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SEPARATION PROPERTIES AT p FOR THE TOPOLOGICAL CATEGORY OF STACK CONVERGENCE SPACES

Mehmet Baran
Erciyes University, Department of Mathematics, Kayseri- TURKEY

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ABSTRACT.

In this paper, an explicit characterizations of each of the separation properties T_0, T_1, PreT_2, and T_2 at a point p is given in the topological categories of stack convergence and constant stack convergence spaces. Moreover , specific relationships that arise among the various T_0, PreT_2, and T_2 structures at p are examined in these categories.

YAKINSAK YIĞIN UZAYLAR KATEGORİSİ İÇİN p DE AYRILMA AKSİYOMLARI

ÖZET. Bu çalısmada, yakınsak yığın uzaylar ve yakınsak sabit yığın uzaylar kategorilerinde p noktasinda T_0, T_1, PreT_2 ve T_2 ayrılma aksi yomlarının herbirinin açık bir karakterizasyonu verildi. Bununla beraber, bu kategorilerde p deki değişik T_0, PreT_2 ve T_2 yapıları arasında ortaya çıkan özel ilişkiler incelendi.

1. INTRODUCTION.

Let A be a set and $\alpha \subset PA$, the set of subsets of A. Define $[\alpha] = \{B \mid B \subset A$ for which there exists $C \in \alpha$ with $C \subset B\}$.

1.1 Definition. A stack on A is a subset α of PA such that $[\alpha] = \alpha$ i.e α is closed under supersets. See [5] p. 345. Let STK(A) denote the set of stacks on A. A stack α is said to be proper iff $\Phi \notin \alpha$.

Let A be a set and K be a function on A whose value K(a) at each a in A is a set of nonempty stacks on A.

1.2 Definition. A pair (A,K) is said to be a Stack Convergence Space if for each a in A,

1. $[a]$ belongs to $K(a)$

2. If α and β are stacks on A and $\alpha \subset \beta$, then $\beta \in K(a)$ if $\alpha \in K(a)$. A morphism $(A, K) \to (B, L)$ is a function $f : A \to B$ such that $f\alpha \in L(f(a))$ if $\alpha \in K(a)$, where $f\alpha$ denotes the stack $\{U | U \subset B$ and $U \supset f(C)$ for some $C \in \alpha\}$. We denote by SCO, the category so formed. See [5] p. 354.

1.3 Definition. The category of Constant Stack Convergence Spaces, $ConSCO$ is the full subcategory of SCO determined by those spaces (A, K), where K is a constant function.

1.4 The discrete structure (A, K) on A in SCO and $ConSCO$ is given by $K(a) = \{\alpha | \alpha \supset [a]\}$, a in A, and $K = \{\alpha | \alpha \supset [b]$ for some b in $A\}$, respectively.

1.5 A source $\{f_i : (A, K) \to (A_i, K_i) \quad i \in I\}$ is an initial lift in SCO if and only if $\alpha \in K(a)$ precisely when $f_i\alpha \in K_i(f_i(a))$ for all i in I. See [4] p. 1374.

1.6 A source $\{f_i : (A, K) \to (A_i, K_i), \quad i \in I\}$ is initial in $ConSCO$ iff $\alpha \in K$ precisely when $f_i\alpha \in K_i$ for all i.

1.7 An epi morphism $f : (A_1, K_1) \to (A, K)$ is final in SCO, iff for each a in A, $\alpha \in K(a)$ implies there exists $\beta \in K_1(a_1)$ such that $f\beta \subset \alpha$ and $f(a_1) = a$. An epi sink $\{i_1, i_2 : (A, K) \to (A_1, K_1)\}$ is final in SCO iff for each a_1 in A_1, $\alpha \in K_1(a_1)$ implies there exists a in A and β in $K(a)$ such that for some $k = 1, 2$, $i_k a = a_1$ and $i_k\beta \subset \alpha$. These are special cases of [4] p. 1375.

1.8 An epi morphism $f : (A_1, K_1) \to (A, K)$ is final in $ConSCO$ iff $\alpha \in K$ implies there exists $\beta \in K$, such that $f\beta \subset \alpha$. An epi sink $\{i_1, i_2 : (A, K) \to (A_1, K_1)\}$ is final in $ConSCO$ iff $\alpha \in K_1$ implies there exists β in K such that $i_k\beta \subset \alpha$ for some $k = 1, 2$. These are special cases of 1.7 .

Let α and β be stacks on X, γ a stack on Y, and $f : X \to Y$ a function.

1.9 Definitions.

$\alpha \cup \beta = \{U | U \subset X$ and $V \subset U$ for some V in α or $\beta\}$

$\alpha \cap \beta = \{U | U \subset X$ and $U \in \alpha$ and $\beta\}$

$f^{-1}(\gamma) = \{U | U \subset X \text{ and } f^{-1}(W) \subset U \text{ for some } W \text{ in } \gamma\}$

$f(\alpha) = \{V | V \subset Y \text{ and } f(U) \subset V \text{ for some } U \text{ in } \alpha\}$

1.10 Lemma. $f(\alpha \cup \beta) = f(\alpha) \cup f(\beta)$ and $f(\alpha \cap \beta) = f(\alpha) \cap f(\beta)$.

Proof. See [2].

Let X be a set and p a point in X. Let $X \bigvee_p X$ be the wedge product of X with itself, i.e. two distinct copies of X identified at the point p. A point x in $X \bigvee_p X$ will be denoted by $x_1(x_2)$ if x is in the first (resp. second) component of $X \bigvee_p X$. Let $X^2 = X \times X$ be the cartesian product of X with itself.

1.11 Definition. The principal p axis map, A_p: $X \bigvee_p X \to X^2$ is defined by $A_p(x_1) = (x_1, p)$ and $A_p(x_2) = (p, x_2)$

1.12 Definition. The skewed p axis map, S_p: $X \bigvee_p X \to X^2$ is defined by $S_p(x_1) = (x_1, x_1)$ and $S_p(x_2) = (p, x_2)$.

1.13 Definition. The fold map at p, $\nabla_p : X \bigvee_p X \to X$ is given by $\nabla_p(x_i) = x$ for $i = 1, 2$.

1.14 Example. If X is the set of real numbers and $p = 0$, then the image of the principal p axis map is just the union of the $x-$ and $y-$ axes, and the image of the skewed p axis map is the union of the diagonal i.e. the line $y = x$ and the $y-$axis.

In this way, we may view the image of A_p and S_p as "axes" in X^2 with origin p.

Let $U : E \to$ Sets be a topological functor [3], X an object in E, and p a point in $UX = B$.

1.15 Definitions.

1. X is $\overline{T}_0$ at p iff the initial lift of the U-source $\{A_p : B \bigvee_p B \to U(X^2) = B^2$ and $\nabla_p : B \bigvee_p B \to UDB = B\}$ is discrete, where DB is the discrete structure on B.

2. X is T_o' at p iff the initial lift of the U-source $\{id : B\bigvee_p B \to U(X\bigvee_p X) = B\bigvee_p B$ and $\nabla_p : B\bigvee_p B \to UDB = B\}$ is discrete, where $X\bigvee_p X$ is the wedge in E i.e. the final lift of the U-sink $\{i_1, i_2 : UX = B \to B\bigvee_p B\}$ where i_1, i_2 denote the canonical injections.

3. X is <u>$Pre\overline{T_2}$ at p</u> iff the initial lift of the U-source $\{S_p : B\bigvee_p B \to U(X^2) = B^2\}$

and the initial lift of the U-source $\{A_p : B\bigvee_p B \to U(X^2) = B^2\}$ agree.

4. X is <u>T_1 at p</u> iff the initial lift of the U-source $\{S_p : B\bigvee_p B \to U(X^2) = B^2$ and $\nabla_p : B\bigvee_p B \to UDB = B\}$ is discrete.

5. X is <u>$PreT_2'$ at p</u> iff the initial lift of the U-source $\{S_p : B\bigvee_p B \to U(X^2) = B^2$ and the final lift of the U-sink $\{i_1, i_2 : UX = B \to B\bigvee_p B\}$ agree.

6. X is <u>$\overline{T_2}$ at p</u> iff X is $\overline{T_o}$ at p and $Pre\overline{T_2}$ at p.

7. X is <u>T_2' at p</u> iff X is T_o' at p and $PreT_2'$ at p.

1.16 <u>Theorem</u>. For the category of topological spaces we have:

1. $\overline{T_o}$ at p is equivalent to T_o' at p and they both reduce to the following (called T_o at p in [1]): for each point x distinct from p, there exists a neighborhood of x missing p or there exists a neighborhood of p missing x.

2. $Pre\overline{T_2}$ at p is equivalent to $PreT'$ at p and they both reduce to the following (called $PreT_2$ at p in [1]): for each point x distinct from p, if the set $\{x, p\}$ is not indiscrete, then there exist disjoint neighborhoods of x and p.

3. $\overline{T_2}$ at p is equivalent to T_2' at p and they both reduce to (called T_2 at p in [1]): for each point x distinct from p, there exist disjoint neighborhood of x and p.

<u>Proof</u>: [1].

1.17 <u>Remark</u>. We define p_1, p_2, ∇_p by $1 + p, p + 1, 1 + 1 : B\bigvee_p B \to B$, respectively where $1 : B \to B$ is the identity map, $p : B \to B$ is constant map at p, and $\pi_i : B^2 \to B$ is the ith projection $i = 1, 2$. Note that $\pi_1 A_p = p_1 = \pi_1 S_p$, $\pi_2 A_p = p_2$, $\pi_2 S_p = \nabla_p$.

2. Separation Properties at p

In this section, we give explicit characterizations of the generalized separation properties at p for the topological categories of Stack Convergence Spaces, SCO and Constant Stack Convergence Spaces, $ConSCO$.

2.1 Theorem. $X = (B, K)$ in SCO is $\overline{T_o}$ at p iff for each $x \neq p$, $K(x) = \{\alpha | \alpha \supset [x]\}$.

Proof. Assume X is $\overline{T_o}$ at p i.e. for any stack σ on the wedge and for any point z in the wedge, $p_1\sigma \in K(p_1z)$, $p_2\sigma \in K(p_2z)$, and $\nabla\sigma \supset [\nabla z]$ iff $\sigma \supset [z]$. Suppose there exists a stack α in $K(x)$ such that $\alpha \not\supset [x]$ for some $x \neq p$. Let $\sigma = i_1\alpha \cup [(p,x)]$. By 1.10: $p_1\sigma = \alpha \cup [p]$, $p_2\sigma = [p] \cup [x]$, and $\nabla\sigma = \alpha \cup [x]$ and consequently $p_1\sigma \in K(x)$, $p_2\sigma \in K(p)$ and $\nabla\sigma \supset [x]$. Hence $\sigma \supset [(x,p)]$ since X is $\overline{T_o}$ at p. Since $x \neq p$, $(x,p) \in i_1\alpha$ and consequently $(x,p) \supset i_1V$ for some $V \in \alpha$. Clearly $V = \{x\}$ i.e. $\alpha \supset [x]$, a contradiction. Hence we must have that for each $x \neq p$, $K(x) = \{\alpha | \alpha \supset [x]\}$.

Conversely, suppose the condition holds and $x \neq p$. If σ satisfies $p_1\sigma \in K(x)$, $p_2\sigma \in K(p)$, and $\nabla\sigma \supset [x]$, then by the assumption $p_1\sigma \supset [x]$ and consequently $\sigma \supset [(x,p)]$. Similarly, if σ satisfies $p_1\sigma \in K(p)$, $p_2\sigma \in K(x)$ and $\nabla\sigma \supset [x]$, then $\sigma \supset [(p,x)]$. If $x = p$ and σ satisfies $p_1\sigma \in K(p), p_2\sigma \in K(p)$, and $\nabla\sigma \supset [p]$ then $\sigma \supset [(p,p)]$. Hence X is $\overline{T_o}$ at p.

2.2 Theorem. $X = (B, K)$ in SCO is T_o' at p iff for each $x \neq p$, $K(x) = \{\alpha | \alpha \supset [x]\}$.

Proof. Assume X is T_o' at p i.e. for any stack σ on the wedge and any point z in the wedge, $\sigma \supset i_k\sigma_1$ for some $\sigma_1 \in K(x)$, where $i_k(x) = z$, $k = 1$ or 2 and $\nabla\sigma \supset [x]$ iff $\sigma \supset [z]$. Suppose there exists $\alpha \in K(x)$ such that $\alpha \not\supset [x]$ for some $x \neq p$. Let $\sigma = i_1\alpha \cup [(p,x)]$. By 1.10, $\nabla\sigma = \alpha \cup [x] \supset [x]$ and $\sigma \supset i_1\alpha$, $i_1(x) = (x,p)$. Hence $\sigma \supset [(x,p)]$ and consequently $(x,p) \supset i_1V$ for some $V \in \alpha$. Thus $V = \{x\}$ i.e. $\alpha \supset [x]$, a contradiction. Therefore we must have $K(x) = \{\alpha | \alpha \supset [x]\}$ for all $x \neq p$.

Conversely if $x \neq p$ and $\sigma \supset i_1\sigma_1$ for some $\sigma_1 \in K(x)$, and $\nabla\sigma \supset [x]$, then by the assumption, $\sigma_1 \supset [x]$ and consequently $\sigma \supset i_1\sigma_1 \supset i_1[x] = [(x,p)]$. Similarly, if

$x \neq p$ and $\sigma \supset i_2\sigma_2$ for some $\sigma_2 \in K(x)$ and $\nabla\sigma \supset [x]$, then $\sigma \supset [(p,x)]$. If $x = p$ and $\sigma \supset i_k\sigma_1$ for some $\sigma_1 \in K(p)$ where $i_k(p) = (p,p)$ for some $k = 1$ or 2, and $\nabla\sigma \supset [p]$, then it follows easily that $\sigma \supset [(p,p)]$. Hence X is T_0' at p.

2.3 Theorem. $X = (B,K)$ in SCO is T_1 at p iff $B = p$.

Proof. Suppose X is T_1 at p i.e. by 1.4 , 1.17 , 1.5 and definition 1.15 for any stack σ on the wedge and any point z in the wedge, $p_1\sigma \in K(p_1z)$, $\nabla\sigma \in K(\nabla z)$, and $\nabla\sigma \supset [\nabla z]$ iff $\sigma \supset [z]$. If $B \neq p$ then there exists $x \in B$ such that $x \neq p$. Let $\sigma = [(p,p)] \cup [(x,p)]$. Note that $p_1\sigma = [p] \cup [x] \in K(p)$, $\nabla\sigma = [p] \cup [x] \in K(x)$, and $\nabla\sigma \supset [x]$. Hence $\sigma \supset [(p,x)]$ since X is T_1 at p. But this is impossible since $x \neq p$. Hence $B = p$. Conversely if $B = p$, then clearly X is T_1 at p (since $\sigma = [(p,p)]$ or $[\phi]$).

2.4 Theorem. $X = (B,K)$ in SCO is $Pre\overline{T_2}$ at p iff for each x in B, $K(x) =$ $STK(B) =$ the set of all stacks on B.

Proof. Suppose X is $Pre\overline{T_2}$ at p i.e. by 1.5 , 1.17 , and definition 1.15 for any stack σ on the wedge and any point z in the wedge if $p_1\sigma \in K(p_1z)$, then $p_2\sigma \in K(p_2z)$ iff $\nabla\sigma \in K(\nabla z)$. We first show that $K(p) = STK(B)$. In order to establish this we need only to show that $[B] \in K(p)$ since every stack contains the set B. Let $\sigma = ["x-axis", "y-axis"]$. Clearly $p_1\sigma = [p] = p_2\sigma \in K(p)$ and consequently $\nabla\sigma = [B]$ is in $K(p)$ (since X is $Pre\overline{T_2}$ at p). Therefore $K(p) = STK(B)$. We next show that $K(x) = STK(B)$ for $x \neq p$. To this end, let $\sigma = [(x,p),(x,p) \cup (p,x), "x-axis"]$. It follows easily that $p_1\sigma = [x]$, $p_2\sigma = [p]$, and $\nabla\sigma = [x]$. Since $p_1\sigma = [x] \in K(p) = STK(B)$ and $\nabla\sigma = [x] \in K(x)$, it follows that $p_2\sigma = [p] \in K(x)$ (since X is $Pre\overline{T_2}$ at p). If $\sigma = ["x-axis", "y-axis"]$, then $p_1\sigma = [p] \in K(x)$, $p_2\sigma = [p] \in K(p)$ and consequently $\nabla\sigma = [B] \in K(x)$ i.e. $K(x) = STK(B)$. The converse is trivial.

2.5 Theorem. $X = (B,K)$ in SCO is $PreT_2'$ at p iff $B = p$.

Proof. Suppose X is $PreT_2'$ at p i.e. by 1.4 , 1.17 , 1.7. and definition 1.15 for any stack σ on the wedge and any point z in the wedge, $p_1\sigma \in K(p_1z)$ and $\nabla\sigma \in K(\nabla z)$ iff $\sigma \supset i_k\sigma_1$ for some σ_1 in $K(x)$, where $i_k(x) = z$, k=1 or 2. If $B \neq p$, then there exists $x \in B$ such that $x \neq p$. Let $\sigma = ["y-axis", (x,p) \cup (p,x)]$. Clearly $p_1\sigma = [p] \in K(p)$ and $\nabla\sigma = [x] \in K(x)$. Hence $\sigma \supset i_1\sigma_1$, for some

$\sigma_1 \in K(x)$ and $i_2(x) = (p,x)$. We show that $\sigma_1 = [B]$. Clearly $[B] \subset \sigma_1$. To show the reverse, if $U \in \sigma_1$, then $i_2U \in i_2\sigma_1 \subset \sigma$, and consequently $i_2U \in \sigma$. Hence $i_2U \supset$ "$y-axis$" i.e. $U = B$. Thus $\sigma_1 = [B]$. Since $\sigma_1 \in K(x)$, it follows that $K(x) = STK(B)$. We next let $\sigma = [(x,p) \cup (p,x)]$. Clearly $p_1\sigma = [x \cup p] \in K(x)$ and $\nabla\sigma = [x] \in K(x)$, and consequently $\sigma \supset i_1\sigma_1$ for some $\sigma_1 \in K(x)$. Hence $i_1V \in i_1\sigma_1 \subset \sigma$ for some $V \in \sigma_1$ and consequently $i_1V \supset (x,p) \cup (p,x)$ which is impossible. Hence $B = p$. Conversely, if $B = p$, then clearly X is $PreT_2'$ at p.

2.6 __Theorem__. $X = (B,K)$ in SCO is $\overline{T_2}$ at p iff $B = p$.

__Proof__. Recall X is $\overline{T_2}$ at p iff X is $\overline{T_o}$ at p and $Pre\overline{T_2}$ at p. Since X is $\overline{T_o}$ at p, $[p] \notin K(x)$ (2.1). But this is a contradiction to $K(x) = STK(B)$ i.e. $Pre\overline{T_2}$ at p (2.4). Hence $B = p$. On the other hand, if $B = p$, then clearly X is $\overline{T_o}$ at p and since $STK(p) = \{[p],[\phi]\} = K(p)$, X is $Pre\overline{T_2}$ at p by 2.4 . Hence X is $\overline{T_2}$ at p.

2.7 __Theorem__. $X = (B,K)$ in SCO is T_2' at p iff $B = p$.

__Proof__. Suppose X is T_2' at p i.e. by definition 1.15 X is T_o' at p and $PreT_2'$ at p.

Then in particular by 2.5 , $B = p$. Conversely, if $B = p$, then clearly X is T_o' at p and $PreT_2'$ at p (2.5). Hence X is T_2' at p.

2.8 __Remark__. $\overline{T_o}$ and T_o' at p agree and T_2' and $\overline{T_2}$ at p agree (2.1 , 2.2 , 2.6 , and 2.7) and $PreT_2'$ at p implies $Pre\overline{T_2}$ at p.

2.9 __Theorem__. $X = (B,K)$ in $ConSCO$ is $\overline{T_o}$ at p iff $B = p$.

__Proof__. Suppose X is $\overline{T_o}$ at p i.e. by 1.4 , 1.17 , 1.6 , and definition 1.15 for any stack σ on the wedge, $p_1\sigma \in K$, $p_2\sigma \in K$, and $\nabla\sigma \supset$ x for some x in B iff $\sigma \supset [z]$ for some z in the wedge. If $B \neq p$, then there exists x in B such that $x \neq p$. Let $\sigma = [$"$x-axis$","$y-axis$",$(x,p) \cup (p,x)]$. Clearly $p_1\sigma = [p] = p_2\sigma$ and $\nabla\sigma = [x]$. Hence $\sigma \supset [z]$ for some z in the wedge. It follows that $z \in \sigma$ which is impossible. Hence $B = p$. Conversely, if $B = p$, then clearly X is $\overline{T_o}$ at p.

2.10 __Theorem__. $X = (B,K)$ in $ConSCO$ is T_1 at p iff $B = p$.

__Proof__. Suppose X is T_1 at p i.e by 1.4 , 1.17 , 1.6 , and definition 1.15 for any stack σ on the wedge. $p_1\sigma \in K$, $\nabla\sigma \in K$, and $\nabla\sigma \supset [x]$ for some $x \in B$ iff $\sigma \supset [z]$ for some z in the wedge. If $B \neq p$, then there exists $x \in B$ such that

$x \neq p$. Let $\sigma = ["y-axis", (x,p) \cup (p,x)]$. Clearly $p_1\sigma = [p] \in K$, $\nabla\sigma = [x] \in K$, and consequently $\sigma \supset [z]$ for some z in the wedge which is a contradiction. Hence $B = p$. The converse is trivial.

2.11 Theorem. $X = (B, K)$ in $ConSCO$ is $Pre\overline{T_2}$ at p iff $K = STK(B)$.

Proof. Suppose $Pre\overline{T_2}$ at p i.e. by 1.6 , 1.17 , and definition 1.15 for any stack σ on the wedge if $p_1\sigma$ in K, then $p_2\sigma$ in K iff $\nabla\sigma \in K$. It is sufficient to show that $[B] \in K$ since every stack contains the set B. Let $\sigma = ["x-axis", "y-axis"]$. Clearly $p_1\sigma = [p] = p_2\sigma \in K$ and consequently $\nabla\sigma = [B] \in K$. Thus $K = STK(B)$. The converse is obvious.

2.12 Theorem. $X = (B, K)$ in $ConSCO$ is $PreT_2'$ at p iff $B = p$.

Proof. Suppose X is $PreT_2'$ at p i.e. by 1.6 , 1.17 , 1.8 , and definition 1.15 for any stack σ on the wedge, $p_1\sigma \in K$ and $\nabla\sigma \in K$ iff $\sigma \supset i_k\sigma_1$ for some σ_1 in K and for k=1 or 2. If $B \neq p$, then there exists $x \in B$ such that $x \neq p$. Let $\sigma = ["x-axis", "y-axis", (x,p) \cup (p,x)]$. Clearly $p_1\sigma = [p] \in K$ and $\nabla\sigma = [x] \in K$ and consequently $\sigma \supset i_k\sigma_1$ for some $\sigma_1 \in K$ and for some k=1 or 2. It readily follows that $\sigma_1 = [B]$ (the proof of 2.5) and consequently $K = STK(B)$. Let $\sigma = [(x,p) \cup (p,x)]$, where $x \neq p$. Clearly $p_1\sigma = [x \cup p] \in K = STK(B)$ and $\nabla\sigma = [x] \in K$. Since X is $PreT_2'$ at p, it follows that $\sigma \supset i_k\sigma_1$ for some $\sigma_1 \in K$. But clearly this is impossible. Hence $B = p$. The converse is obvious.

2.13 Theorem. $X = (B, K)$ in $ConSCO$ is $\overline{T_2}$ at p iff $B = p$.

Proof. This follows easily from 2.9 , 2.11 , and definition 1.15 .

2.14 Theorem. $X = (B, K)$ in $ConSCO$ is T_2' at p iff $B = p$.

Proof. Suppose X is T_2' at p i.e by definition 1.15 X is T_0' at p and $PreT_2'$ at p. In particular, by 2.12 , $B = p$. Conversely if $B = p$, then clearly X is T_0' at p and X is $PreT_2'$ at p (2.12). Hence X is T_2' at p.

2.15 Remark. $\overline{T_2}$ at p and T_2' at p are identical and $PreT_2'$ at p implies $Pre\overline{T_2}$ at p.

REFERENCES

[1]. M. Baran, Separation Properties, Indian J. Pure Appl. Math. (to appear).

[2]. M.Baran, Stacks and Filters, Turkish J. Math., DCGA (to appear).

[3]. H. Herrlich, Topological Functors, Gen. Top. Appl. 4 (1975) 125-142.

[4]. L.D. Nel, Initially Structured Categories And cartesian Closedness, Can. Journal of Math., Vol. XXVII, No. 6, (1975), 1361-1377.

[5]. F. Schwarz, Connections Between Convergence And Nearness, Lecture Notes in Math. No. 719, Springer-Verlag (1978), 345-354.