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NEW EXPANSIONS IN THE ECONOMIC PRODUCTION QUANTITY MODEL CONTAINING DEFECTIVE PRODUCTS

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ABSTRACT: The basic assumption in classical production quantity models is that all of the products produced are free of defect. But in real life, this assumption is not always valid. EPQ models were examined for the situation where the proportion of defective products is a random variable and fits a certain probability distribution. When the literature reviewed, it is seen that the scanning speed of defective products is accepted as equal to the production rate. In this study, a model was developed under the condition that the scanning speed is smaller than the production rate. In addition, a sample problem solution was made with the assumption that the defective product ratio is random variable and corresponds to the normal distribution and uniform distribution.

Keywords: EPQ, Defective product, Scan rate

1. INTRODUCTION

Inventory models have been developed to meet the ever-evolving challenges of real-world applications. The economic production quantity model is the most common model that is used to find the optimum production quantity for each cycle, taking into account the production, preparation, inventory and stock-free costs in general [1]. Traditional economic order/production quantity (EOQ/EPQ) models assume that all products are excellent. However, in case that defective products are often present in manufacturing processes, free of defect manufacturing cannot accurately reflect real-life problems. Forwhy in any production system, the defect caused by the different difficulties that arise in the production process is a natural phenomenon. Part of the manufactured goods is defective for many reasons, such as malfunctions of the facilities, worker errors and poor-quality material. Therefore, detecting defective products is one of the most important problems faced by production units. As a natural consequence of the assumption that defective products can be produced in a production process, the necessity of inspection, scanning, control, etc. concepts in production systems have emerged. Defective items may also be sold at the same starting price after rework in case that they can be repaired. Repairable items need to be reworked, and they are usually kept in a warehouse until the time of their reworking process. The reworking process can take place after the normal production process or when the inventory of excellent products drops to zero. Due to the difference between the maintenance costs of defective and free of defect products, the time it takes to start the reworking process is crucial given the impact of the inventory system on the total cost. Customer satisfaction and retention of defective products, as well as the costs associated with the return of defective products and after-sales services, need to be considered. However, this activity should be optimally organized, because there is a cost sanction on top of the screening and reworking costs of a defective product, and that is the cost of reputational damage or punishment. [2].

This study will help managers to choose the best policy for the economic production quantity-reprocessing process based on production system conditions.

2. LITERATURE REVIEW

The first studies involving the rework of defective products [3] include the study in which the system produces excellent products up to a certain point and comes out of control and started to produce defective products until the end of production. It is assumed that the period that has passed until the end of system getting out of control fits the exponential distribution and no pre-purchase is allowed. In this study, time needed to perfect the defective products by reworking has not been taken into consideration. (they are considered zero) [3-4]. Salameh and Jaber discuss a realistic situation where defective products are identified through a scanning process and sold with discount after the cycle. That is why model presented by Salameh and Jaber attracted attention in literature since its wide applicability on scanning and quality control in model inventory systems [5]. Hayek and Salameh developed an economic production quantity model for situations in which defective product rate fits to uniform distribution. It is assumed that defective products are reworked with a fixed rate when production stops. Not having stock is allowed in the model [6]. Chiu et. al. deal with defining an optimum working time for economic production quantity (EPQ) model with scrap, rework and stochastic machine malfunctions in this article. It formulates production-stock cost statements, machine malfunction scenarios and scenarios in which no malfunction occurs. They combine two functions and find the optimum cycle duration [7]. Eroğlu and Özdemir developed Salameh and Jaber model for conditions allowing not having stock [8]. Eroğlu et alia developed an economic production model for condition when a fixed measurable feature of the products is present in normal distribution and defective products are grouped as repairable, low quality and scrap in their study [1]. Maddah and Jaber worked over the model of Salameh and Jaber and calculated optimum expected annual profit using Renewal Reward Theory (RRT) [9]. Khan et al. expanded the model of Salameh and Jaber to involve the condition where defective products are sold or changed. They take Type 1 and Type 1 malfunctions deriving from misgrading of units during scanning [10]. Wee et al., worked on EPQ models for defective products allowing scanning rate to be slower than or equal to production speed [11]. Haydar et al. designed a scanning process carried out in production process, during production and end of production in their model. Defective products produced are grouped as discounted sale and rework. In each condition demand during production are provided solely from flawless products. This means that system scans products while selling. Thus, scanning rate is the same as demand rate before the end of production. After the production ends, a scanning process will begin for all remaining products [12]. Kumar et al. expands traditional EPQ model with a defective product inventory that has a demand rate depending on advertisement. In this model, production speed is considered as a decision variable. This article also handles the issue of sale of defective products after 100% elimination process with discount in single batch. Scanning speed is considered different than production speed and changed proportionally to production speed [13]. Al-Salamah used an EPQ model that has defective quality and random machine malfunction depending on heuristic artificial bee colony and put optimal batch size by calculating multivariate expected average cost function [14].

Most of the production quantity models that have defective production a rework processes reviewed in literature ignore scanning effect. A study adopting the approach in which scanning speed is equal to or slower than production speed and higher than demand rate has been found [11]. Additionally, defective product production rate is considered as a function of production speed.

A gap in literature has been identified regarding to the review of the relationship between scanning rate and production rate.

3. MATHEMATICAL MODEL

Inventory models involving defective products attracted great attention in literature. Most of the existing studies ignored the relationship between scanning speed and production speed within production cycle and considered speed as equal. Products are produced in single line and a specific amount of these products are defective. Defective product rate is a random variable and considered as suitable to a specific probability distribution. Repairable products are repaired through rework process.

3.1. Functionality of Process and Related Necessary Considerations

[1] Demand rate of manufactured products are known and fixed.
[2] Production of manufactured products are known and fixed.
[3] Raw materials and unit holding cost of the manufactured products are known and fixed.
[4] Raw materials and installation cost of the manufactured products are known and fixes.
[5] Examination and repair costs are fixed.
[6] Inspections are faultless.
[7] α units of production is made within production unit of time and defect rate of the production is p. Defect rate is a random variable and density function is $f(p)$.
[8] Demand for flawless products is β in unit of time and at least $\beta/1-p$ products need to be inspected to meet the demand an meet the demand with flawless products. Ignoring the fact that the scanning speed needs to be higher than $\beta/1-p$ rate, demand cannot be covered with flawless products.
[9] Not having stock is not allowed.
[10] Quality control cost through the production period is higher than the quality control cost after production period, $d1$ >$d2$.

3.2. Model Parameters

a: Production speed in a unit of time
α_1: Repair speed in a unit of time
β: Demand speed in a unit of time
c: Unit production cost
c_r:Unit rework (repair) cost
d_1:Scanning cost during production
d_2: Scanning cost of the defective product after production stops
h: Holding cost in a unit of time
h_1: Holding cost of defective products in a unit of time
p: $f(p)$probability density function and defective product probability
$f(p)$: Probability density function of P
K: Production establishment cost (fixed)

x: Scanning speed in a unit of time (quantity of separating defective and flawless products from each other)
y: Optimum product quantity produced during a production cycle
T: production cycle $T = t_1 + t_2 + t_3 + t_4$
TC : total cost in a production cycle
$ETC(y)$ expected total cost in a production cycle
$ETCU(y)$ expected total cost in a unit of time

3.3. Model Functionality Under $\beta/1-p \leq x \leq \alpha$ Consideration

If scanning rate x is lower than production rate a, whole products that are manufactured at_1 will not scanned during production period. t_1x of the y products will be scanned during production period and remaining $y - t_1x$ will be scanned during scanning period t_2. Defective products will be stocked with px speed during production and scanning periods. After the scanning period, defective products will be repaired with a_1 speed and added to solid products inventory. Change of inventory

Level over time is provided on figure (1) and (2).

Figure 1. Change of Produced Product Inventory a Cycle Over Time

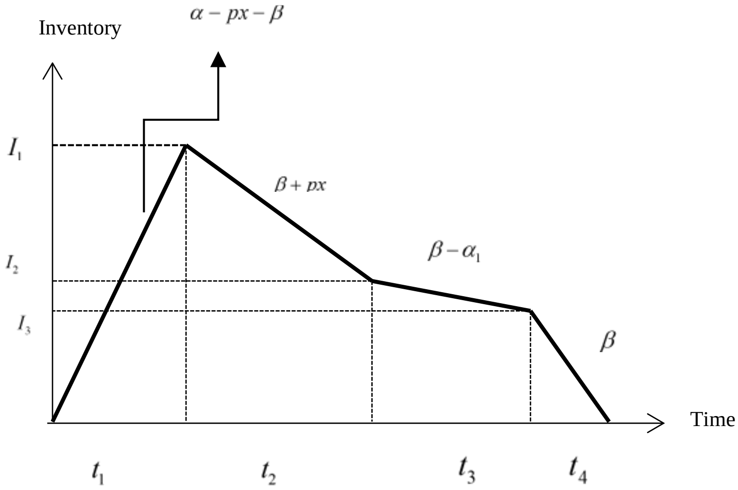
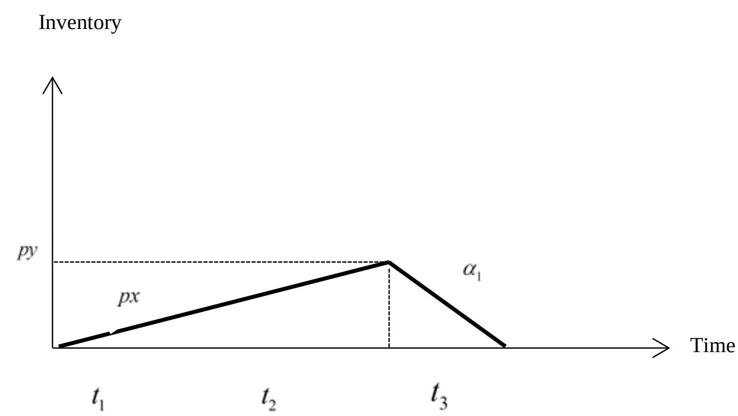


Figure 2. Change of Defective Product Inventory for a Cycle Over Time



Products are produced during t_1 with α speed and products are examined with x speed to separate defective and flawless products. As a result of examination with x speed, defective products that are produced with px speed are taken out of inventory on image 1 and added to defective product inventory on image 2. Since y products are produced through t_1 with α speed;

Taking Image (1) and (2) into consideration;

Production time is,

$$t_1 = y/\alpha \tag{1}$$

and from image 1

$$t_1 = \frac{I_1}{\alpha - px - \beta} \tag{2}$$

The following is obtained from Equation (1) and (2)

$$I_1 = \left(1 - \frac{\beta}{\alpha} - \frac{px}{\alpha}\right) y \tag{3}$$

In t_1, the amount of unexamined product is $y - xt_1 = \left(1 - \frac{x}{\alpha}\right) y$.

Screening time after production as unexamined products will be examined at x rate in t_2 (screening time after production) is as follows:

$$t_2 = \frac{\left(1 - \frac{x}{\alpha}\right) y}{x} = \left(\frac{1}{x} - \frac{1}{\alpha}\right) y \tag{4}$$

In t_2, the defective products detected with px rate as a result of examination are deducted from the inventory in figure 1 and added to the inventory of defective products in figure 2. The defective and excellent products are completely separated at the end of this process. The following is obtained from Figure 1;

$$t_2 = \frac{I_1 - I_2}{px + \beta} \tag{5}$$

And the following is obtained from equation (4) and (5);

$$I_2 = \left(1 - p - \frac{\beta}{x}\right) y \tag{6}$$

In $t_1 + t_2$, the defective products accumulate at px rate and reach to the amount of py as $py = (t_1 + t_2)px$.

In t_3, the defective products are repaired at α_1 rate and become excellent. Then;

Rework Time (rework time) is as follows;

$$t_3 = \frac{py}{\alpha_1} \tag{7}$$

$$t_3 = \frac{I_2 - I_3}{\beta - \alpha_1} \quad \text{is obtained from figure 1.} \tag{8}$$

And the following is obtained from equation (7) and (8)

$$I_3 = \left(1 - \frac{\beta}{x} - \frac{p\beta}{\alpha_1}\right) y \tag{9}$$

As the I_3 amount of product will meet the demand within t_4, The equation of $t_4 = \frac{I_3}{\beta} = \left(\frac{1}{\beta} - \frac{1}{x} - \frac{p}{\alpha_1}\right) y$ (10)

is obtained.

As all defective products are repaired and become excellent, the amount of excellent product in a cycle is equal to the production amount y in the cycle. As it will be obtained by proportioning the amount of excellent product to the demand rate if the cycle time is t ,the following applies;

$$t = \frac{y}{\beta} \tag{11}$$

As the total cost TC in a cycle will be the sum of the purchase cost, repair cost, examination cost, ordering cost, and inventory cost, the following applies:

$$\begin{aligned} TC &= cy + c_r py + d_1 x t_1 + d_2(y - x t_1) + K + h\left[\frac{t_1 I_1}{2} + \frac{t_2(I_1+I_2)}{2} + + \frac{t_3(I_2+I_3)}{2} + \frac{t_4 I_3}{2}\right] \\ &\quad + h_1\left[\frac{(t_1+t_2+t_3)py}{2}\right] \\ &= \left[c + c_r p + (d_1 - d_2)\frac{x}{\alpha} + d_2\right] y + K + \left\{\frac{h}{2}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) + \frac{(h_1-h)}{2}\left(\frac{p}{x} + \frac{p^2}{\alpha_1}\right)\right\} y^2 \end{aligned} \tag{12}$$

As $E[.]$ is the expected value operator, the expected total cost ETC per cycle is as follows;

$$ETC = \left[c + c_r E[p] + (d_1 - d_2)\frac{x}{\alpha} + d_2\right] y + K + \left\{\frac{h}{2}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) + \frac{(h_1-h)}{2}\left(\frac{E[p]}{x} + \frac{E[p^2]}{\alpha_1}\right)\right\} y^2 \tag{13}$$

The below expected total cost per unit time $ETCU$ is obtained by proportioning the expected value of the cycle time $\left(E[t] = \frac{y}{\beta}\right)$ to the expected total cost per cycle ETC;

$$\begin{aligned} ETCU = \frac{ETC}{E[t]} &= \left[c + c_r E[p] + (d_1 - d_2)\frac{x}{\alpha} + d_2\right] \beta \\ &\quad + \frac{K\beta}{y} + \left\{\frac{h}{2}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) + \frac{(h_1-h)}{2}\left(\frac{E[p]}{x} + \frac{E[p^2]}{\alpha_1}\right)\right\} \beta y \end{aligned} \tag{14}$$

If the $ETCU$ function is differentiated with respect to the y variable and set to zero to find the optimum order amount, the following equations are obtained.

$$\frac{dETCU}{dy} = -\frac{K\beta}{y^2} + \left\{\frac{h}{2}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) + \frac{(h_1-h)}{2}\left(\frac{E[p]}{x} + \frac{E[p^2]}{\alpha_1}\right)\right\} \beta = 0 \tag{15}$$

$$y = \sqrt{\frac{2K}{h\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) + (h_1 - h)\left(\frac{E[p]}{x} + \frac{E[p^2]}{\alpha_1}\right)}} \tag{16}$$

3.4 Illustrative Example

A factory produces a single item A. The number of daily demands for item A is 1200 and the production capacity is 5000 per day. There are defects in production for various reasons and it is considered that the defect rate complies with a certain probability distribution. The examination (inspection) is made during production and defective products are separated. Daily examination capacity is 4000 items. While the unit examination cost during production is 1.2 TRY, the unit examination cost after production is 1 TRY. All defective products are repaired and become excellent. The daily repair capacity is 600 items. Unit repair cost is 10 TRY and unit production cost is 50 TRY. The preparation cost is 16700 liras, and the stock cost of defective and excellent products is 3 and 2 liras, respectively.

Therefore, model parameters are as follows:

$\beta = 1200, \alpha = 5000, x = 4000, d_1 = 1.2, d_2 = 1, \ \alpha_1 = 600$

$c_r = 10, c = 50, K = 16700, h_1 = 3, h = 2.$

If normally distributed with an average of $\mu = 0.06$ and variance of $\sigma^2 = 0.0001$, the defect rate is $E[p] = \mu = 0.06, E[p^2] = \mu^2 + \sigma^2 = 0.0037$. Then, the economic production amount with the equation of (15) is y=5092.64 unit, while the expected total cost with the equation of (14) is $ETCU = 69982.17\ lira$

The cycle time is *t=4.2439 days*. Also, t_1*=1.0185 day*, t_2*=0.2546 day*, t_3*=0.5093 day*, t_4*=2.4614 day*, I_1*=3625.96 unit*, I_2*=3259.29 unit*, I_3*=2953.73* is obtained.

If the defect rate complies with the uniform distribution and the probability density function is the following;

$$f(p) = \frac{100}{3}, \quad 0.04 \leq p \leq 0.07$$

It would be $E[p] = \mu = 0.055, E[p^2] = 0.0031$. Then, with the equation of (16) economic production amount is y=5097.10 unit, the expected daily total cost is $ETCU = 69915.30\ lira$ and cycle time is *t=4.2476* with the equation of (15). Also t_1*=1.0194 day*, t_2*=0.2549 day*, t_3*=0.4672 day*, t_4*=2.5062 day*, I_1*=3649.52 unit*, I_2*=3287.63 unit*, I_3*=3007.29* is obtained.

4. RESULTS AND SUGGESTIONS

Screening is conducted at the end of the production to detect the defective parts in a production process of a single product likely to produce a defective product at a random rate. Once detected, the defective products are repaired at a fixed rate before being returned to inventory. The assumption of continuous screening during production complicates analysis and is not practical for most production systems especially with low probability of defective products and high production rate, thus making continuous screening during production quite expensive. The decisions about when and where to position screening stations have great importance in practice. The decision to position screening after or during production has importance. All the above-mentioned literature neglects the screening time required to identify defective components. Therefore, the study contributes to the literature. The efficiency of screening rises to prominence while lower screening costs suggest longer screening time. It has also been observed that reducing defect rates or return rates and increasing the customer satisfaction without a significant increase in system costs or new investments may be achieved by adjusting operational parameters. The model can be improved by adding screening errors related to the screening rate being greater than the production rate and the stockless production.

REFERENCES

[1] Eroğlu A, Sütçü, A, Sulak, H,(2008). Rassal Olarak Kusurlu Üretim Yapan Üretim Süreçleri İçin Ekonomi Üretim Miktarı Modeli: [Economy Production Quantity Model for RandomlyImperfectProduction Processes.] *Gazi Üniversitesi Mühendislik Mimarlık. Fakultesi Dergisi*, v.23, n.4, p.23-929.

[2] Hauck, Z, Rabta, B, Reiner, G (2021). Analysis of screening decisions in inventory models with imperfect quality items. *International Journal of Production Research*, v.59, n.21, p.6528-6543, doi.org/10.1080/00207543.2020.1818862.

[3] Rosenblatt, M,J, Lee, H,L, (1986). Economic production cycles with imperfect production processes. *IIE Trans. Taylor and Francis Online*. v.18, p. 48-55, doi.org/10.1080/07408178608975329.

[4] Moussawi-Haidar, L, Salameh, M, Nasr, W. (2016). Production lot sizing with quality screening and rework. *Applied Mathematical Modelling*. v.*40* n.4 p.3242-3256, doi.org/10.1016/j.apm.2015.09.095.

[5] Salameh, M.K, Jaber M.Y,(2000). Economic production quantity model for items with imperfect quality. *Int. J. Production Economics*, v.64, p.59-64, doi.org/10.1016/S0925-5273(99)00044-4.

[6] Hayek, P.A, Salameh, M.K, (2001). Production lot sizing with the reworking of imperfect quality items produced. *Production Planning and Control*. v.12, n.6, p.584-590, doi.org/10.1080/095372801750397707

[7] Chiu,S.W, Wang, S.L, Chiu, Y.P, (2007). Determining The Optimal Run Time for EPQ Model With Scrap, Rework, and Stochastic Breakdowns**.** *European Journal of Operational Research*, v.180, n.2, p. 664-676, doi.org/10.1016/j.ejor.2006.05.005.

[8] Eroğlu, A, Gültekin Ö, (2007). An Economic Order Quantity Model With Defective İtems and Shortages. *International Journal of Production Economics*, v.106, n.2, p.544-549, doi.org/10.1016/j.ijpe.2006.06.015.

[9] Maddah, B and Jaber, MY. 2008. Economic order quantity for items with imperfect quality: revisited. *International Journal of Production Economics*, v.112, n.2, p.808–815, doi.org/10.1016/j.ijpe.2007.07.003.

[10] Khan, M, Jaber, Y, Bonney, M, (2011). An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*, v.133, n.1, p. 113-118, doi.org/10.1016/j.ijpe.2010.01.023.

[11] Wee, H.M., Wang, W. T., Yang, P. C, (2013). A production quantity model for imperfect quality items with shortage and screening constraint. *International Journal of Production Research*, v.51, n.6, p.1869-1884, doi.org/10.1080/00207543.2012.718453.

[12] Maddah, B., Salameh, M.K, Moussawi-Haidar, L, (2010). Order Overlapping: A Practical Approach for Preventing Shortages During Screening. *Computers and Industrial Engineering* , v.58, n.4, p.691-695, doi.org/10.1016/j.cie.2010.01.014.

[13] Manna, A. K., Dey, J. K., Mondal, S. K, (2017). Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand. *Computers & Industrial Engineering*, v.104, p.9-22, doi.org/10.1016/j.cie.2016.11.027.

[14] Al-Salamah, M, (2018). Economic Production Quantity with the Presence of Imperfect Quality and Random Machine Breakdown and Repair Based on the Artificial Bee Colony Heurstic. *Applied Mathematical Modelling*, v.63, n.1, p.68–83, doi.org/10.1016/j.apm.2018.06.034.