Cilt 11, Sayı 3, Sayfa 123-128, 2022 Volume 11, Issue 3, Page 123-128, 2022

Araştırma Makalesi https://doi.org/10.46810/tdfd.1134722 Research Article

Türk Doğa ve Fen Dergisi
Turkish Journal of Nature and Science
www.dergipark.gov.tr/tdfd

A Generalized Thermoelastic Behaviour of Isotropic Hollow Cylinder

Mehmet EKER[1*], Durmuş YARIMPABUÇ[2]

[1] Tarsus University, Faculty of Engineering, Department of Mechanical Engineering, Mersin, Türkiye
[2] Osmaniye Korkut Ata University, Faculty of Arts and Sciences, Department of Mathematics, Osmaniye, Türkiye
Mehmet EKER ORCID No: 0000-0002-6785-1710
Durmuş YARIMPABUÇ ORCID No: 0000-0002-8763-1125

Corresponding author: mehmeteker@tarsus.edu.tr

(Received: 23.06.2022, Accepted: 14.09.2022, Online Publication: 29.09.2022)

Keywords
Generalized coupled thermoelasticity, Lord-Shulman theory, Hollow cylinder, Pseudospectral Chebyshev method (PCM)

Abstract: In this study, the thermoelastic behavior of a thick-walled homogeneous cylinder based on Lord-Shulman theory is investigated. It is assumed that the inner and outer surfaces of the cylinder are traction-free, and the outer surface is maintained at a reference temperature while the inner surface is subjected to a time-dependent internal temperature load. The governing equations in coupled form are solved with the pseudospectral Chebyshev method. The numerical approach is validated with benchmark results in the literature. The temperature, radial and tangential stress distributions are examined for three different nondimensional times to represent the time-varying effects of the applied instantaneous temperature load. The effect of the coupled term in Lord-Shulman theory for different high temperatures is examined and the difference between the coupled and uncoupled solution in different time periods is tabulated and the difference is highlighted.

İzotropik İçi Boş Silindirin Genelleştirilmiş Termoelastik Davranışı

Anahtar Kelimeler
Bağlaşımlı termoelastisite, Lord-Shulman Teorisi, Kalın cidarlı silindir, Pseudospektral Chebyshev yöntemi

Öz: Bu çalışmada, içi boş homojen bir silindirin Lord-Shulman teorisine dayalı termoelastik davranışı incelenmiştir. Silindirin iç ve dış cidarlara etki eden etki eden mekanik bir yükün olmadığı, dış cidarda sabit bir sıcaklık, iç cidarın ise zamana bağlı bir iç sıcaklık yüküne maruz kaldığı varsayılmıştır. Bağlaşımlı formdaki bünye denklemleri, pseudospektral Chebyshev yöntemiyle çözülmüştür. Kullanılan sayısal yaklaşım, literatürde mevcut referans sonuçlarla doğrulanmıştır. Sıcaklık, radyal ve teğetsel gerilme dağılımları, uygulanan anlık sıcaklık yükünün zamanla değişen etkilerini temsil etmek için üç farklı zaman için incelenmiştir. Lord-Shulman teorisinde yer alan bağlaşımlı terimin farklı yüksek sıcaklıklar için etkisi incelenmiştir. Farklı zamanlar için bağlaşımlı ve bağlaşımsız çözümler arasındaki fark vurgulanmış ve tablo olarak sunulmuştur.

1. INTRODUCTION

In the advanced applications, engineering structures can work under challenging thermal shock loads such as in the nuclear blast environment, pulsed laser and electromagnetic radiation. Thermal stresses and deformations would develop from a sudden increase in temperature in an elastic medium, significantly altering the mechanical behavior of thermo-elastic materials. As a result, in thermoelastic problems, the coupling between the temperature and displacement fields becomes critical [1]. The conventional heat conduction equation is the foundation of the classical (uncoupled) thermoelasticity theory. The theory was established by Biot [2]. Because of its parabolic character of the energy equation, the conventional heat conduction theory posits that thermal disturbances propagate at infinite speeds. This prediction may be appropriate for most engineering purposes, but it is a physically unfeasible assumption, especially at extremely low temperatures near absolute zero or for extremely short time periods. In order to overcome physical discrepancy of infinite speed prediction for thermal disturbances, the generalized theories of thermoelasticity were formulated. These theories offer the wave type heat propagation with finite speed. This phenomenon is mostly described as second sound [3]. The Lord–Shulman (LS) theory [4], which modifies Fourier's law of heat conduction by introducing one thermal relaxation time, is one of the well-known generalized theories. In the theory, the parabolic type heat equation is replaced by a hyperbolic one which ensures finite speeds of propagation for both heat and elastic waves. The detail for the other generalized thermoelastic theories and recent studies can be found Shakeriaski et al.'s review in [5].

Reformulation of classical thermoelastic equations by Lord-Shulman dates back to 1967. They presented wave-form thermal equation instead of the law of Fourier and defined the term relaxation time. The relaxation time represents the needed time-lag to establish steady state heat conduction in a volume element if the element is exposed to thermal shock. Derivation of thermoelastic formulation based on generalized thermoelasticity was done by Furukawa et al. [6-7] for an infinite body with a circular cylindrical hole and for an infinite solid cylinder. They obtained the temperature and thermal stresses with one relaxation time. A coupled thermoelastic problem in finite domain was analyzed by Hosseini Tehrani and Eslami [8-9] with boundary element method. They studied the effect of relaxation times and coupling coefficient on the thermal and elastic wave propagation. Bases on generalized coupled thermoelasticity, a disk problem solved by Bagri and Eslami [10] by utilizing transfinite element method. They obtained the thermal and stress wave propagation through the radius of disk and illustrated the coupling coefficient effect on the results. A unified coupled thermoelasticity formulation was proposed by same authors [11] based on the Lord–Shulman (L–S), Green–Lindsay (G–L), and Green–Naghdi (G–N) models for isotropic homogeneous cylinders and spheres. The structures are considered as subject to thermal shock on their inner walls. The governing equations were solved analytically into Laplace domain and transformed to the time domain by a numerical Laplace inversion. Application of Lord-Shulman theory was also extended to functionally graded cylinders. In this respect, Bagri and Eslami [12] utilized transfinite element method and the numerical inverse Laplace technique under uniform thermal shock to introduce unified formulation based on Lord Shulman, Green-Lindsay and Green-Naghdi methods. The dynamic response of functionally graded thermoelastic thick-walled hollow cylinders due to time-dependent heat flux studied by Sharma et al. [13] with an analytical method. They studied the effect of inhomogeneity for strains, stresses, displacement and temperature by the help of Lord-Shulman theory. The theory was also used for the thermoelastic problem of clamped axisymmetric infinite hollow cylinders under thermal shock with variable thermal conductivity by Zenkour [14]. He investigated effects of variable thermal conductivity and time parameters on radial displacement, temperature, and stresses. Thermoelastic interactions in the context of Lord-Shulman theory of a hollow cylinder compared the optimal homotopy analysis solution to the exact solution by Abbas and Abd-elmaboud [15]. They illustrated temperature, displacement and radial stress distribution graphically and discussed the effect of the relaxation time on the results.

Performing the thermoelastic behaviour analyzes in the literature, some researchers neglected the coupled terms, while others made calculations by taking these terms into account. However, the field of study examining the differences between stresses in these two different analyzes in the literature remained untouched. The originality of this work is that it examines the difference between stresses in analyzes with and without coupled terms. In this study, coupled thermoelastic behavior of a hollow homogeneous copper cylinder subjected to a time dependent internal temperature load is carried out. Among the generalized thermoelastic theories, Lord-Shulman theory with one relaxation time is used. The non-dimensional governing equations are obtained in coupled form and solved numerically with pseudospectral Chebyshev method (PCM). The temperature, radial and tangential stress distributions are presented graphically for three different nondimensional times. The effect of the coupled term in the Lord-Shulman theory on the solution for different high temperatures is examined and the difference between the coupled and uncoupled solution in different time periods is emphasized.

2. GOVERNING EQUATIONS

In the framework of Lord-Shulman theory, a linear, homogeneous thermoelastic continuum governing equations is employed for an isotropic thick-walled cylinder with $a \leq r \leq b$. Here a and b are the inner and outer radius respectively. The schematic illustration of the cylinder is given in Figure 1. Under the plane strain condition and cylindrical symmetry consideration, the equation of motion in the absence of body forces is given by [15],

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho \frac{\partial^2 u}{\partial t^2} \tag{1}$$

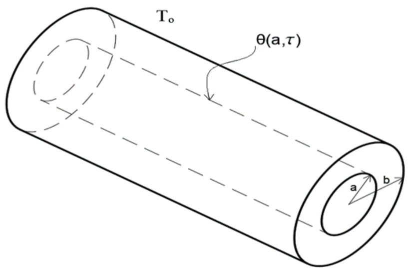


Figure 1. Schematic illustration of the homogeneous hollow cylinder

where ρ is the mass density, u displacement component, r radius. σ_{rr} and $\sigma_{\theta\theta}$ are the radial and circumferential stresses respectively.

$$\sigma_{rr} = (\lambda + 2\mu)\frac{\partial u}{\partial r} + \lambda \frac{u}{r} - \gamma(T - T_0) \tag{2}$$

$$\sigma_{\theta\theta} = \lambda \frac{\partial u}{\partial r} + (\lambda + 2\mu)\frac{u}{r} - \gamma(T - T_0) \tag{3}$$

where T is the absolute temperature, T_0 the reference uniform temperature of the body, λ, μ are elastic parameters, γ the thermal elastic coupling tensor in which $\gamma = (3\lambda + 2\mu)\alpha$. Since cylindrical symmetry is taken into account under plane strain conditions, the only nonzero component of the displacement vector is u_r, and u can be represented by $u_r = u(r,t)$.

The energy equation without heat sources is given as below [15],

$$k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e T + T_0\gamma\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right) \quad (4)$$

where c_e is the specific heat at constant strain, k the thermal conductivity and τ_0 is relaxation time proposed by Lord-Shulman theory.

For convenience, the following non-dimensional variables are introduced [15]:

$$\eta = c_1\chi\, r,\quad U = c_1\chi\, u,\quad \tau = c_1{}^2\chi\, t,$$
$$\bar{\tau}_0 = c_1{}^2\chi\,\tau_0,\ S_{rr} = \frac{1}{\lambda+2\mu}\sigma_{rr},\ S_{\theta\theta} = \frac{1}{\lambda+2\mu}\sigma_{\theta\theta}, \quad (5)$$
$$\theta = \frac{T-T_0}{T_0},\quad c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}},\quad \chi = \frac{\rho\, c_e}{k}$$

The aforementioned governing equations are reduced to following equations in considerations of the non-dimensional variables described in Equation (5).

$$\frac{\partial^2 U}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial U}{\partial \eta}-\frac{U}{\eta^2}-\beta\frac{\partial\theta}{\partial\eta}=\frac{\partial^2 U}{\partial\tau^2} \quad (6)$$

$$\frac{\partial^2\theta}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial\theta}{\partial\eta}=\left(\frac{\partial}{\partial\tau}+\bar{\tau}_0\frac{\partial^2}{\partial\tau^2}\right)\left(\theta+\epsilon\left(\frac{\partial U}{\partial\eta}+\frac{U}{\eta}\right)\right) \quad (7)$$

in a similar way stress equations can be obtained as,

$$S_{rr} = \frac{\partial U}{\partial\eta}+\xi\,\frac{U}{\eta}-\beta\,\theta \quad (8)$$

$$S_{\theta\theta} = \xi\frac{\partial U}{\partial\eta}+\frac{U}{\eta}-\beta\,\theta \quad (9)$$

where $\beta = \frac{T_0\gamma}{\rho c_1{}^2}$, $\epsilon = \frac{\gamma}{\rho c_e}$, $\xi = \frac{\lambda}{\rho c_1{}^2}$. The boundary conditions in which the inner and outer surfaces of the cylinder are traction-free and the outer surface is maintained at a reference temperature, while the inner surface is subjected to a time-dependent internal temperature load, are given in dimensionless form as follows:

$$S_{rr}(a,\tau) = 0,\qquad \theta(a,\tau) = \theta_1 e^{-\alpha\tau}$$
$$S_{rr}(b,\tau) = 0,\qquad \theta(b,\tau) = 0 \quad (10)$$

where θ_1 is constant and α is an exponent of diminished heat flux.

3. SOLUTION PROCEDURE

The solution of physical variables can be performed by following the decomposition methodology below [15].

$$U(\eta,\tau) = U(\eta)\, e^{\omega\tau},\qquad \theta(\eta,\tau) = \theta(\eta)\, e^{\omega\tau}$$
$$S_{rr}(\eta,\tau) = S_{rr}(\eta)\, e^{\omega\tau},\ S_{\theta\theta}(\eta,\tau) = S_{\theta\theta}(\eta)\, e^{\omega\tau} \quad (11)$$

where ω is the angular frequency. After decomposition, the system of differential equations can be rewritten in the following form.

$$\frac{\partial^2 U}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial U}{\partial\eta}-\frac{U}{\eta^2}-\omega^2 U-\beta\frac{\partial\theta}{\partial\eta}=0 \quad (12)$$

$$\frac{\partial^2\theta}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial\theta}{\partial\eta}-(\omega+\bar{\tau}_0\omega^2)\left(\theta+\epsilon\left(\frac{\partial U}{\partial\eta}+\frac{U}{\eta}\right)\right)=0 \quad (13)$$

$$S_{rr} = \frac{dU}{d\eta}+\xi\,\frac{U}{\eta}-\beta\,\theta \quad (14)$$

$$S_{\theta\theta} = \xi\frac{dU}{d\eta}+\frac{U}{\eta}-\beta\,\theta \quad (15)$$

and the boundary conditions will take following form

$$S_{rr}(a) = 0,\qquad \theta(a) = \theta_1 e^{-(\omega+\alpha)\tau}$$
$$S_{rr}(b) = 0,\qquad \theta(b) = 0 \quad (16)$$

Then, the system of ordinary differential equations given in Equation (12-16) is solved with pseudospectral Chebyshev method in coupled form.

3.1. Coupled Solution

In the Lord Shulman's approach, it is presumed that if the body is strained, transient stress change will accompany these strains, and interrelatedly, the temperature changes induce thermal strains [16]. For this reason, the equation of motion for the displacement and the heat conduction equation have to be solved simultaneously. Then, the coupled form of these dimensionless equations may be expressed in compact form as:

$$\frac{\partial^2 Y}{\partial\eta^2}+\mathrm{L}_\eta\frac{\partial Y}{\partial\eta}-\mathrm{L}_\mathrm{C}\mathrm{Y}=0 \quad (17)$$

where $\mathrm{Y} = [\theta\ U]^\mathrm{T}$ and the linear coefficient matrices $(\mathrm{L}_\eta, \mathrm{L}_\mathrm{C})$ are,

$$\mathrm{L}_{\eta} = \begin{bmatrix} \frac{1}{\eta} & -\epsilon(\omega+\bar{\tau}_0\omega^2) \\ -\beta & \frac{1}{\eta} \end{bmatrix},$$

$$\mathrm{L}_{C} = \begin{bmatrix} -(\omega+\bar{\tau}_0\omega^2) & \frac{-\epsilon(\omega+\bar{\tau}_0\omega^2)}{\eta} \\ 0 & -\frac{1}{\eta}-\omega^2 \end{bmatrix} \quad (18)$$

3.2. Numerical Solution of the Problem

The pseudospectral Chebyshev approach is based on first-kind Chebyshev polynomials. In the method, a solution is found in the interval specified in the problem. Chebyshev Gauss-Lobatto points, which have a denser point distribution at the boundary points than the midpoints, are used for obtaining high accuracy results. In accordance with the equation below, these points are evenly spaced on the semicircle.

$$\eta_j = \cos\left(\frac{j\pi}{n}\right), \qquad (j = 0,1,\dots,n) \quad (19)$$

The pseudospectral Chebyshev Model is employed to perform the coupled thermoelastic analysis of hollow cylinders under the time depended temperature loading by referring to the study of Trefethen [17], Fornberg [18] and Gottlieb [19] that depends on discretization of the governing equation (17) with respect to the spatial variable using the pseudospectral Chebyshev method. With reference to collocation points, the first order $(n+1)x(n+1)$ Chebyshev differentiation matrix will be created and denoted by D. First-order Chebyshev differentiation matrix D provides highly precise approximation to $U'(\eta_j)$, $\theta'(\eta_j)$, $U''(\eta_j)$, $\theta''(\eta_j)$ ..., simply by multiplying the differential matrix with vector data $U'(\eta_j) = (D\,U)_j$, $\theta'(\eta_j) = (D\,\theta)_j$ $u''(\eta_j) = (D^2\,U)_j$, $\theta''(\eta_j) = (D^2\,\theta)_j$ suchlike where $\boldsymbol{U} = [U_0,\dots,U_n]^T$ and $\boldsymbol{\theta} = [\theta_0,\dots,\theta_n]^T$ discrete vectors data at positions η_j. The Chebyshev differentiation matrix computing process and codes as an m-file can be found in prominent references see e.g., (Trefethen, [17]), where the collocation points η_j are numbered from right to left and defined in $[-1,1]$. The approach may be used to any interval with a minor modification. Accordingly, the coupled linear thermoelastic equation for the cylinder (17) is simply converted into a linear system by using the pseudospectral Chebyshev collocation approach as below:

$$L\,\boldsymbol{Y} = 0 \quad (20)$$

where

$$L = D^2 + \mathrm{L}_{\eta}D - L_c. \quad (21)$$

Boundary conditions (16) are imposed to this linear system (20) by only replacing the first and last row of the system matrix L with the appropriate values. Then, the non-trivial solution of the dimensionless temperature and displacement distributions can be found by solving the linear system with any decomposition method.

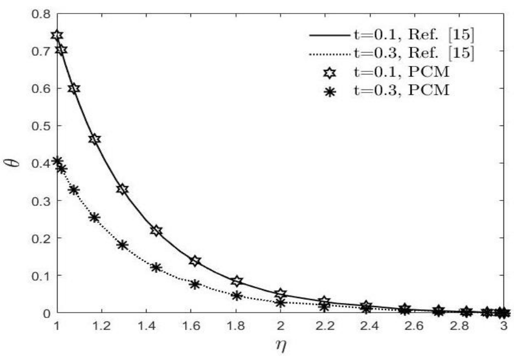
4. RESULTS AND DISCUSSION

In this study, thermoelastic interactions of a hollow copper cylinder based on Lord-Shulman theory are analyzed by the pseudospectral Chebyshev method. It is assumed that the outer surface is kept at the reference temperature while the inner surface is subject to temperature degradation over time. Analyzes are made by increasing the internal temperatures up to $T_1 = 500\,K$, which is the elastic limit of the copper material ($T_0 = 293\,K$). In the numerical calculations, following physical properties are used for copper [15],

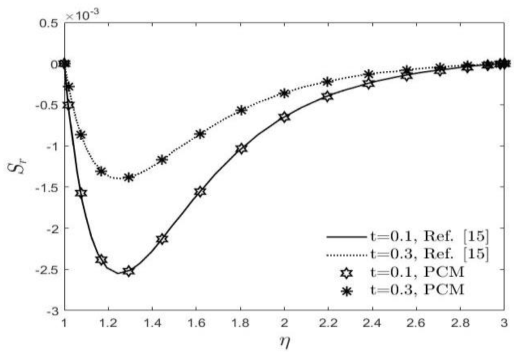
Table 1. Mechanical and thermal material properties of copper

Pro	Value	Unit	Pro	Value	Unit
λ	$7.76\,x\,10^{10}$	$kg\,ms^{-2}$	c_e	$3.831\,x\,10^{2}$	$m^2K^{-1}s^{-2}$
μ	$3.86\,x\,10^{10}$	$kg\,ms^{-2}$	ρ	$8.954\,x\,10^{3}$	$kg\,m^{-3}$
K	$3.68\,x\,10^{2}$	$kg\,m\,K^{-1}s^{-3}$	α	$17.8\,x\,10^{-6}$	$1\,K^{-1}$

The other quantities for the numerical calculation are chosen as $\omega = 5$, $\alpha = 3$, $\Omega = 5$ and $\tau_0 = 0.02$ [15]. Before proceeding to solution of the current problem, the numerical method is tested with the benchmark solution [15] available in the literature. The dimensionless temperature and radial stress distributions at certain times are compared on the graph (Figure 2) with analytical results. In comparison, the PCM solution results obtained using 16 data points are found to be in good agreement with the analytical results. According to these results, the solution to the current problem is continued with PCM.



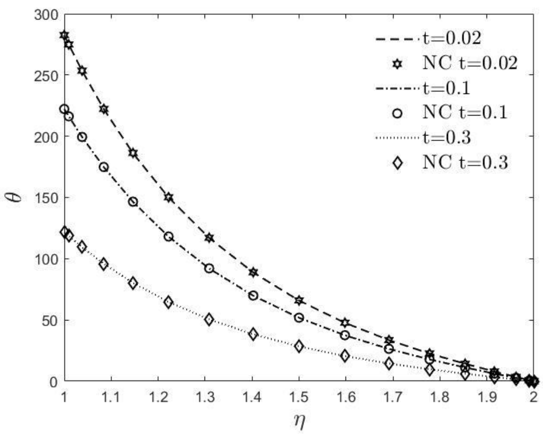
(a) Temperature distribution



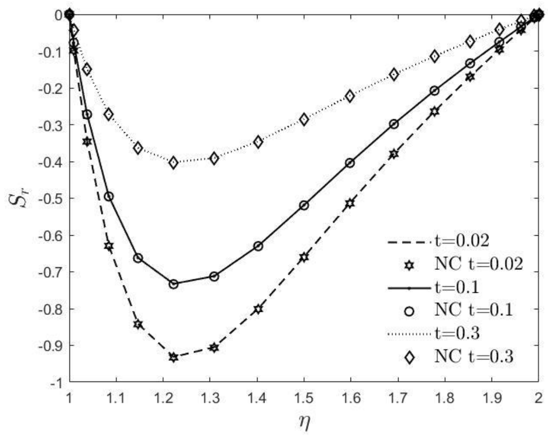
(b) Radial stress distribution

Figure 2. Distribution of temperature and radial stress in a hollow cylinder along the thickness at certain times

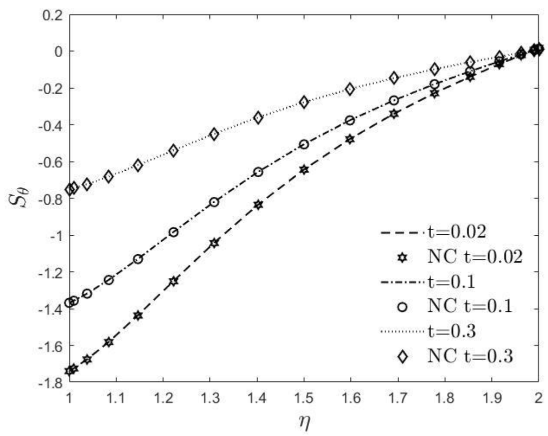
The non-dimensional numerical results of temperature, and stresses are given graphically for different values of time in Figure 3 for coupled and noncoupled (NC) conditions. In order to see the time-varying effect of the applied temperature load as the boundary condition, all results are plotted at $t = 0.02, 0.1$ and 0.3 time frames. These frames depict the various phases of the temperature load. In Figure 3(a), nondimensional temperature distributions are shown. In the early stage of loading ($t = 0.02$) the inner wall temperature is almost at the highest value that can be reached with the applied load. It is observed that this value decreases as time progresses to $t = 0.1$ and 0.3 nondimensional times. The dimensionless temperature distribution along the wall thickness decreases exponentially and reaches to zero at the outer wall in accordance with the boundary conditions. Since the results are given graphically, the difference between the coupled and uncoupled results is not clearly observed at this stage. The radial stress distributions satisfying the traction-free boundary conditions at both ends are presented in Figure 3(b). It is seen from the figure that the stress magnitudes are proportional to the temperature distributions. It is observed that the highest stress values occur at the beginning of loading (t=0.02) and after a certain period of time, these values decrease. The stresses reach their highest values in the first quarter of the wall ($\eta \cong 1.25$) from the inside to the outside. Similarly, it is observed in Figure 3(c) that the greatest tangential stress occurs on the inner wall in the provinces where the temperature effect is greatest. With the advancing time, the slope of the tangential stress curve decreases with the wall thickness. It can be also observed that all stress results occur in the compression direction Figure 3(b-c) as the cylinder is subjected to internal thermal load.



(a) Temperature distribution



(b) Radial stress distribution



(c) Tangential stress distribution

Figure 3. Distribution of temperature, radial and tangential stresses in a hollow cylinder along the thickness at certain times

Further research is done to see the effect of the coupled term on the results. The infinity norm is used to calculate the maximum difference between the results obtained from the coupled and uncoupled solutions ($||S_{coupled} - S_{uncoupled}||_\infty$) at three different temperatures ($\theta = 300, 400, 500$) and at different time periods ($t = 0.02, 0.1, 0.2$). The gradient variation of the coupled terms of radial and tangential stresses are given in Table 2 and Table 3, respectively. As can be seen from the Table 2-3, the stress values are larger in the calculations made by including the coupled terms, so to stay on the safe side in the analysis, the coupled terms should be included in the analysis. In the early stages of thermal loading, the differences are greatest for all time periods and for temperature magnitudes. In addition, as the temperature effect increased ($T = 500\ K$), it is observed that the difference between the coupled and the uncoupled solution increased. This indicates the need for coupled solutions under sudden loading conditions. This difference tends to decrease as time progresses ($for\ t = 0.1, 0.2$).

Table 2. The gradient variation of the coupled terms to radial stress

Time	$\theta = 300$	$\theta = 400$	$\theta = 500$
	$\sigma_r x10^{-4}$	$\sigma_r x10^{-4}$	$\sigma_r x10^{-4}$
0.02	13.080977	17.441303	21.801629
0.1	10.289861	13.719815	17.149768
0.3	5.647195	7.529594	9.411992

Table 3. The gradient variation of the coupled terms to tangential stress

Time	$\theta = 300$	$\theta = 400$	$\theta = 500$
	$\sigma_\theta x10^{-4}$	$\sigma_\theta x10^{-4}$	$\sigma_\theta x10^{-4}$
0.02	17.624611	23.499481	29.374352
0.1	13.864010	18.485347	23.106683
0.3	7.608730	10.144973	12.681217

5. CONCLUSION

This article examines the combined thermoelastic behavior based on the Lord-Shulman theory of a hollow homogeneous copper cylinder whose inner surface is subjected to a time dependent internal temperature load. The coupled governing equations are solved by using the pseudospectral Chebyshev method. The dimensionless temperature, radial and tangential stress variation along the radius of the cylinder and in some time frames are obtained and shown in the figures. The influence of coupled and uncoupled conditions is demonstrated on the thermoelastic responses of the copper cylinder at different times, which determines the intensity of the thermal load. It has been noticed that in all cases, the highest temperature and stress values occur at the beginning of the instantaneous thermal loading and these values decrease with time. When the differences between the coupled and uncoupled solution results are revealed with the help of the infinity norm, it is observed that the stress values are higher in the calculations made by including the coupled terms. According to this analysis, the highest difference in the results is obtained in the early times of the instantaneous highest thermal loading, while the lowest difference is obtained in the later time period at the lowest temperature. For a more precise analysis, it may be recommended to choose the coupling analysis for problems involving instantaneous loading or high temperature difference. Besides, it can be stated that the PCM is a useful, sensitive numerical solution method and can be adopted simply for solving such problems.

REFERENCES

[1] Ai ZY, Ye ZK, Yang JJ. Thermo-mechanical behaviour of multi-layered media based on the Lord-Shulman model. Comput Geotech. 2021;129.
[2] Biot MA. Thermoelasticity and irreversible thermodynamics. J. Appl. Phys. 1956;27:240–253.
[3] Zamani A, Hetnarski RB, Eslami MR. Second sound in a cracked layer based on Lord–Shulman theory. J. Therm. Stresses. 2011;34(3):181-200.
[4] Lord HW, Shulman Y. A generalized dynamical theory of thermo-elasticity. J. Mech. Phys. Solids. 1967;15(5):299–309.
[5] Shakeriaski F, Ghodrat M, Escobedo-Diaz J, Behnia M. Recent advances in generalized thermoelasticity theory and the modified models: a review. J. Comput. Des. Eng. 2021;8(1):15-35.
[6] Furukawa T, Noda N, Ashida F. Generalized thermoelasticity in an infinite solid with a hole. J. Therm. Stresses. 1989;12(3):385–402.
[7] Furukawa T, Naotake N, Fumihiro A. Generalized thermoelasticity for an infinite solid cylinder. JSME Int J., Ser. A. 1991;34(3):281-286.
[8] Hosseini-Tehrani P, Eslami MR. Boundary element analysis of coupled thermoelasticity with relaxation times in finite domain. AIAA Journal. 2000;38(3):534–41.
[9] Hosseini-Tehrani P, Eslami MR. Boundary element analysis of finite domains under thermal and mechanical shock with the Lord-Shulman theory. J. Strain Anal. Eng. Des. 2003;38(1):53–64.
[10] Bagri A, Eslami MR. Generalized coupled thermoelasticity of disks based on the Lord–Shulman model. J. Therm. Stresses. 2004;27(8):691–704.
[11] Bagri A, Eslami MR. A unified generalized thermoelasticity; solution for cylinders and spheres. Int. J. Mech. Sci. 2007;49(12):1325–35.
[12] Bagri A, Eslami MR. A unified generalized thermoelasticity formulation; application to thick functionally graded cylinders. J. Therm. Stresses. 2007;30(9-10):911-930.
[13] Sharma JN, Sharma PK, Mishra KC, Dynamic Response of Functionally Graded Cylinders due to Timedependent Heat Flux. Meccanica. 2016;51(1):139-154.
[14] Zenkour, AM. Generalized thermoelasticity theories for axisymmetric hollow cylinders under thermal shock with variable thermal conductivity. J. Mol. Eng. Mater. 2018;6(3-4):1850006.
[15] Abbas IA, Abd elmaboud Y. Analytical solutions of thermoelastic interactions in a hollow cylinder with one relaxation time. Math. Mech. Solids. 2017;22(2):210-223.
[16] Carter JP, Booker JR. Finite element analysis of coupled thermoelasticity. Comput Struct. 1989;31(1):73-80.
[17] Trefethen, LN. Spectral Methods in Matlab, 10, SIAM, Philadelphia, PA, 2000.
[18] Fornberg, B. A Practical Guide to Pseudospectral Methods, 1, Cambridge University Press, Cambridge, 1998.
[19] Gottlieb D. The Stability of Pseudospectral-Chebyshev Methods. Math. Comput. 1981;36(153):107-118.