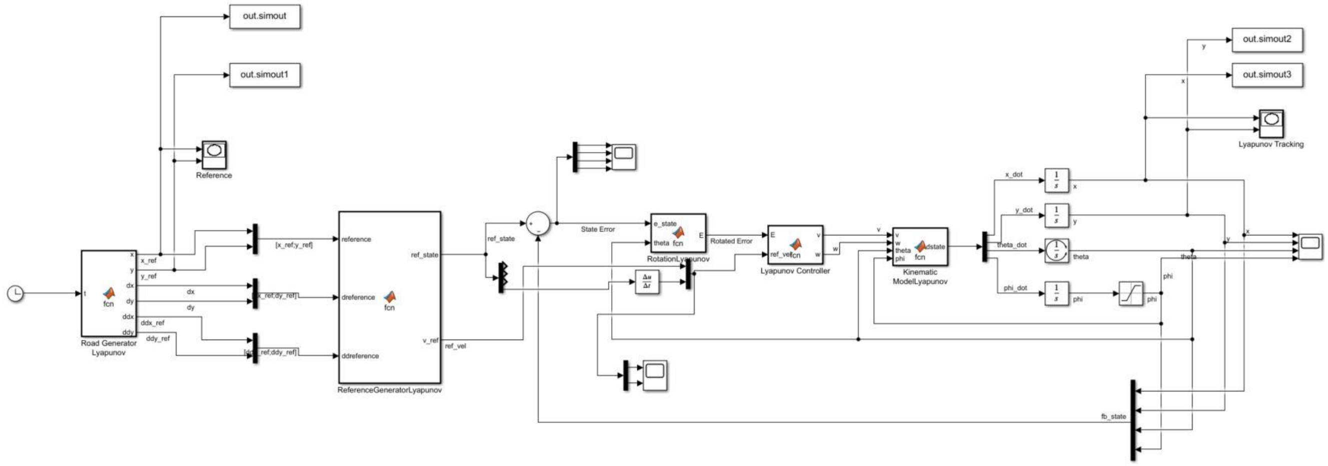

Figure 3. Lyapunov Control Scheme

Figure 2 shows the Simulink scheme of kinematic bicycle model controlled via LQR technique. A similar block scheme is created for Lyapunov based control technique in Simulink environment and can be seen in Figure 3.

Simulations are done for 10 seconds with fixed step of 0.001 seconds. Solver selection is ODE4(Runge-Kutta). Results are plotted together with the reference trajectory and can be seen in Figure 4.

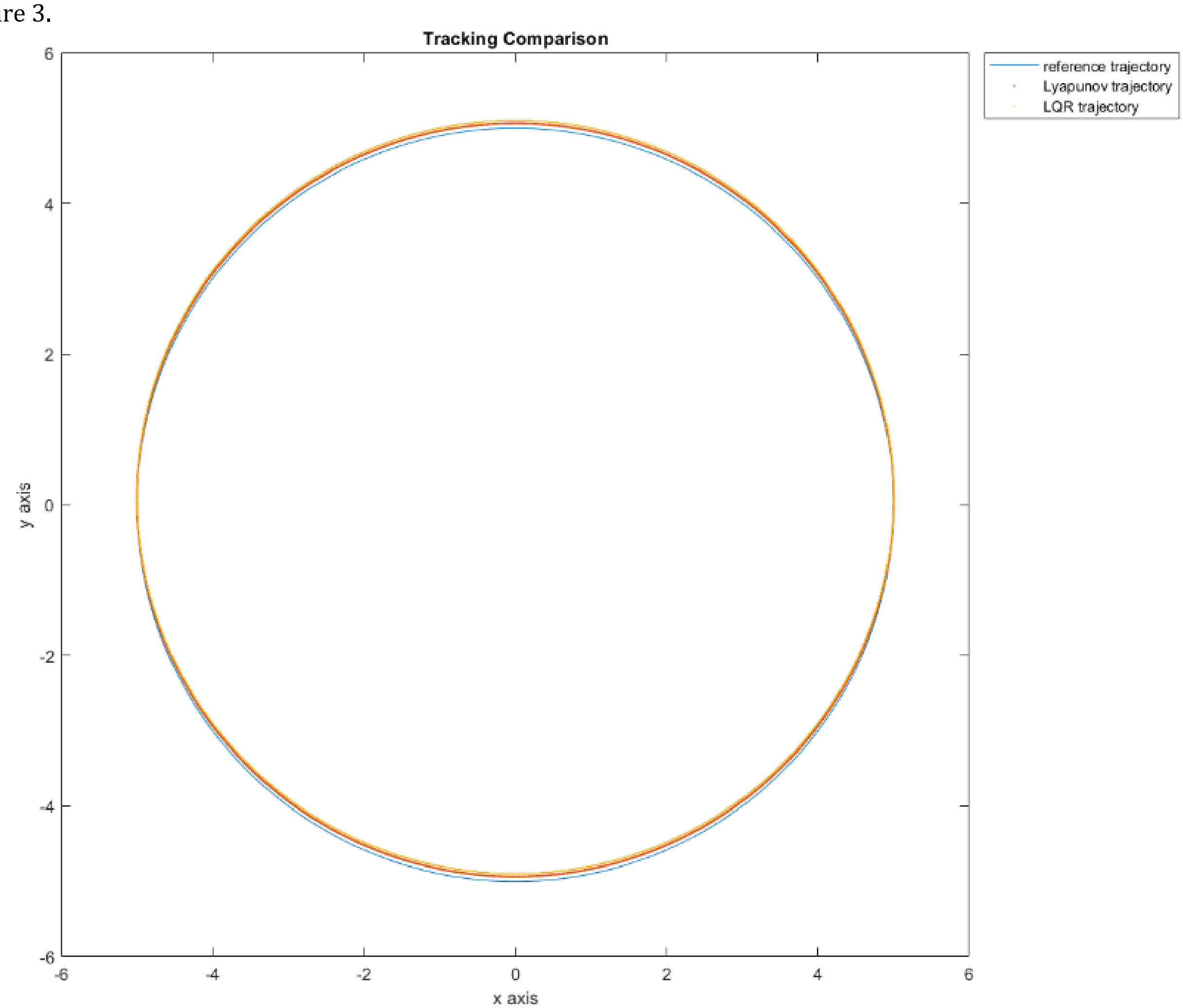

Figure 4. Tracking Comparison of Two Controllers

Table 1. Performance Comparison Through Indicators

Indicator	Lyapunov Based Controller	Linear Quadratic Regulator (LQR)
Cumulative Deviation	4.5506 m	9.0552 m
Mean Deviation on x-axis	-3.0346×10^{-4} m	-0.0378 m
Mean Deviation on y-axis	-0.0322 m	-0.0570 m
Variance of Deviation on x-axis	5.1747×10^{-4} m	0.0017 m
Variance of Deviation on y-axis	5.1758×10^{-4} m	0.0018 m

From first inspection of Figure 4 it is hard to evaluate performance of both control approaches. Therefore, a deviation metric is defined and considered as performance indicator. This deviation metric is measuring distance between reference trajectory point and kinematic bicycle model's rear wheel contact point to the ground and the performances of both controllers can be seen in Table 1. Deviation metric is expressed in Equation 53.

$$d = \sqrt{(x_r - x)^2 + (y_r - y)^2} \quad (53)$$

For evaluation of performance, deviation from trajectory on each time step is collected. Then, cumulative deviation from reference trajectory is calculated together with mean and variance on x and y axes. These values are shared to the reader in previous table. Evolution of deviations in both axes for each control strategy can be seen in Figure 5.

Table 1 and Figure 5 give the meaningful performance differences. Importance of every single indicator can be clarified. First indicator is the cumulative deviation. Since Equation 53 collects the deviation value from reference trajectory for every time step, summed deviation is the most important indicator of the tracking performance. As this indicator goes to zero, performance is to be said get better. However, this indicator is not the single performance indicator because of time dependency of this deviation. As shown in Figure 5, the deviation from reference trajectory changes over time along both axes of the cartesian coordinate system. Minimized variation around zero deviation from reference trajectory is the best expression of the performance analysis. Therefore, minimum deviation from reference trajectory can be defined statistically.

Both control techniques have very small mean deviations on both cartesian coordinate axes. However, variances around this mean value separates two scenarios. In Figure 5 this consideration becomes more visible. While oscillations around zero are similar in terms of frequency, their amplitude show how cumulative error gets greater for LQR control approach.

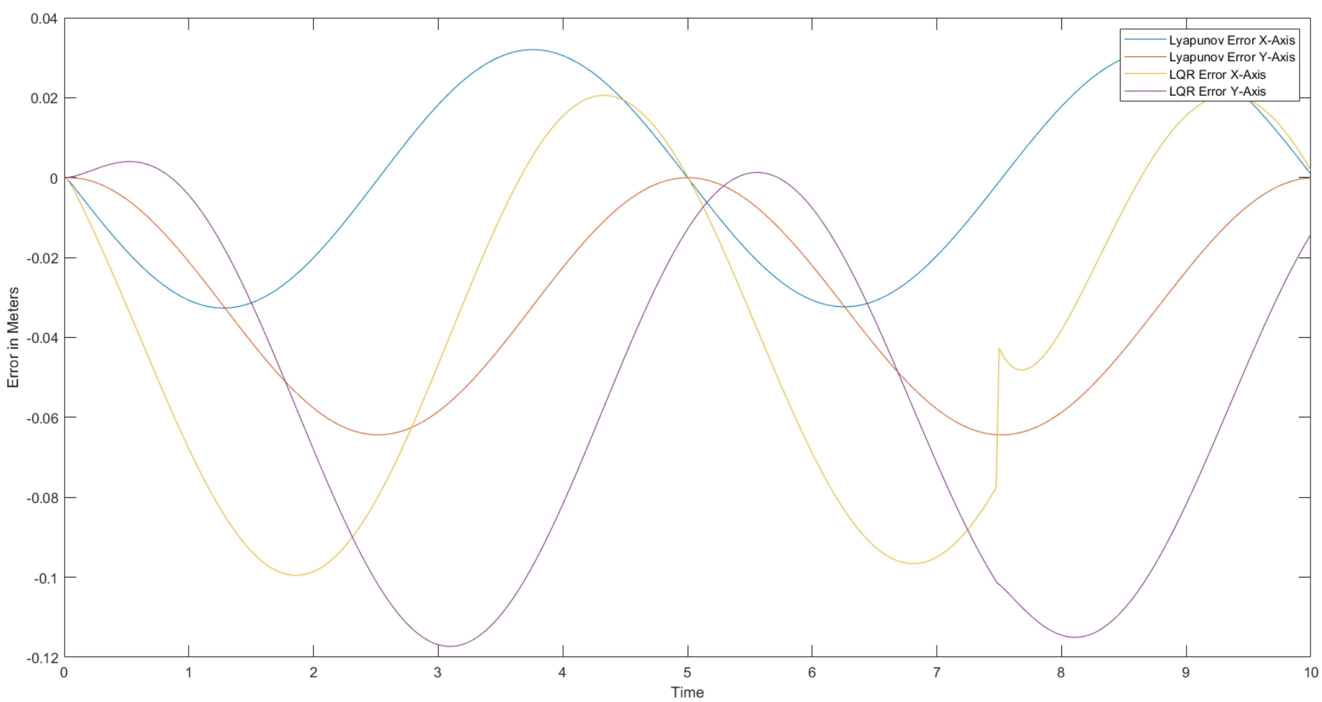

Figure 5. Tracking Deviations on Each Axis for Both Controllers

Another perspective which is compared in this paper is the performance difference of a linear and a nonlinear controller approach on kinematic bicycle model. Both techniques are well known and widely used approaches. Lyapunov based controller action needs an additional term other than the states of the system which is the reference speed input. However, LQR only utilizes a predefined gain matrix for a simpler feedback action. This situation gives another conclusion for the compared control approaches. Since Lyapunov based control action utilizes reference linear velocity in its feedback calculation, it is possible to define a time-varying positive term and apply this controller for time dependent cases. To be more clear, this controller would be able to track varying radius trajectories. On the other hand, LQR control strategy would not be able to calculate such a feedback matrix because of variable system matrix entities.

4. Conclusion

In this research two different types of control strategies are derived and applied to kinematic bicycle model. Control strategies rely on Linear Quadratic Regulator and Lyapunov Stability Theorem. For both control strategies, stability is searched and guaranteed by designed feedback actions. Reference trajectory was a circle with radius of 5 meters, which had to be completed in 10 seconds. Reference speed inputs were decided to be constants. Kinematic bicycle model was able to follow the reference trajectory with small deviations for suggested control settings. Performance comparison is made and seen that Lyapunov based control strategy performed a better tracking result. Future work can be done for obstacle avoidance and globally asymptotically controllers. As a future work, for testing the performance of the developed controller in real environment, an experimental study on a wheeled robot model is planned. Noise cancellation, sensor measurement and data filtering would be another challenging task for realization.

It is evident that Lyapunov based control strategy performed better than LQR approach. Both techniques have at least a mean deviation of order 10^{-2} from reference trajectory. Hence, both strategies are resulting reliably good performance. Moreover, LQR strategy should have a better performance under noisy measurement application or uncertainties due to its lower dependency on variables. Future studies can be constructed under this idea.

However, both techniques need to be applied around reference trajectory and a preliminary system matrix analysis and design is necessary. Moreover, an obstacle avoidance strategy and a global planner must be added before deploying such controller. By this way it would be possible to get a global control approach.

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