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Modeling Love with 4D Dynamical System

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ABSTRACT Dynamic modeling of romantic relationships explains the development of love/hate feelings between two people over time with a system of differential equations. Rather than postulating an individual's emotions as a one-component feeling of love, this study assumed two-component feelings of intimacy and passion. As a result of this assumption, relationship dynamics are represented by a four-dimensional system of equations. The possible outcomes of this new 4D model were compared with the results of the classical 2D model and it was seen that they could give very different outputs. In addition, situations that cannot be explained by classical models such as the end of passion in long-term relationships, relationships that turn from friendship to love, and the reunion of couples after separation are interpreted.

KEYWORDS
Dynamics of love
Fixed points
Human behaviour
Linear systems
Mathematical sociology

INTRODUCTION

The dynamic systems approach has been used for many years in applied mathematics and physics to model the behavior of many physical systems, such as the motion of planets (position and velocity), mass-spring systems, oscillators, and electrical circuits. Afterward, it expanded its field of use in the field of engineering and gained a respectable place. An example of this is the solution of flow models, which are in the form of partial differential equations, by converting them to systems of ordinary differential equations (Shah *et al.* 2022) (Shahzad *et al.* 2023) (Bilal *et al.* 2021) (Qureshi *et al.* 2022). In the last few decades, its use has become widespread in disciplines such as biology, economics, and psychology (Richardson *et al.* 2014). This study primarily scanned the dynamic modeling efforts of the study of romantic relationships in the literature, which are included in the field of psychology, and aimed to make psychological theories and dynamical approaches consistent with each other.

Psychologists have developed various theories to explain love. According to Rubin, liking and loving are separate emotions. He defines liking with feelings such as being appreciated, admired, enjoying, spending time, and wanting to be with the partner. He defines love as a more intense emotion with strong desires for physical contact and intimacy (Rubin 1970). The color wheel model was developed by Lee in 1973 and according to Lee, there are three main styles of love. These are EROS with physical and emotional passion, LUDUS with playful style, and STORGE, which combines family love with friendship (Lee *et al.* 1988). In 1987, Hazan and Shaver put forward the attachment theory, three styles of adult attachment; she described them as "anxious-indecisive" who fears that her partner does not love her, "avoidant" who has difficulty developing trust, and "safe" who has no fear of abandonment (Hazan and Shaver 2017). According to Hatfield, there are two basic types of love, compassionate and passionate. While mutual trust and respect are at the forefront of tender love, deep feelings and sexual attraction are at the forefront of passionate love (Hatfield *et al.* 1988).

One of the most popular love theories is the triangular love theory developed by Sternberg in 1986. Three basic components of love in this theory are named intimacy, passion, and commitment. Intimacy encompasses the emotions that lead to the experience of warmth in a loving relationship. Passion represents emotions that lead to sexual attraction and romance. Finally, the commitment component describes the determination to maintain the relationship for a long time (Sternberg 1986).

There are also psychological studies on the relationship between intimacy and passion. Baumeister and Bratslavsky reported that passion is a function of the change in intimacy, hence the time derivative of intimacy, and that there may be a positive or negative correlation between passion and intimacy within the same relationship(Baumeister and Bratslavsky 1999). In addition, Aykutoğlu and Uysal state that they found evidence for the existence of a relationship between intimacy and passion, and that physical attraction is effective on this relationship (Aykutoğlu 2015) (Aykutoğlu and Uysal 2017).

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Table 1 Several models expressing the love dynamics between couples or love-triangle among triples.

1	$\dot{x}_1 = a_1 x_1(t) + b_1 x_2(t)$, $\dot{x}_2 = b_2 x_1(t) + a_2 x_2(t)$.	(Sprott 2004), (Erbaş 2022) and (Sunday *et al.* 2012)
2	$\dot{x}_1 = -\alpha_1 x_1(t) + \beta_1 x_2(t) + \gamma_1 A_2$, $\dot{x}_2 = -\alpha_2 x_2(t) + \beta_2 x_1(t) + \gamma_2 A_1$.	(Rinaldi 1998b) and (Bielczyk *et al.* 2012)
3	$\dot{x}_1 = -\alpha_{10} x_1(t) + \beta_{10} x_2(t) + F_{10}(t)$, $\dot{x}_2 = -\alpha_{20} x_2(t) + \beta_{20} x_1(t) + F_{20}(t)$.	(Wauer *et al.* 2007) and (Chen *et al.* 2016)
4	$\dot{R}_J = aR_J + b(J-G)$, $\dot{J} = cR_J + dJ$, $\dot{R}_G = aR_G + b(G-J)$, $\dot{G} = eR_G + fG$.	(Sprott 2004)
5	$\dot{R} = aR + bJ(1-\lvert J\rvert)$, $\dot{J} = cR(1-\lvert R\rvert) + dJ$.	(Sprott 2004)
6	$\dot{R} = aR + bJ(1-\lvert J\rvert) + y(t)$, $\dot{J} = cR(1-\lvert R\rvert) + dJ + f(t)$.	(Huang and Bae 2018a)
7	$\dot{x} = R^L(y(t)) + R^A(A)(1 + B(x(t))) - \alpha x(t)$	(Rinaldi and Della Rossa 2020)
8	$\dot{x}_1 = ax_1 + bx_2(1-\varepsilon x_2^2)$, $\dot{x}_2 = dx_2 + cx_1(1-\varepsilon x_1^2)$.	(Barley and Cherif 2011)
9	$\dot{x}_1 = -\alpha_{10} x_1 + \beta_{10}\frac{x_2}{1+\varepsilon_0\lvert x_2\rvert} + F_{10}$, $\dot{x}_2 = -\alpha_{20} x_2 + \beta_{20}\frac{x_1}{1+\varepsilon_0\lvert x_1\rvert} + F_{20}$.	(Wauer *et al.* 2007)
10	$\dot{x}_1 = -\alpha_1 x_1(t) + \rho_1 A_2 + R_1(x_2)$, $\dot{x}_2 = -\alpha_2 x_2(t) + \rho_2 A_1 + R_2(x_1)$.	(Rinaldi *et al.* 2013a), (Rinaldi *et al.* 2010), (Rinaldi *et al.* 2013b), (Liao and Ran 2007), (**?**)
11	$\dot{L} = -\alpha_1 L + \beta_1\left[P(1-(P/\gamma)^2) + A_P\right]$ $\dot{P} = -\alpha_2 P + \beta_2\left[L + \frac{A_L}{1+\delta Z}\right]$ $\dot{Z} = -\alpha_3 Z + \beta_3 P$	(Rinaldi 1998a)
12	$\dot{R}_J = aR_J + b(J-G)\left(1-(J-G)^2\right)$, $\dot{J} = cR_J\left(1-R_J^2\right) + dJ - mJG$, $\dot{R}_G = aR_G + b(G-J)\left(1-(G-J)^2\right)$, $\dot{G} = eR_G(1-R_G^2) + fG - nJG$.	(Liu and Chen 2015)

Table 2 continuation of Table 1

13	$\dot{R}_J = aR_J + b(J-G)(1-\|J-G\|)$, $\dot{J} = cR_J(1-\|R_J\|) + dJ$, $\dot{R}_G = aR_G + b(G-J)(1-\|G-J\|)$, $\dot{G} = eR_G(1-\|R_G\|) + fG$.	(Ahmad and El-Khazali 2007)
14	$D_t^\alpha u(t) = -\rho_1 u(t) + \sigma_1 v(t)\left(1-\epsilon v^2(t)\right) + \varphi_1$ $D_t^\alpha v(t) = -\rho_2 v(t) + \sigma_2 u(t)\left(1-\epsilon u^2(t)\right) + \varphi_2$	(Owolabi 2019), (Goyal *et al.* 2019) and (Ozalp and Koca 2012)
15	$d^\alpha R/dt^\alpha = aR + bJ(1-J) + 5\sin(\pi t)$ $d^\beta J/dt^\beta = cR(1-R) + dJ$	(Huang and Bae 2018b)
16	$d^\alpha R_J/dt^\alpha = aR_J + b\, sgn(J-G)$, $d^\beta J/dt^\beta = c\, sgn(R_J) + dJ$, $d^\gamma R_G/dt^\gamma = aR_G + b\, sgn(G-J)$, $d^\eta G/dt^\eta = e\, sgn(R_G) + fG$.	(Ahmad and El-Khazali 2007)
17	$D^{2\alpha}x_1(t) = -\alpha_1 x_1(t) + \beta_1(x_2-x_3)\left(1-\varepsilon(x_2-x_3)^2\right) + \gamma_1$ $D^{2\alpha}x_2(t) = -\alpha_2 x_2(t) + \beta_2 x_4\left(1-\varepsilon x_1^2\right) + \gamma_2$ $D^{2\alpha}x_2(t) = -\alpha_2 x_3(t) + \beta_3 x_4\left(1-\varepsilon x_4^2\right) + \gamma_3$ $D^{2\alpha}x_4(t) = -\alpha_1 x_4(t) + \beta_1(x_3-x_2)\left(1-\varepsilon(x_3-x_2)^2\right) + \gamma_4$	(Koca and Ozalp 2014)
18	${}^C D_t^{\Theta,\rho} M_r(t) = \beta_a + L_r^2 - L_i^2 + \beta_c M_r$, ${}^C D_t^{\Theta,\rho} M_i(t) = 2L_r L_i + \beta_c M_i$, ${}^C D_t^{\Theta,\rho} L_r(t) = \beta_b + M_r^2 - M_i^2 + \beta_d L_r$, ${}^C D_t^{\Theta,\rho} L_i(t) = 2M_r M_i + \beta_d L_i$.	(Kumar *et al.* 2021) and (Jafari *et al.* 2016)

Dynamical modeling of romantic relationships has led researchers in physics, mathematics, and engineering to the idea of explaining the evolution of romantic relationships with a system of differential equations. There is a consensus that the first attempt at this was a short article published by Strogatz. Strogatz suggested the following system of equations in the study of Romeo and Juliet in which he tried to predict the love relationship (Strogatz 1988):

$$\frac{dR}{dt} = -aJ \; and \; \frac{dJ}{dt} = bR \tag{1}$$

In Equation 1, R(t) represents the quantity of Romeo's love/hate for Juliet and, J(t) represents the quantity of Juliet's love/hate for Romeo at time t. Here, a and b are the positive parameters that characterize their romantic styles of the couple. After Strogatz, researchers proposed more complex models to obtain the love/hate evolution of individuals in a romantic relationship as a function of time. In general, the models in the literature can be generalized by Equation 2 (Erbaş 2022).

$$\frac{dx}{dt} = f(x,y,t) \; and \; \frac{dy}{dt} = g(x,y,t) \tag{2}$$

In this equation, the left-hand sides of the equation are the derivatives with respect to time and the right-hand sides are functions (f_1 and f_2) that explain the rate of change in their feelings with instantaneous feelings and time. The models proposed by various authors in their studies are summarized in Table 1. Same equations are combined in the table so notations of the authors may be different from their original papers. The reader who wants to reach detailed information about these studies can refer to the references in the last column.

The first five rows in Table 1 are examples of homogeneous or in homogeneous first-order systems of linear equation. Rows 5-10 are nonlinear first order systems with two unknowns, rows 14 and 18 are systems of fractional order equations. In the literature, systems of equations with more than two unknown functions have also been used. The system of equations seen in row 11 deals with the relationship of Laura and Petrarch. $L(t)$ indicates Laura's feelings towards *Petrarch*, $P(t)$ indicates Petrarch's feelings toward *Laura*, and $Z(t)$ indicates *Petrarch's poetic inspiration*.

The four-dimensional models in rows 4, 12, 13 and 16-18 model three-person romantic relationships, called love triangles. In the love triangle model, Sprott assumes that Romeo has a mistress named Guinevere and that she and Juliet do not know each other. In this case, Romeo has two different affections or interests (R_J and R_G), while Juliet and Guinevere only have feelings for Romeo (J and G).

While systems of fractional differential equations are used between lines 14-18, the feelings of individuals are modeled with complex numbers in the last line. In line 18, Jafari and Sprott represent feelings with complex numbers, while their fractional derivatives are assigned as nonlinear functions of feelings (Jafari *et al.* 2016). In studies of the dynamic analysis of love, the individual's feeling for his partner is called love for positive values and hate for negative values. In models that express emotions with complex numbers, Jafari and Sprott consider the individual's feelings as a state of indecision in which love and hate coexist (Jafari *et al.* 2016). But even this is far from the fact that the individual has two-dimensional emotions that affect each other.

The triangular theory of love, which has a wide research area in psychology, sees emotions in a romantic relationship as two components (intimacy and passion) that affect each other. Although this distinction has been accepted in the school of psychology, there is, to our knowledge, no work in the school of mathematics where dynamic calculations are made. To fill this gap in the literature and to help interdisciplinary reconciliation between psychology and mathematics, in this study, the dynamics of the relationship between individuals with two-dimensional feelings were modeled with a four-dimensional differential equation system. This study, which is a first in this respect, aimed to construct and test the simplest linear model. If this model is developed;

- Relationships that minimal models cannot explain can be explained more comprehensively,
- Relationships evolving from friendship to love can be predicted,
- The situations of couples who come together after a long separation are predictable.

MATHEMATICAL MODEL

In the studies on the dynamic modeling of love, the emotions of the individual were handled as one-dimensional in the love/hate range. This study aims to linearly model how individuals' romantic relationships evolve in a two-dimensional emotional state. For this reason, a two-component emotional state, which is the intimacy of the individual to his/her partner and the deeper passions he feels, has been determined as a function of time. The intimacy and passion of x to his/her partner y are shown with $x_i(t)$ and $x_p(t)$ respectively. For example, if the individual is in the emotional state of $(x_i, x_p) = (2, -1)$, she has sincere feelings towards her partner and wants to spend time with him, but has no passions and desires with him. That is, she does not feel romantic or sexual desire.

Throughout the study, the emotions denoted by x and y will represent the emotions of the fictional couples named Xena and Yorgo. x_i and x_p will show Xena's intimacy and passion for Yorgo, and y_i and y_p will show Yorgo's intimacy and passion for Xena. Since an individual's state of emotion is handled in two components, four qualitative combinations can occur. These combinations can be interpreted as follows according to the signs of the components.

$(x_i, x_p) = (+, +)$: Xena feels warm, close, and passionate toward Yorgo. She enjoys spending time and being with him and desires romance/sexuality with him.

$(x_i, x_p) = (+, -)$: Xena has a good time with Yorgo and likes him friendly. But they have no romantic or sexual desires toward him.

$(x_i, x_p) = (-, +)$: Xena does not find Yorgo close or sincere, and even finds him boring. But she is fascinated by Yorgo's charm and desires him.

$(x_i, x_p) = (-, -)$: Xena does not feel intimacy or passion towards Yorgo. He does not enjoy spending time with her and does not desire romance or sex with her.

In the two-component feeling of love modeling, it is clear that a romantic relationship can be expressed with a total of four functions. The simplest dynamic modeling of these four functions is the linear differential equation system. In Equation 3, the intimacy and passion of individuals x and y and their relationship with the rate of change of these feelings are shown.

$$\frac{d}{dt}\begin{pmatrix} x_i \\ y_i \\ x_p \\ y_p \end{pmatrix} = \begin{pmatrix} a_{xx} & a_{xy} & b_{xx} & b_{xy} \\ a_{yx} & a_{yy} & b_{yx} & b_{yy} \\ c_{xx} & c_{xy} & d_{xx} & d_{xy} \\ c_{yx} & c_{yy} & d_{yx} & d_{yy} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ x_p \\ y_p \end{pmatrix} \tag{3}$$

In this equation, the parameters that make up the coefficients matrix was defined in Table 3. After these parameters and initial feelings of the couple are determined, readers can use the MATLAB script in the Appendix to visualize the future of the relation. How to comment on the visualizations is expressed in the Results section.

RESULTS AND DISCUSSION

Results of some possible scenarios

To set an example for the model described above, a scenario was prepared considering the characteristics of Xena and Yorgo when they met. According to this scenario, the romantic styles of Xena and Yorgo are shown in Table 3. The explanations of the parameters in the tables are given. In the first interview, it was assumed that both of them started with neutral emotions and the initial condition was chosen as (0,0,0,0). The development of the relationship starting from this condition over time is shown in Figure 1a. As can be seen from this figure, as Xena's sense of intimacy increases, her passions decrease over time. On the contrary, Yorgo's passion for her increases as he gets colder from Xena.

One of the interesting features in this scenario is the influence of individuals' impressions on each other. When a small change is made in the values shown in Table 3, the course of the relationship changes. If Yorgo had found Xena sympathetic, that is, if f_{yx} was one instead of zero, the development of the relationship would have been as in Figure 1b. In Figure 1b, while Yorgo's intimacy and passions increase over time.

Table 3 Romantic styles of Xena and Yorgo.

$a_{xx} = -0.2$	Forgetting coefficient of the intimacy of Xena to Yorgo.
$a_{xy} = -0.4$	If Yorgo's intimacy increases, Xena's will decrease, and if it decreases, it will increase. If Xena's partner shows closeness/interest to her, Xena gradually loses her sense of intimacy.
$b_{xx} = +0.5$	If Xena's passion increases, her sense of intimacy increases, and if it decreases, it decreases. She is intimate with someone she is passionate about. She might just want to fall in love.
$b_{xy} = -0.2$	As Yorgo's passion for Xena increase, Xena's closeness to Yorgo decreases. When she realizes that Yorgo is not in love, Xena increases her intimacy. Maybe she doesn't want someone in love with her.
$c_{xx} = +0.3$	Her passion increase when Xena feels close. Men with whom she does not feel close are not attractive, but men with whom she feels sincere can be attractive.
$c_{xy} = +0.7$	Intimate men are very attractive to Xena. Her passion for men who do not behave closely is significantly reduced.
$d_{xx} = -0.1$	Forgetting coefficient of the passion of Xena for Yorgo.
$d_{xy} = +0.4$	As Yorgo's passion grows, so does Xena's. A man who acts romantic may attract her.
$f_{xy} = +1.0$	Xena's impression of intimacy or friendship with Yorgo. Xena finds Yorgo intimate and friendly. She enjoys being friends and spending time with him.
$g_{xy} = -1.0$	Xena's impression of glamorousness or attractiveness about Yorgo. Xena does not find Yorgo romantically or sexually attractive.
$a_{yy} = -0.2$	Forgetting coefficient of the intimacy of Yorgo to Xena.
$a_{yx} = +0.6$	If Yorgo's intimacy increases, Xena's will decrease, if it decreases, it will increase. If Yorgo's partner shows intimacy/interest to him, Yorgo increases his sense of intimacy.
$b_{yy} = -0.5$	If Yorgo's passion increases, his sense of intimacy decreases, and if it decreases, it increases. He is intimate with someone he is not passionate about. He might just want not to fall in love.
$b_{yx} = +0.6$	As Xena's passion for Yorgo increases, Yorgo's intimacy with Xena increases. When he realizes that Xena is not in love, Yorgo decreases his intimacy. Maybe he wants someone in love with her.
$c_{yy} = -0.3$	His passion decreases when Yorgo feels close. Women with whom he does not feel close are attractive, but women with whom he feels intimacy are not attractive.
$c_{yx} = -0.1$	Intimate women are not attractive to Yorgo. His passion for women who are close to him weakens a little.
$d_{yy} = -0.1$	Forgetting coefficient of the passion of Yorgo to Xena.
$d_{yx} = -0.4$	As Xena's passion increases, Yorgo's decreases. A romantic woman does not attract him.
$f_{yx} = +0.0$	Yorgo's impression of intimacy or friendship with Xena. Yorgo found Xena neither sympathetic nor antisympathetic.
$g_{yx} = +1.0$	Yorgo's impression of glamorousness or attractiveness about Xena. Yorgo finds Xena attractive. He desires her romantically and sexually.

Xena's emotions show a similar development. However, as Xena's intimacy decreases initially and then increases, her passion initially increases and then decreases. But they still enter into an ideal relationship together. While the relationship shown in the first case goes to the incompatible regions (2^{nd} and 3^{rd} quadrants) in every sense, in the second case it goes to the best region. In Figure 1a, while Zeyna wants to remain friends and not have emotional relations, Yorgo wants an emotional relationship and refuses to meet. In Figure 1b, they both enter into a friendly and passionate relationship. What makes such a big difference is that Yorgo doesn't find Xena sympathetic. It can be said that the impressions of individuals on each other affect the relationship very sensitively.

In another scenario, the romantic styles of our fictional characters Xena and Yorgo are parameterized as in Table 4. In the relationship that started according to these parameters, it can be understood from Figure 2a that Xena wants friendship without a romantic relationship and Yorgo seeks love, not intimacy. As an example of Yorgo's strategy for obtaining Xena, when the a_{yx} parameter is increased to -3, the relationship evolves as in figure 2b. So if Yorgo exaggerates his avoidance of friendship, it may cause Xena to desire him because a friendly man is not attractive to Xena, but a man who avoids intimacy with her is attractive ($c_{xy} = -1$).

Suppose that individuals with romantic styles in Table 3 meet again after a long separation, but during this time their impressions of each other, not their characteristics, have changed. Now both of them have neutralized the impression of being close to each other ($f_{xy} = f_{yx} = 0$), but their attraction has become $g_{xy} = 0.5$ and $g_{yx} = 0.7$. How such an encounter would progress is shown in Figure 3a. We can see that their feelings of intimacy are gradually developing, but Yorgo's passion first increases and then decreases, while Xena's passion increases steadily. In other words, ex-lovers reconcile and become friends, but it can be said that while Xena wants to try again, Yorgo does not take kindly to this.

When the matrix in Equation 3 is grouped and divided into four sub-matrices, 2x2 matrices A, B, C, and D in Eqs.4 and 5 are obtained. Matrices A and D respectively model the effect of intimacies on intimacies, while matrix D represents the effect of passions on passions. The B matrix represents the influence of passions on intimacy, and the C matrix represents the influence of intimacy on passions. Situations, where there is no reciprocal cross-effect on intimacies and passions, can be expressed by Equations 4 and 5. In general, the studies seen in the literature are 2-dimensional. The two-dimensional work done by ignoring the cross-interactions and the four-dimensional study with the cross-interaction introduced in this study are visualized with the data in Table 2, and the results are compared in Figure 3b. According to Figure 3b, intimacy and passion follow a very different course when cross interactions come into play.

$$\begin{bmatrix} \overrightarrow{I'}_{2x1} \\ \overrightarrow{P'}_{2x1} \end{bmatrix} = \begin{bmatrix} \hat{A}_{2x2} & \hat{B}_{2x2} \\ \hat{C}_{2x2} & \hat{D}_{2x2} \end{bmatrix} \begin{bmatrix} \overrightarrow{I}_{2x1} \\ \overrightarrow{P}_{2x1} \end{bmatrix} + \begin{bmatrix} \overrightarrow{f}_{2x1} \\ \overrightarrow{g}_{2x1} \end{bmatrix} \tag{4}$$

If B_{2x2}=C_{2x2}=0, then

$$\begin{aligned} \overrightarrow{I'}_{2x1} &= \hat{A}_{2x2} \overrightarrow{I}_{2x1} + \overrightarrow{f}_{2x1}, \\ \overrightarrow{P'}_{2x1} &= \hat{D}_{2x2} \overrightarrow{P}_{2x1} + \overrightarrow{g}_{2x1}. \end{aligned} \tag{5}$$

Discussion

When the results obtained here are discussed, perhaps the first issue that comes to mind is how to determine the parameters that determine romantic styles. The parameters explained in Table 2 can be obtained by surveys to be applied to individuals, by observation, or by examining the past relationships of individuals. But as this would be a thorny work in psychology, it is not covered here. The signs of these parameters will give the romantic style of the individual, but the question of how much is quite difficult to answer.

Another issue that needs to be discussed is that the relationship dynamics are linear. One might argue that a complex subject such as human behavior cannot be modeled with linear systems. Suggesting more complicated non-linear equations will of course give results closer to reality, but the fact that a four-dimensional structure is tried for the first time and measurement difficulties in human emotions made it necessary to start with the simplest model, the linear model. Moreover, the approximation of nonlinear systems by linearizing them around fixed points is a commonly used approximation technique. The model described here can be said to be a model that approximately explains the evolution of a short-term relationship around neutral emotions. It would be appropriate to interpret the evolution of the relationship for the first few days or a week or two. Otherwise, in long-term developments, the model will deviate too much from reality.

To model the chaotic nature of love, it is necessary to construct two-dimensional nonlinear and non-homogeneous or at least three-dimensional nonlinear dynamic systems (Sprott 2010). Although nonlinear studies have been tried in the studies in the literature, it does not seem possible for a system with two unknowns to create a chaotic system by itself. However, it has been determined that chaos occurs when non-homogeneous terms are added.

In addition, since the love triangle models are expressed with a system of equations with four unknowns, it has been observed that four-dimensional nonlinear models can produce chaos (Kacar *et al.* 2018; Wang *et al.* 2022). However, when the individual's feelings for his partner are divided into two intimacy and passion, a nonlinear four-dimensional system alone can produce chaos. There is no need for two-dimensional homogeneous systems as Huang and Bae have pointed out (Huang and Bae 2018a) or for the four-dimensional nonlinear love triangle dynamics as Liu and Chen (Liu and Chen 2015). Even the simplest love affair can be chaotic, with no outside human influence or involvement of a third party.

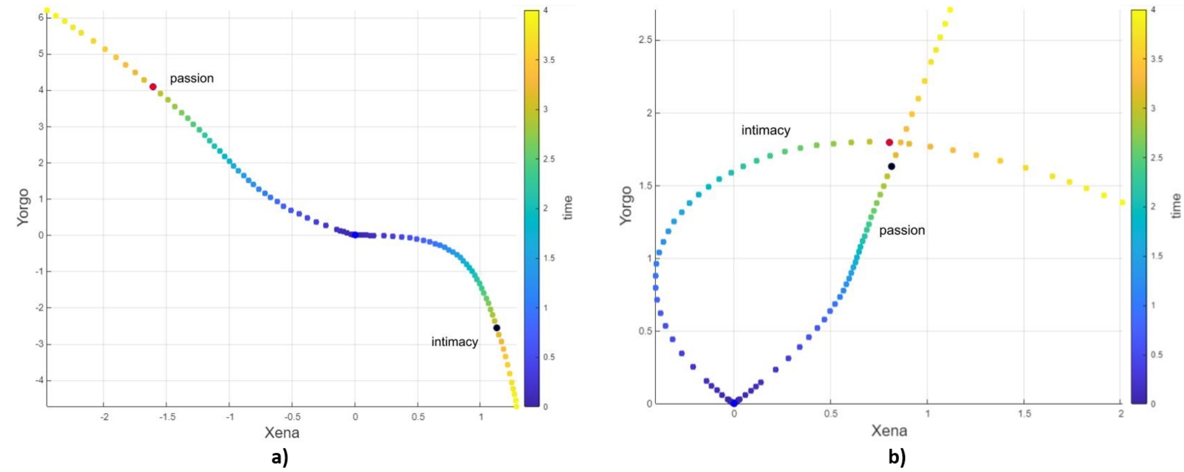

Figure 1 Evolution of the intimacy (black dot) and passion (red dot) between Xena and Yorgo according to the parameter in Table 3 with **a)** $f_{yx} = 0$ and **b)** $f_{yx} = +1.0$.

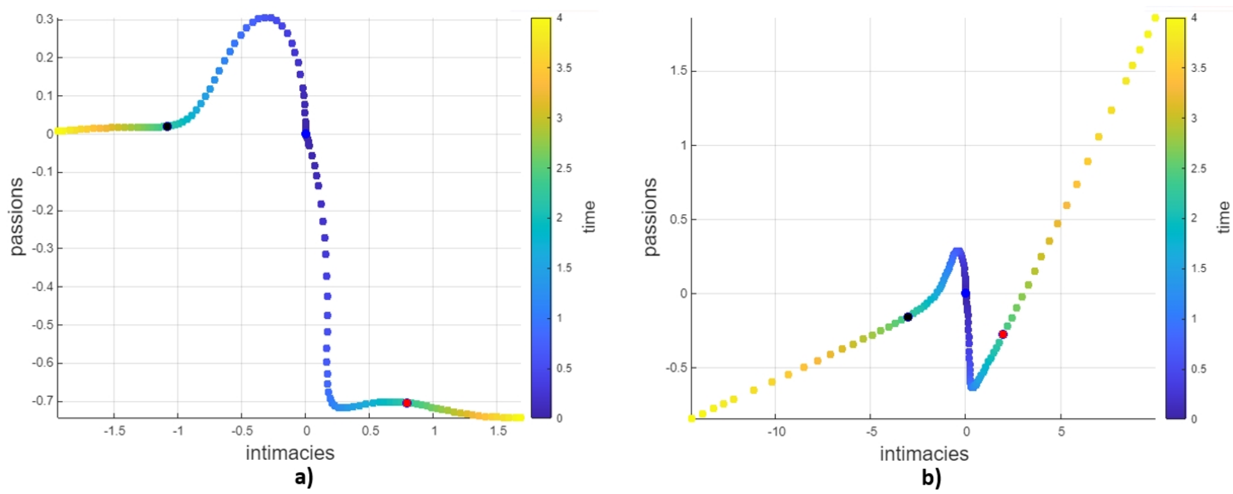

Figure 2 Evolution of the intimacy (black dot) and passion (red dot) between Xena and Yorgo according to the parameter in Table 4 with **a)** $a_{yx} = -1.0$ and **b)** $a_{yx} = -3.0$.

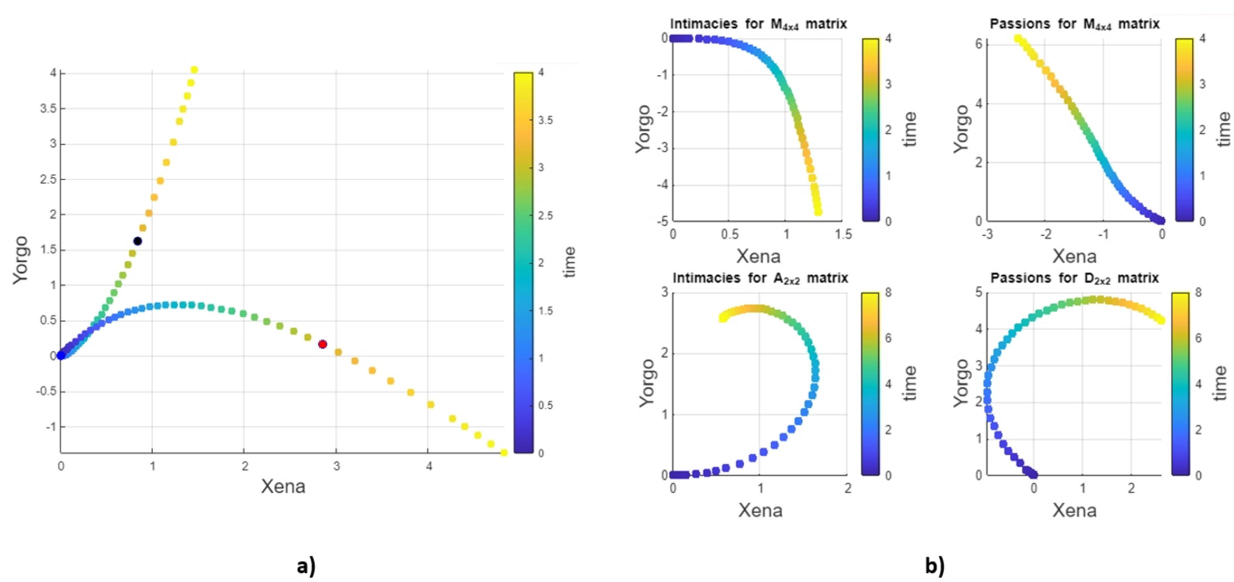

Figure 3 a) Intimacy (black dot) and passion (red dot) after a long separation ($f_{xy} = f_{yx} = 0$, $g_{xy} = 0.5$, $g_{yx} = 0.7$), **b)** Grouping the matrix according to Eqs.4 and 5.

Table 4 Romantic Styles for Scenario 2

Xena	a_{xx}=-1	a_{xy}=-1	b_{xx}=+1	b_{xy}=-2	c_{xx}=-1	c_{xy}=-1	d_{xx}=-1	dxy=-1	fxy=+1	gxy=-1
Yorgo	ayy=-1	**ayx=-1**	byy=-1	byx=+1	cyy=+1	cyx=+1	dyy=-1	dyx=+1	fyx=0	gyx=+1

CONCLUSION

As a result, in this study, emotions in a romantic relationship are discussed in two dimensions, intimacy, and passion, which are predicted by the triangular love theory. In cases where an individual's sense of intimacy influences his passion, results appear very different from those predicted by classical one-dimensional emotion models. In addition, it has been seen that the parameters that determine the romantic styles of individuals and their impressions of each other can affect the future of the relationship quite sensitively. Finally, it can be said that romantic relationships, which are seen as fragile in one-dimensional emotion models, are more fragile in two-dimensional models.

APPENDIX

```
clear all; clc; close all;
% Input the parameters and initial conditions
axx= -0.2; axy= -0.4; bxx= +0.5; bxy= -0.2; cxx= +0.3; cxy= +0.7;
dxx= -0.1; dxy= +0.4; fxy=0; gxy=0.5;
ayy= -0.2; ayx= +0.6; byy= -0.5; byx= +0.6; cyy= -0.3; cyx= -0.1;
dyy= -0.1; dyx= -0.4; fyx=0; gyx=0.7;
xi0=-0.0; yi0=-0.0; xp0=+0.0; yp0=+0.0;
% Matrix and calculations. Do not type anything below
A=[axx axy bxx bxy;ayx ayy byx byy;cxx cxy dxx dxy;cyx cyy dyx dyy]; B=[fxy;fyx;gxy;gyx];
f = @(t,x) A*[x(1);x(2);x(3);x(4)]+B;
[t,xa] = ode45(f,[0 4],[xi0 yi0 xp0 yp0]);
% Xena vs Yorgo. Black dot:intimacy, Red dot:passion
subplot(2,2,1)
%figure(1);
s1=scatter(xa(:,1),xa(:,2),40,t,'filled'); grid on; ax = gca;ax.XDir = 'normal';view(-31,14); xlabel('Xena','FontSize',16); ylabel('Yorgo','FontSize',16);
cb = colorbar;cb.Label.String = 'time'; cb.Label.FontSize = 14;hold on;
s2=scatter(xa(:,3),xa(:,4),40,t,'filled'); grid on; ax = gca;ax.XDir = 'normal';view(-31,14);
cb = colorbar;cb.Label.String = 'time'; cb.Label.FontSize = 14;
hold on;L1 = plot(xa(50,1),xa(50,2), 'ob', 'MarkerSize',7, 'MarkerFaceColor','black'); axis tight;
hold on;L2 = plot(xa(50,3),xa(50,4), 'ob', 'MarkerSize',7, 'MarkerFaceColor','red'); axis tight;
hold on; L3 = plot(xa(1,1),xa(1,2), 'ob', 'MarkerSize',7, 'MarkerFaceColor','blue');
hold on; L4 = plot(xa(1,3),xa(1,4), 'ob', 'MarkerSize',7, 'MarkerFaceColor','blue'); view(2);
% Intimacy vs Passion. Black dot:Yorgo, Red dot:Xena
subplot(2,2,2)
%figure(2);
s3=scatter(xa(:,1),xa(:,3),40,t,'filled'); grid on;
ax = gca;ax.XDir = 'normal';view(-31,14); xlabel('intimacies','FontSize',16); ylabel('passions','FontSize',16);
cb = colorbar;cb.Label.String = 'time'; cb.Label.FontSize = 14;hold on;
s4=scatter(xa(:,2),xa(:,4),40,t,'filled'); grid on;
ax = gca;ax.XDir = 'normal';view(-31,14);
cb = colorbar;cb.Label.String = 'time'; cb.Label.FontSize = 14;
hold on;L1 = plot(xa(50,1),xa(50,3), 'ob', 'MarkerSize',7, 'MarkerFaceColor','r'); axis tight;
hold on;L2 = plot(xa(50,2),xa(50,4), 'ob', 'MarkerSize',7, 'MarkerFaceColor','black'); axis tight;
hold on; L3 = plot(xa(1,2),xa(1,4), 'ob', 'MarkerSize',7, 'MarkerFaceColor','blue');
hold on; L4 = plot(xa(1,1),xa(1,3), 'ob', 'MarkerSize',7, 'MarkerFaceColor','blue'); view(2);
% Intimacy of Xena vs intimacy of Yorgo vs passion of Xena.
subplot(2,2,3)
%figure(3);
s5=scatter3(xa(:,1),xa(:,2),xa(:,3),40,t,'filled'); grid on;
ax = gca;ax.XDir = 'normal';view(-31,14); xlabel('intimacy of Xena','FontSize',16); ylabel('passion of Xena','FontSize',16); zlabel('passion of Xena','FontSize',16);
cb = colorbar;cb.Label.String = 'time'; cb.Label.FontSize = 14;hold on;
% Intimacy of Xena vs intimacy of Yorgo vs passion of Yorgo.
subplot(2,2,4)
%figure(4);
s6=scatter3(xa(:,1),xa(:,2),xa(:,4),40,t,'filled'); grid on;
ax = gca;ax.XDir = 'normal';view(-31,14); xlabel('intimacy of Xena','FontSize',16); ylabel('passion of Yorgo','FontSize',16); zlabel('passion of Yorgo','FontSize',16);
cb = colorbar;cb.Label.String = 'time'; cb.Label.FontSize = 14;
```

Conflicts of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Availability of data and material

Not applicable.

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