Kocaeli Üniversitesi **Sosyal Bilimler** Enstitüsü **Dergisi** (7) 2004 / 1 : 93-103

Volatility in the Growth Rate of Real GNP : Evidence from Turkey

Nilgün Çil Yavuz*
Burak Güriş**

Abstract: This paper empirically investigates the volatility in the growth rate of real GNP for Turkey based upon quarterly data covering the period 1987: I-2003: II. Conditional volatility is estimated using the well-known Generalized Autoregressive Conditional Heteroscedastic (GARCH) model. The empirical results show that, although there were important events for the period, their effects on the growth rate had been no persistent.
Keywords: GARCH Model, Conditional Variance, Growth Rate

1. Introduction

Many economic and financial series, such as foreign exchange rates, returns on stocks, growth rates and inflation rates, exhibit time-varying volatility. Autoregressive conditional Heteroscedastic (hereafter ARCH) model introduced by Engle (1982) and the extension to generalized ARCH (hereafter GARCH) model by Bollerslev (1986) have been widely used to model volatility of economic and financial time series. Since the ARCH and the GARCH models provide a favorable framework to study time-varying volatility in the time series, they have become standard tool in econometrics ever since Engle and Bollerslev first reported them.

This paper empirically analyzes the volatility in growth rate of real Gross National Product (hereafter GNP) in Turkey by generalized autoregressive conditional Heteroscedastic [GARCH(1, 1)] model. The estimation model is specified as a

* Asst. Prof., Istanbul University, Dept. of Econometrics
** Resch.Asst., Istanbul University, Dept. of Econometrics

GARCH (1, 1) process, which has been widely used in the literature since Bollerslev-Wooldridge (1992). This narrow specification is supported by Engle (1993) Lagrange multiplier test results, which reveal strong evidence of ARCH effects of a high order.

It is often debated whether policymakers should take into account the volatility of macroeconomic variables. One of the important macroeconomic variables is growth rate that is calculated from real GNP. If the volatility of the real growth rate of an economy is constant, a confidence interval for a real GNP forecast would be a unique function of the sample variance and standard deviation. However, shocks affecting the growth rate will lead to changes in the volatility of the growth rate, i.e. the sample variance or standard deviation would not be constant (Hamori, 2000; 143-144).

2. Theoretical framework: GARCH model

In traditional econometric models, the variance of disturbance term is assumed to be constant. However, many time series exhibit periods of unusually large volatility followed by periods of relative tranquility. In such states, the assumption of a constant variance is no longer valid (Enders, 1995: 139). First time Engle (1982) shows that it is possible to model simultaneously both the mean and variance of a series that is inflation in the U.K. It is called the autoregressive conditional heteroskedastic (ARCH) model in which the unconditional variance is constant but the conditional variance is not constant, by Engle.

The ARCH models are used to model and forecast the conditional variance. In each case the variance of the dependent variable is specified to depend upon past values of the dependent variable. The ARCH (p) model is specified as follows:

$$y_t = \pi_0 + \sum_{i=1}^{k} \pi_i y_{t-i} + \varepsilon_t \qquad (2.1)$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \qquad (2.2)$$

where $\omega > 0$ and $\alpha_i \geq 0$ for $i = 1,.....p$, $p \geq 1$

In equation (2.2) σ_t^2 is the conditional variance of ε_t. To ensure that σ_t^2 is

strictly positive for all realizations of ε_t, it is required that $\omega > 0$ and $\alpha_i \geq 0$ for $i = 1,.....p$. Equation (2.2) also shows that the conditional variance is the weighted average of the squared values of past residuals.

The GARCH model allows for both autoregressive and moving average components in the heteroskedastic variance. In this model, conditional variance depends on the conditional variance in the previous period as well as lagged disturbance.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (2.3)$$

$\omega > 0$; $\alpha_i \geq 0$ for $i = 1,.....p$, $p \geq 1$; $\beta_j \geq 0$ for $j = 1,.....q$, $q \geq 0$

These conditions are to ensure that σ_t^2 is strictly positive for all realizations of ε_t,

According to Equation (2.3), the conditional variance today (σ_t^2) depends upon three factors:

- The mean (ω)
- Past news about volatility which is taken to be the lag the squared residual from mean equation (ε_{t-i}^2 ; the ARCH term)
- Past forecast variance (σ_{t-j}^2 ; the GARCH term). If the coefficients on σ_{t-j}^2 in the variance equation are statistically different from zero, significant GARCH effect appears to exist in the data.

If p and q in equation (2.3) are equal one, it is called the simple GARCH (1, 1) process that is special condition of the GARCH model. Empirical studies show that this model is adequate in modeling volatilities of most economic and financial time series. Consistent estimates of the parameters are obtained using the quasi – maximum likelihood procedure suggested by Bollerslev and Wooldridge (1992). The variance equation of the GARCH (1, 1) process as follows:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \qquad (2.4)$$

where $w \geq 0$; α, $\beta > 0$ to ensure strictly positive conditional variance and $\alpha + \beta < 1$.

By successively substituting for the lagged conditional variance into Eq.(2.4) the following expression is found:

$$\sigma_t^2 = \frac{w}{1-\beta} + \alpha \sum_{i=1}^{\infty} \beta^{i-1} \varepsilon_{t-i}^2 \qquad (2.5)$$

The GARCH variance is like a sample variance. But an ordinary sample variance would give each of the past squares an equal weight rather than declining weights. However, GARCH variance emphasizes the most recent observations. Since σ_t^2 is the one period ahead forecast variance which is determinated by past information, it is called the conditional variance (Bhar & Hamori, 2003: 225).

Before estimating the conditional variance of the series, it is necessary to examine the residuals of the mean equation for time-varying volatility (Kontonikas, 2003). The standard test is a Lagrange multiplier[1] (LM) test developed by Engle (1982). And then, Bollerslev (1986) suggested the LM test for testing a GARCH model against a higher order ARCH model. LM. Bollerslev (1986) reported that the LM test for GARCH (p, q) is equivalent to a test for *r*th order ARCH (where p + q = r).

Valid inference and reliable parameter estimates from the GARCH model require that the variable/variables in the system be stationary, at least in their conditional means (Lee, 2002: 178).

3. Data

The data used in this paper is based on the quarterly real GNP in Turkey. The data was obtained from Central Bank of the Republic of Turkey. The sample period is the first quarter of 1987 through the second quarter of 2003. Real GNP variable is seasonally adjusted by using Census X-11 method based on moving average principle. The quarterly growth rate is calculated as follows:

$$y_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \times 100 \qquad (3.1)$$

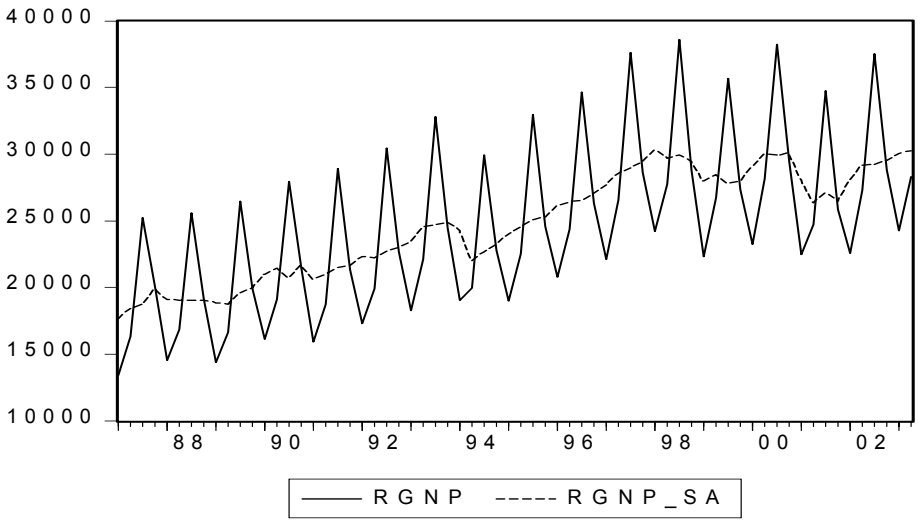
Where Y_t is real GNP (at fixed 1987 prices) at time t.

[1] In testing for the presence of ARCH effect, the LM test involves regressing the squared OLS residuals from the fitted ARMA model on a constant and q lagged values. The R^2 from the regression multiplied by the number of observations TR^2 follows a χ^2 distribution. Estimation under the null hypothesis only is required which if rejected indicates the presence of ARCH effects. The likelihood ratio test requires estimation under both the null and alternative.

4. Empirical Results

The real GNP (RGNP) and seasonally adjusted real GNP (RGNP_SA) series are illustrated in Figure1. The mean values of real GNP and adjusted real GNP have been increasing over time. But, these considerable decreases in GNP are detected in the both of the series.

Figure 1 : Real Gross National Product (1987 prices)



These decreases are caused by political and economic events, and natural catastrophe that are the economical shock in 1994, the earthquake shock in 1999 and financial crises in 2001 years. The objective of this paper is to examine volatility persistence in the growth rate of real GNP in the presence of these breaks.

Table 1 : Summary statistics on growth rate

Mean	0,87	Skew ness	-1,05
Standard Deviation	3,02	Kurtosis	4,35

Table 1 presents the several descriptive statistics on the growth rate for Turkey. The first row shows average of growth rate. The second row is the series' standard deviation. Thus, variance, which can be interpreted as unconditional volatility, is equal to 9.1 value, but is biased estimator. The last two rows show statistics for testing normality. Result of Jarque-Bera[2] test for normality is 17.03 (P-value = 0.0002). The null hypothesis of normality is rejected by the Jarque-Bera (JB) asymptotic test for the growth rate. Ljung-Box test statistics indicates significant autocorrelation in growth rate. Result of Ljung-Box (Q) test is equal to13.841. At the 0.05 significant level, the critical value of χ^2 with four degrees of freedom is 9.487. Hence the null hypothesis of no autocorrelation is rejected.

Figure 1 describes the behavior of growth rate. Since it is necessary that the growth rate be stationary for valid inference and reliable parameter estimates from GARCH model, augmented Dickey Fuller (ADF) and Phillips Perron (PP) tests are used to determinate whether the growth rate is stationary.

Figure 1 : Growth rate: 2nd quarter of 1987-2nd quarter of 2003

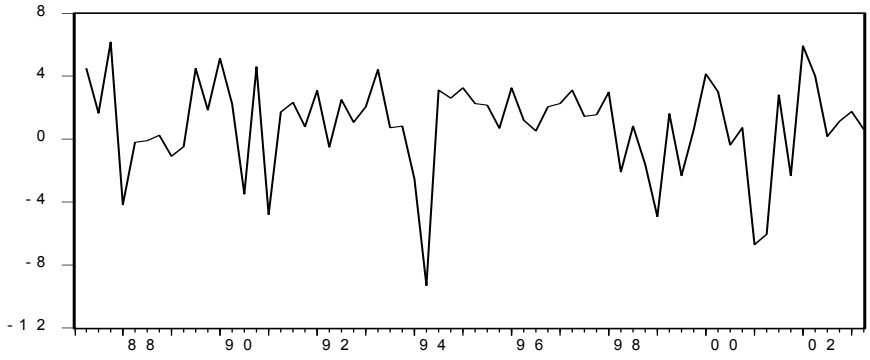


Table 2 displays the results of augmented Dickey Fuller (ADF) and Phillips Perron (PP) tests for unit root for the 1987:1-2003:2 sample period.

[2] Jarque-Bera for normality is defined by $T\left[S^2/6+(K-3)^2/24\right]$ which is asymptotically distributed as χ^2 (2). The 1% critical value equal 9.21.

Table 2 : Unit root test on real growth rate

	ADF	PP
Case 1	-5.947180*(3)**	-7.675730*(3)***
Case 2	-6.006314*(3)	-7.644880*(3)
Case 3	-4.988528*(3)	-7.276491*(3)

Notes: Case 1 shows that the auxiliary regression is run with a constant. Case 2 shows that auxiliary regression is run with a constant and time trend. Case 3 shows that auxiliary regression is run without any deterministic term.

(*) Implies that the null hypothesis of the existence of a unit root is rejected at a %1 significance level.

(**) The lag lengths are chosen according to Akaike Information Criteria (AIC) for ADF tests.

(***) PP tests are estimated for alternative Bartlett kernel truncation lags.

The ADF and PP statistics reveal evidence against that unit-root null hypothesis for the growth rate variable for levels. Thus, real growth rate is stationary variable [I (0)]. Before estimating the conditional variance of the real growth rate, it must be checked whether there is autocorrelation in the residuals of the conditional mean equation. For the conditional mean equation, assuming that the growth rate follows autoregressive-moving average (ARMA) process, and is a function of autoregressive lags and moving average terms. The equational form of the ARMA (p, q) model is:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \delta + \varepsilon_t - \theta_1 \varepsilon_{t-1} - ..... - \theta_q \varepsilon_{t-q}$$

where, y_t = growth rate ($t=1,2,.....T$) ; ϕ_i = parameters of the autoregressive factors ($i=1,……,p$) ; θ_k = parameters of the moving average factors ($k=1,……,q$) ; δ = constant;

ε_t= white noise. First, Box-Jenkins techniques were used to reduce the set of prospective ARMA specifications. Two functions, the autocorrelation function (ACF) and the partial autocorrelation function (PACF), were used to assist in the identification stage of the model. The ACF and PACF of residuals should be indicative of a white-noise process. To further assist in the identification of the correct ARMA model, the general information criteria, Akaike- Schwartz information criteria were used. The best-fitting ARMA specification having lowest Akaike- Schwartz information criteria for the conditional mean equation is as following.

Table 3 : OLS estimates of growth rate

Dependent Variable : y_t			**Diagnostic Statistics**		
Variable	Coefficient	Prob. Value	R^2 = 0.32	Log like hood = -140.28	
c	0.8547	0.0076	Q(4)=1.734	Q(8)=4.701	Q(16)=7.448
y_{t-2}	-0.4318	0.0010	Q^2(4)=2.095	Q^2(8)=5.361	Q^2(16)=11.084
y_{t-3}	0.2432	0.0347	T*R^2(1)=0.921	T*R^2(4)=1.836	T*R^2 (8)=5.230
y_{t-4}	-0.3028	0.0134			
. . ε_{t-2}	0.8051	0.0000			
. . ε_{t-3}	-0.3790	0.0000			

Notes : Q and Q^2 represent the Ljung-Box test statistic for the joint significant of autocorrelations of standardized and squared standardized residuals respectively for the first 4, 8 and 16 lags. Under the null hypothesis of no serial correlation, distributed as χ^2 (4), χ^2 (8) and χ^2 (16). The 5% critical values are 9.487, 15.507 and 26.296 respectively.

- In practice the maximum number of sample autocorrelations and partial autocorrelations to use is T/4. Since observation number is 61, maximum lag length for Q and Q^2 tests is used 16.
- TR^2 shows ARCH Lagrange Multiplier (ARCH-LM) test statistics for serial correlation for the first 1, 4 and 8 lags. The 5% critical values for χ^2 (1), χ^2 (4) and χ^2 (8) are 3.841, 9.487 and 15.507 respectively.

It can be seen from the results in Table 3 that all estimated coefficients in chosen model are significant at conventional levels. This condition implies that the characteristic roots are inside the unit circle. Ljung-Box (Q) statistics indicate that the residual series appear to be white-noise. The Ljung-Box (Q and Q^2) statistics of the residuals at 4.8 and 16 lags shows that there no autocorrelation in residuals. This means that the fitted model is reasonably well specified. Thus model is adequate. The Lagrange multiplier (ARCH-LM) for the presence of ARCH disturbances shows and it can be seen that for the growth rate the null hypothesis of no ARCH errors (i.e. homoscedastic process) is not rejected at the 5 % level. A battery of diagnostic tests indicates that the residuals are serially uncorrelated.

Although the model appears adequate, there are two periods (1994: 2, 2001: 1) of unusual volatility that is characteristic of a GARCH process. Estimation results for the GARCH (1,1) model displayed in Table 4.

Table 4 : Estimate of the GARCH (1,1) model

(1) $y_t = 1.346 - 0.445y_{t-2} + 0.318y_{t-3} - 0.219y_{t-4} + 0.841\varepsilon_{t-2} - 0.361\varepsilon_{t-3} + \varepsilon_t$

(4.576) (3.400) (2.744) (2.127) (17.222) (7.870)

(2) $\sigma_t^2 = 2.209 + 0.572\varepsilon_{t-1}^2 + 0.137\sigma_{t-1}^2$

(1.162) (1.747) (0.398)

Log L= -137,5764

JB = 1.603, P-value = 0.448

*The numbers in parentheses are the absolute values of the t-statistics.

The parameters in the model are estimated using the maximum likelihood procedure, as described in Engle (1982) and Bollerslev (1986) and results are reported in table 4. Estimates for the conditional mean and conditional variance of real growth rate are reported in equation (1) and equation (2) respectively. The estimated coefficients in the GARCH (1, 1) -M model are similar to the OLS coefficients reported in Table 3. The GARCH (1, 1) parameters in the conditional variance are stable because they sum to less than one. The coefficient on the lagged, squared residuals (ε_{t-1}^2) is significant at 10 percent level (t = 1.747). The coefficient on the lagged error variance (σ_{t-1}^2) in the equations is insignificant, indicating that the real growth rate shocks have no persistent effect on real growth rate.

Table 5 : Diagnostics tests for residuals of GARCH (1, 1) model

Lagrange Multiplier test	**Ljung-Box test**
$TR^2(4)$ = 2.907 (0.573)	Q (8) = 8.003 (0.046)
$TR^2(8)$ = 8.022 (0.431)	Q (16) = 9.846 (0.544)
	Q^2 (8) = 9.919 (0.019)
	Q^2 (16) = 18.331(0.074)

Notes : The figures in brackets are the P-value.

-The 5 % critical values are χ^2 (4) = 9.487and χ^2 (8)= 15.507 for Lagrange Multiplier serial correlation test.

-The 5 % critical values are χ^2 (8)= 15.507, χ^2 (16)= 26.296 for Ljung-Box serial correlation test.

The normality test (Jarque-Bera) is significant which is consistent with the hypothesis that the residual from GARCH model is normally distributed.

Diagnostic tests on the residuals and its square are reported at Table 5. Ljung-Box (Q and Q^2) test statistics clearly indicate that there is no the serial correlation in the conditional variance. Lagrange multiplier test also indicates that the residuals are serially uncorrelated. As a result the model appears adequate.

5. Summary and Conclusion

This study has examined the volatility in GNP growth rate for Turkey. Since the reliability of the empirical study depends on the statistical validity and appropriate interpretation of the underlying model, the paper extensively examined the statistical properties of the baseline GARCH model. The empirical results show that, shocks have no persistent effect on real growth rate. The other words, the effects of economical and political events and the earthquake shock had no changing effect on the growth rate volatility in the long term. However, the shocks have caused breaks on real GNP in the short term.

Reel Büyüme Hızında Dalgalanmalar: Türkiye Örneği

Özet : Bu çalışmada Türkiye' nin reel GSMH'ın büyüme hızının volatilitesi 1987: I-2003: II dönemi için üçer aylık veriler kullanılarak ampirik olarak araştırılmıştır. Koşullu volatilite, genelleştirilmiş otoregresif koşullu heteroskedasite modeli (GARCH) aracılığı ile tahmin edilmiştir. Ampirik sonuçlara göre ilgili periyotta önemli olayların varlığına rağmen , bunların büyüme hızına etkisi kalıcı olmamıştır.

Anahtar Kelimeler : GARCH Modeli, Koşullu Varyans, Büyüme Hızı

References

Bhar, R. and S Hamori (2003). "Alternative Characterization of the Volatility in the Growth Rate of Real GDP". **Japan and World Economy** 15: 223-231.

Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heretoskedasticity". **Journal of Econometrics** (31): 307-327.

Bollerslev, T. and J. Wooldridge (1992). "Quasi Maximum Likelihood Estimation of Dynamic Models with Time Varying Covariance". **Econometric Reviews** (11): 143-172.

Enders, W. (1995). **Applied Econometric Time Series**. New York: John Wiley and Sons.

Engle, R. F. and T. P. Bollerslev (1986). "Modeling the Persistence of Conditional Variances".

Econometric Reviews (5): 1-50.

Engle, R. F. (1983). "Estimates of the Variance of U.S. Inflation Based Upon the ARCH Model". **Journal of Money, Credit and Banking** (29): 266-301.

Engle, R. F. (1982), "Autoregressive Conditional Heteroscedasticiyt with Estimates of the Variance of United Kingdom Inflation". **Econometrica**, Vol. 50, No. 4, July: 987-1007.

Hamori, S. (2000). "Volatility of Real GDP: Some Evidence from the United States, the United Kingdom and Japan". **Japan and World Economy** 12: 143-152.

Kontanikas, A. (2003). "Inflation and Inflation Uncertainty in the United Kingdom, Evidence from GARCH Modeling". **Economic Modelling**, Article in Press

Lee, J. (2002). "The Inflation-Output Variability Tradeoff and Monetary Policy: Evidence from a GARCH Model". **Southern Economic Journal** 69, July: 175-188.