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STRONGLY-CONSERVATIVE SEQUENCE-TO-SERIES MATRIX TRANSFORMATIONS

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SUMMARY

In this note, the class (f:cs) of strongly-conservative matrices has been characterized. Besides this, a corollary which characterizes the class $(f:cs)_r$ of strongly-multiplicative matrices and a theorem of Steinhaus type which is stated as "a matrix can not be both strongly-multiplicative and coercive" have been given.

DİZİDEN-SERİYE KUVVETLİ KONSERVATİF MATRİS DÖNÜŞÜMLERİ

ÖZET

Bu çalışmada, kuvvetli konservatif matrislerin (f:cs) sınıfı karakterize edildi. Bundan başka, kuvvetli çarpımsal matrislerin $(f:cs)_r$'sınıfını karakterize eden bir sonuç ile "bir matris hem kuvvetli çarpımsal ve hem de coercive olamaz" diye ifade edilen Steinhaus tipi bir teorem verildi.

1. INTRODUCTION

Let m denote the linear space of all bounded real sequences. The shift operator S on m is defined by $(Sx)_n=x_{n+1}$. A Banach limit L is defined as a non-negative linear functional on m ([1],p.32) such that L(Sx) = L(x) and L(e) = 1, where e = (1,1,1,..). A sequence $x \varepsilon$ m is said to be almost convergent to the generalized limit x_o if all Banach limits of x is x_o [3]. This is denoted by f-lim$x=x_o$. It is proved by Lorentz [3] that f-lim$x=x_o$ if and only if $\lim_p(x_n+.....+x_{n+p-1})/p=x_o$ uniformly in n. It is well-known that a convergent sequence is almost convergent such that its limit and its generalized limit are equal. By f, we denote the linear space of all almost convergent sequences.

An infinite matrix $A = (a_{nk}),(n,k=0,1,\ldots)$ defines a transformation from the sequence space λ into the sequence space μ if for each $x\in\lambda$, the matrix product $Ax = (\sum_k a_{nk}x_k)$ exists and is in μ. Here and afterwards $\sum_k$ will denote the summation from k=0 to ∞. By $(\lambda:\mu)$, we denote the class of all such matrices. If there is some notion of limit or sum in λ and μ, then we write $(\lambda:\mu;P)$ to denote the subclass of $(\lambda:\mu)$ which preserves the limit or sum. Further, $A\in(\lambda:\mu)$ is said to be strongly-multiplicative r if $\lim Ax = r(f\text{-}\lim x)$ for each $x\in\lambda$, where $\lambda = f, fs$ and $\mu = c, cs$. We should note here that c denotes the linear space of all convergent sequences and cs,fs also denote the linear spaces of all convergent, almost convergent series, respectively. By $(\lambda:\mu)_r$, we denote the class of all such matrices. It is now trivial in the case r=1 that the class $(\lambda:\mu)_r$, coincides with the class $(\lambda:\mu;P)$ and thus it is immediate that $(\lambda:\mu;P)\subset(\lambda:\mu)_r\subset(\lambda:\mu)$.

Theorems of Steinhaus type are firstly formulated by Maddox [4], with above notations, as follows : Let λ, μ, ω be sequence spaces and suppose $\omega\supset\lambda$. Then we shall call a result of the form $(\lambda:\mu;P)\cap(\omega:\mu) = \Phi$ a theorem of Steinhaus type.

The class (f:c;P) was characterized by Lorentz [3]. Later, the characterization of the class (f:c) was given by Sıddıqi [6]. The classes (fs:c;P) and (fs:cs;P) were characterized by Öztürk [5] and Başar-Çolak [2], respectively. By strong-conservativity of any summability method, we mean the method belonging to one of the classes (f:c), (fs:c), (fs:cs) or (f:cs). The object of this note is to characterize the class (f:cs) and in this way to fill up a gap in the existing literature. Moreover, a theorem of Steinhaus type which asserts that a strongly-multiplicative sequence-to-series transformation can not simultaneously be coercive, has been stated and proved.

2. SEQUENCE-TO-SERIES TRANSFORMATIONS

In this section, we give necessary and sufficient conditions on the infinite matrix $A=(a_{nk})$ in order that A should transform f into cs. To do this, we require the following lemma due to Sıddıqi [6] which characterizes the class (f:c):

Lemma 2.1. The method A transforms f into c if and only if

$$\sup_n \sum_k |a_{nk}| < \infty \, , \tag{2.1}$$

$$\lim_n a_{nk} = a_k \text{ for each } k, \tag{2.2}$$

$$\lim_n \sum_k a_{nk} = a \text{ exists}, \tag{2.3}$$

$$\lim_n \sum_k |\Delta(a_{nk}-a_k)| = 0, \text{ where } \Delta(a_{nk}-a_k) = a_{nk}-a_k-(a_{n,k+1}-a_{k+1}). \tag{2.4}$$

Now we have the following:

Theorem 2.2. The method A transforms f into cs if and only if

$$\sup_n \sum_k \Big| \sum_{i=0}^{n} a_{ik} \Big| < \infty, \tag{2.5}$$

$$\sum_n a_{nk} = a_k \text{ for each } k, \tag{2.6}$$

$$\sum_n \sum_k a_{nk} = a, \tag{2.7}$$

$$\lim_n \sum_k \Big| \sum_{i=0}^{n} \Delta(a_{ik}-a_k) \Big| = 0. \tag{2.8}$$

Proof. Necessity. Let $A\in(f:cs)$ and $x\in f$. Since e^k and e are in f, the necessities of (2.6) and (2.7) are easily obtained by taking $x=e^k$ and $x=e$, respectively. Where e^k is the sequence whose only non-zero terms is a 1 in the k^{th} place.

Now, consider the following equality obtained from the m^{th} partial sums of $\sum_{i=0}^{n}(Ax)_i$ by reversing the order of summation:

$$\sum_{i=0}^{n}\sum_{k=0}^{m} a_{ik}x_k = \sum_{k=0}^{m}\left(\sum_{i=0}^{n} a_{ik}\right)x_k \ ; \ n,m=0,1,\ldots \tag{2.9}$$

which yields by letting $m\to\infty$ that

$$\sum_{i=0}^{n}\sum_{k} a_{ik}x_k = \sum_{k}\left(\sum_{i=0}^{n} a_{ik}x_k\right) \ ; \ n=0,1,\ldots \tag{2.10}$$

Letting $n\to\infty$ in (2.10), we see that $B=(b_{nk})\in(f:c)$, where $b_{nk}=\sum_{i=0}^{n} a_{ik}$ for all n, k. Thus, $B=(b_{nk})$ satisfies (2.1), (2.4) and these are respectively equivalent to (2.5), (2.8).

Sufficiency. Suppose the conditions (2.5)-(2.8) hold and $x\in f$. Again consider the matrix $B=(\sum_{i=0}^{n} a_{ik})$ in (2.9). Therefore, it is immediate that "$B=(b_{nk})$ satisfies (2.1), (2.2), (2.3) and (2.4) if and only if $A=(a_{nk})$ satisfies (2.5), (2.6), (2.7) and (2.8), respectively." Hence, $B\in(f:c)$ and this yields by letting $n\to\infty$ in (2.10) that Ax cs and this step concludes the proof.

As an easy consequence of Theorem 2.2, we have

Corrolary 2.3. a) $A\in(f:cs)_r$ if and only if (2.5) holds, (2.6) and (2.8) hold with $a_k=0$ for each k and (2.7) also holds with a=r.

b) $A\in(f:c_os)$ if and only if (2.5) holds, (2.6) and (2.8) hold with $a_k=0$ for each k and (2.7) also holds with a=0, where c_os denotes the linear space of those series converging to zero.

We now give a lemma due to Stieglitz-Tietz [7] which characterizes the class (m:cs):

Lemma 2.4. The method A transforms m into cs if and only if (2.6) holds and

$$\sum_k \left| \sum_{i=0}^{n} a_{ik} \right| \tag{2.11}$$

converges, uniformly in n.

Now, we can give the next theorem of Steinhaus type about the strongly -multiplicative and coercive matrix classes:

Theorem 2.5. The classes $(f:cs)_r$ and $(m:cs)$ are disjoint.

Proof. Suppose now that the converse of this is true and $A \in (f:cs)_r \cap (m:cs)$. Then, by condition (2.6) of Corollary 2.3.a), we have

$$\lim_n \left| \sum_{i=0}^{n} a_{ik} \right| = 0$$

Combining this with (2.11) we get that

$$\lim_n \sum_k \left| \sum_{i=0}^{n} a_{ik} \right| = \sum_k \left| \sum_i a_{ik} \right| = 0$$

which contradicts (2.7) of Corollary 2.3.a).

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