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ON THE POLAR INERTIA MOMENTUMS OF ORBITS FORMED UNDER CLOSED PLANAR MOTIONS

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SUMMARY:

In this paper,for the 1-parameter closed planar motion it is shown that all the fixed points of the moving plane whose trajectory curves have equal polar inertia momentum lie on the same circle of the moving plane.For the different values of the polar inertia momentums it is obtained different circles such that the centers of them are the same which is the Steiner point. And it is given another formula for the polar inertia momentum of the trajectory curve (X) in the fixed plane which doesn't depend on the components of X. Moreover it is seen that the difference of the polar inertia momentums of moving centrode with respect to a point on a line segment chosen on the moving plane and one of the end points of its doesn't depend on the motion.

KAPALI DÜZLEMSEL HAREKETLER ALTINDA OLUŞAN YÖRÜNGELERİN KUTUPSAL ATALET MOMENTLERİ ÜZERİNE

ÖZET

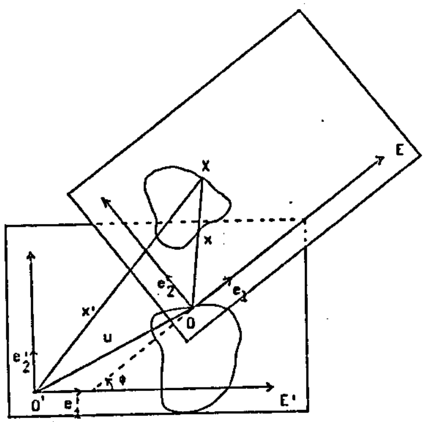
Bu çalışmada 1-parametreli kapalı düzlemsel hareketler altında hareketli düzlemin,yörünge eğrileri aynı kutupsal atalet momentine sahip olan sabit noktalarının geometrik yerinin, hareketli düzlemin bir çemberi olduğu görülür. Atalet momentinin muhtelif değerleri için hareketli düzlemin farklı yarıçaplı çemberleri elde edilir, öyle ki bu çemberlerin hepsinin de merkezi Steiner noktasıdır. Yörünge eğrilerinin kutupsal atalet momentleri için, seçilen noktanın bileşenlerinden bağımsız olan bir formül elde edildi.Ayrıca hareketli pol eğrisinin, uç noktaları aynı eğriyi çizen bir doğru parçası üzerindeki bir nokta ve uç noktalardan herhangi birine göre kutupsal atalet momentleri farkının hareketten bağımsız olduğu da gösterildi.

1-INTRODUCTION

In this paper, we are going to consider the planes as being complex planes, that is, each point X of the plane is to be observed as the representative of a complex number $x=x_1+ix_2$. An 1-parameter motion of plane is described in complex number notation by

(1) $x' = xe^{i\phi} + u'$

where x is a complex number which describes a point given in the so-called moving plane, say E, and x' is the complex number corresponding to the same point in the so-called fixed plane, say E'.



Moreover $u'=u'(t)$ and $\phi=\phi(t)$ are functions of a real single parameter t. And u' represents the origin of moving system expressed in the system of the fixed plane. Let the complex number $u=u_1+iu_2$ represent the origin of fixed system expressed in the system of moving plane. Then we have

(2) $u' = -ue^{i\phi}$.

If T is the smallest positive number satisfying the following equalities

(3) $$u_j(t+T)=u_j(t)\ ,\ j=1,2$$ $$\phi(t+T)=\phi(t)+2\pi\nu$$

then the motion given by (1) is called 1-parameter closed planar motion with the period T and rotation number ν [1]. Such a motion is going to be shown by B. For the moving pole points P and fixed pole points P' we have

(4) $p=u-idu/d\phi \quad , \quad p'=u+idu'/d\phi$

where $p=p_1+ip_2$ and $p'=p'_1+ip'_2$. On the other hand we obtain that

(5) $x\bar{x}=x\bar{x}-\bar{u}x-u\bar{x}+u\bar{u}$

where $\bar{x}$ and $\bar{u}$ are the complex conjugates of x and u, respectively.

2-ON THE POLAR INERTIA MOMENTUMS OF ORBITS

Let X be a fixed point in the moving plane E. Thus (1) defines a parametrized closed curve (X) in E' which is called the trajectory curve of X under the motion B. The polar inertia momentum of the curve (X), T_X, is given by

(6) $T_X = \oint x\bar{x}d\phi$

where the integration is taken along the closed curve (X) in E'. By using (4),(5) and (6) we obtain

(7) $T_X=x\bar{x}\oint d\phi-2x_1\oint(p_1d\phi-du_2)-2x_2\oint(p_2d\phi+du_1)$.

The Steiner point $s=s_1+is_2$ of the moving centrode (P) for the distribution of mass with the density $d\phi$ is given by

(8) $s_j=(\oint p_jd\phi)/(\oint d\phi) \quad , \quad j=1,2$

where the integrations are taken along the closed curve (P). Considering (3) and (8) in (7) we get

(9) $T_X=2\pi\nu(x\bar{x}-\bar{s}x-s\bar{x})+T_0$

where T_0 is the polar inertia momentum of the trajectory curve of the origin of the moving system [2]. We may rewrite (9) such as

(10) $x_1^2+x_2^2-2s_1x_1-2s_2x_2+(T_0-T_X)/(2\pi\nu)=0$

If we consider all the fixed points of the moving plane such that their trajectory curves have equal polar inertia momentum, then (10) shows a circle in the moving plane. Moreover the center of this circle is the Steiner point $s=s_1+is_2$. Hence we can give the following theorem:

Theorem 1: All the fixed points of the moving plane whose trajectory curves have equal polar inertia momentum lie on the same circle in the moving plane. For the different values of the polar inertia momentums, we obtain different circles whose centers are the same which is the Steiner point.

Now let us consider the circle given by

(11) $x_1^2+x_2^2-2s_1x_1-2s_2x_2+T_0/(2\pi\nu)=0$

and X be a fixed point of the moving plane which doesn't belong to the circle given by (11). $T_X/(2\pi\nu)$ is the power of the point X with respect to the circle given by (11). Hence we can give the following theorem:

Theorem 2: Let us consider a fixed point X of the moving plane and the trajectory curve (X) of X. Then the polar inertia momentum T_X of the closed curve (X) can be given by

$$T_X=2\pi\nu\lambda$$

where λ is the power of the point X with respect to a circle such that the center of this circle is the Steiner point and it is determined with the property that the power of X with respect to this circle is zero.

The area enclosed by the trajectory curve (X) is given by

$$F_X=1/2\oint(x_1'dx_2'-x_2'dx_1').$$

We see that

(12) $F_X=\pi\nu(x\bar{x}-\bar{s}x-s\bar{x})+F_0$

where F_0 is the area enclosed by the trajectory curve of the origin of moving system [3]. From (9) and (12) we get

(13) $T_X-T_0=2(F_X-F_0).$

For the area F_X we have also the following formula, [1]

(14) $F_X=F_P'-F_P+1/2\ T_{P/X}$

where F_P' and F_P are the areas enclosed by the fixed centrode (P') and the moving centrode (P), respectively and

$$T_{P/X}=\oint a^2 d\phi \quad , \quad a=|p-x|$$

$T_{P/X}$ can be considered as the polar inertia momentum of the moving centrode (P) with respect to the fixed point X. If Y is another fixed point of the moving plane then, from (14) we obtain

(15) $F_X-F_Y=1/2\ (T_{P/X}-T_{P/Y})$

Hence we can give the following corollary:

Corollary 1: Let X and Y be two fixed points of the moving plane. The areas of the trajectory curves of X and Y are the same if and only if the polar inertia momentums of the moving centrode with respect to the points X and Y are the same.

For the polar inertia momentums of the trajectory curves of X and Y, from (13) and (14) we have

(16) $T_X-T_Y=(T_{P/X}-T_{P/Y})$

Thus we can give the following corollaries:

Corollary 2: Cosider an one-parameter closed planar motion and two fixed points X and Y of the moving plane. If the polar inertia momentums of (X) and (Y) are the same then the polar inertia momentums of the moving centrode with respect to X and Y are the same.

Corollary 3: Let K,L,M and N be four fixed points of the moving plane such that two of them, say K and L, move on the same curve (K) while the other two move on another curve (M) and let the segments KL and MN meet in X.Then K,L,M and N lie on the same circle of moving plane if and only if the polar inertia momentums of the moving centrode with respect to these four points are the same.

Proof: If the polar inertia momentums of the moving centrode are the same then we have

$$T_{P/K}-T_{P/X}=T_{P/M}-T_{P/X}.$$

Cosidering corollary 1.we see that

$$F_K-F_X=F_M-F_X.$$

From the Holditch's theorem we obtain that

(17) $\overline{KX}.\overline{XL}=\overline{MX}.\overline{XN}$

The last equation shows that K,L,M and N lie on the same circle of the moving plane. Conversely, if (17) is satisfied then it can be easily obtained that the polar inertia momentums of the moving centrode with respect to K,L,M and N are the same. This is to be shown.

Taking Y=0 in (15) we get

(18) $F_X-F_0=1/2\ (T_{P/X}-T_{P/0})$

Joining (18) and (13) we obtain

$$T_X=T_0-T_{P/0}+T_{P/X}$$

The last equation is very important because it doesn't depend on the coordinates of the chosen fixed point X.

Now, let us consider two different fixed points X and Y of the moving plane such that during the motion both of them draw the same closed curve. And let us choose another fixed point Z on the line segment XY. From (15) we obtain

$$1/2\ (T_{P/X}-T_{P/Z})=F_X-F_Z=\pi ab$$

or

$$T_{P/X}-T_{P/Z}=2\pi ab$$

where $a=\overline{XZ}$ and $b=\overline{YZ}$. Hence we can give the following corollary:

Corollary 4: The difference of the polar inertia momentums of the moving centrode with respect to the points X and Z doesn't depend on the motion but only on the distances of Z to the end points.

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