Kütahya Dumlupınar University
Institute of Graduate Studies

Journal of Scientific Reports-A
E-ISSN: 2687-6167
Number 50, September 2022

RESEARCH ARTICLE

PD CONTROLLER DESIGN and STABILITY ANALYSIS for SYSTEMS HAVING FRACTIONAL ORDER DELAY

Münevver Mine ÖZYETKİN[1*]

[1]Aydın Adnan Menderes University, Faculty of Engineering, Department of Electrical and Electronics Engineering, Aydın, m.ozyetkin@adu.edu.tr, ORCID: 0000-0002-3819-5240

Receive Date:05.07.2022 *Accepted Date: 22.08.2022*

ABSTRACT

Fractional order systems (FOS) are one of the subjects that have been studied extensively. Such systems are difficult and complex structures to be analyzed mathematically. The degree of difficulty is even greater when the system has time delay. Considering the studies on FOS, either the transfer function of the system is of fractional order, or the controller has a fractional order structure. Time delay is common in practical systems. Studies mostly focused on classical time delay term. However, time delay can be of a fractional order. Studies for such systems are quite limited as the mathematical analysis part is complex.

In this study, systems having fractional order delay, are examined. By using the stability boundary locus (SBL) method, the necessary equations for calculating all stable PD controller parameters for such systems are obtained. It is necessary to test whether the parameters selected from the obtained stability region provide stability. However, stability analysis of such systems is very problematic. For this reason, an approximation method previously proposed by the author is used. Thus, the system is transformed to a fractional order structure. Then, the stability analysis of the FOS can be easily done. It is seen that the obtained equations and the approximation method proposed by the author for the stability test provide quite reasonable results.

Keywords: *Fractional Order System, Time Delay, PD Controller, Stability Region*

1. INTRODUCTION

FOS have transfer functions expressed by fractional order derivatives and/or fractional order integrals. Although this is a very old topic, it has been mostly studied by the mathematicians [1]. With the development of modern physics, the application research of fractional order operators has increased [1]. The memory feature of such systems and the emergence of solution techniques also have an effect on this. The voltage-current relation of a semi-infinite lossy RC line is given as an example of FOS [2].

A control system can have both fractional order dynamics and be controlled by a fractional order controller. Many important studies have been done on the fractional order PID (Proportional Integral Derivative) controllers. Some important examples can be found in [3–5]. Since the PID controller is a

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widely known controller structure, there are many studies on this subject, and it is still being studied. In this regard, [6–9] can be examined for some important studies based on the stability region concept. Despite its performance limitations, the PD (Proportional Derivative) controller is widely used for robot position control due to simplicity [10]. Position controllers often include PD control. Robot force control systems are commonly included PD or PID controllers [10]. The adaptive PD controller for robot manipulators were studied in [11]. The PD controller design for the active suspension system was made in [12]. However, studies on PD in the literature are still limited.

Time delays are frequently encountered in industrial applications such as heat exchanges, distillation units, mining processes, and steel production, etc. [13]. Time-delay processes range from biological systems to mechanical systems, including economic or electric fields [13]. The classical time delay is known as $\exp(-s\tau)$ in the s domain. However, there are cases where the time delay is fractional order. For example, the transfer function of the terminated resistance-capacitance line is $\exp(-\sqrt{sT})$ [14]. Here, T is distributed lag. For simplicity, the normalized form $\exp(-\sqrt{s})$ is often used [14]. In addition to the electrical transmission line, certain types of delays in servo mechanisms and other mechanical and thermal phenomena also appear to have transfer functions of interest [14]. Studies have mainly focused on the stability of a class of distributed systems. Transfer functions of such systems involve $\sqrt{s}$ and/or $\exp(-\sqrt{s})$. As noted in [15], many circuits, processes, or systems such as thermal processes, hole diffusion of transistors, electromagnetic devices, and transmission lines have distributed parameters and/or delay elements. Stability analysis of linear distributed parameter systems (DBS) is complex since the Laplace domain representation of these systems include either the square root sign of s or other irrational or transcendental functions [15,16]. Mathematical descriptions of linear DBS with a distributed lag include double valued functions of s in the form of $\sqrt{s}$ and $\exp(\sqrt{s})$. Although it is a very interesting subject, studies in the literature are quite limited. Some of these studies can be summarized as follows. In [17], Pontryagin's stability criterion is extended to systems with distributed delay. In [15], the algebraic stability test procedures are extended to a certain class of DBS with multiple delays. An algebraic stability test procedure such as the Routh is extended to a certain class of DBS with a distributed lag in [16]. When both studies are examined, it is seen that the $\sqrt{s}$ plane is considered. The basis on which it is based is the Riemann surface, and it is seen that the stability in this regard is defined by the Brin criterion [16]. The Brin criterion is an important step in the stability definition of the FOS. In [18], analytical stability bound has been obtained using the Lambert function W for a class of delayed fractional-order differential equations with constant coefficients. A numerical algorithm for testing the BIBO stability of fractional delay systems (FDS) is presented in [19]. BIBO stability of a class of FDS with commensurate orders and multiple commensurate delays of retarded type is studied in [20]. A study was carried out on FOS of retarded type with two independent delays, and the stability regions in spaces of delays were determined in [21]. Recently, a method using frequency domain data to obtain time response of FOS has been reported in [22]. Thanks to this method computation of time responses of FOS is possible. These studies involve very complex procedures and are not easy to understand. Also, these studies do not provide universally acceptable practical algebraic criteria or algorithms. In [23], the author proposed an approximation method to investigate stability of the systems having integer order and non-integer order delay. Using this approximation, the time delay in fractional structure can be converted into a fractional order transfer function (FOTF) and the procedures applied for FOTFs can be applied to these systems. In addition, in this study, PID design has been made for systems with fractional order time delay (FOTD), As stated before, the studies related on this topic is very restricted. Therefore, there is a need for new research on this subject. This study is the first application of the approximation method presented in [23] for PD controllers. The equations obtained here are completely original.

This study is aimed to calculate all stable PD controller parameters for FOTD processes using the stability region method. In addition, an approximation method previously proposed by the author is briefly presented to test whether the parameters selected from the obtained stability region provide stability.

2. MATERIALS and METHODS

In this section, PD controller is designed for systems with FOTD. The necessary equations have been obtained based on the SBL method. There is extensive literature information about this method. For some of them [6,8,9,24–27] can be examined. To the best of the author's knowledge, there are no studies on PD controller design and stability regions for systems with FOTD. The author's work on PID design on this subject can be seen in [23]. This section also presents an approximation method required for stability test.

2.1. Obtaining PD Parameters Using Stability Region Method

Let the transfer function (TF) of the system given in Figure 1 be defined as follows.

$$G_p(s) = G(s)e^{-(\tau s)^{\alpha}} = \frac{N(s)}{D(s)}e^{-\tau^{\alpha}s^{\alpha}} \tag{1}$$

The TF of the PD controller is defined by Eq. (2),

$$C(s) = k_p + k_d s \tag{2}$$

The characteristic equation (CE) of the system is obtained as follows.

$$\Delta(s) = 1 + C(s)G_p(s) = 1 + (k_p + k_d s)\frac{N(s)}{D(s)}e^{-\tau^{\alpha}s^{\alpha}} = D(s) + (k_p + k_d\, s)N(s)e^{-\tau^{\alpha}s^{\alpha}} \tag{3}$$

Let us express the TF of the system as follows.

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega N_o(-\omega^2)}{D_e(-\omega^2) + j\omega D_o(-\omega^2)} = \frac{N_e + j\omega N_o}{D_e + j\omega D_o} \tag{4}$$

For simple notation, the expression $(-\omega^2)$ will not be written in the next equations. In this case, the CE is obtained as follows.

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$$\begin{aligned}
\Delta(j\omega) &= (D_e + j\omega D_o) + (k_p + j\omega k_d)(N_e + j\omega N_o)\left(e^{-\tau^{\alpha}(j\omega)^{\alpha}}\right) \\
&= (D_e + j\omega D_o) + [(k_p + j\omega k_d)(N_e + j\omega N_o)](e^{-\left[(\cos\frac{\pi}{2}\alpha + j\sin\frac{\pi}{2}\alpha)\omega^{\alpha}\tau^{\alpha}\right]}) \\
&= (D_e + j\omega D_o) + (k_p + j\omega k_d)(N_e + j\omega N_o)e^{-(\cos\frac{\pi}{2}\alpha)\omega^{\alpha}\tau^{\alpha}}e^{-j(\sin\frac{\pi}{2}\alpha)\omega^{\alpha}\tau^{\alpha}} \\
&= (D_e + j\omega D_o) + (k_p + j\omega k_d)(N_e + j\omega N_o)e^{-(\cos\frac{\pi}{2}\alpha)\omega^{\alpha}\tau^{\alpha}} \\
&\times[\cos(\omega^{\alpha}\tau^{\alpha}(\sin\frac{\pi}{2}\alpha)) - j\sin(\omega^{\alpha}\tau^{\alpha}(\sin\frac{\pi}{2}\alpha))]
\end{aligned} \tag{5}$$

Let Eqs. (6) and (7) be expressed as follows.

$$e^{-(\cos\frac{\pi}{2}\alpha)\omega^{\alpha}\tau^{\alpha}} = e^{-m} \tag{6}$$

$$\omega^{\alpha}(\sin\frac{\pi}{2}\alpha)\tau^{\alpha} = n \tag{7}$$

Using these equations, the CE is rearranged as follows.

$$\begin{aligned}
\Delta(j\omega) &= (D_e + j\omega D_o) + (k_p + j\omega k_d)(N_e + j\omega N_o)e^{-m}[\cos(n) - j\sin(n)] \\
&= D_e + k_p N_e e^{-m}\cos(n) - \omega^2 k_d N_o e^{-m}\cos(n) + \omega k_p N_o e^{-m}\sin(n) + \omega k_d N_e e^{-m}\sin(n) \\
&+ j[\omega D_o + \omega k_p N_o e^{-m}\cos(n) + \omega k_d N_e e^{-m}\cos(n) - k_p N_e e^{-m}\sin(n) + \omega^2 k_d N_o e^{-m}\sin(n)]
\end{aligned} \tag{8}$$

If Eq. (8) is set to zero, the real and imaginary parts are obtained as follows.

Real Part:

$$\mathrm{Re}\,\Delta(j\omega) = k_p[N_e e^{-m}\cos(n) + \omega N_o e^{-m}\sin(n)] + k_d[\omega N_e e^{-m}\sin(n) - \omega^2 N_o e^{-m}\cos(n)] = -D_e \tag{9}$$

Imaginary Part:

$$\mathrm{Im}\,\Delta(j\omega) = k_p[\omega N_o e^{-m}\cos(n) - N_e e^{-m}\sin(n)] + k_d[\omega^2 N_o e^{-m}\sin(n) + \omega N_e e^{-m}\cos(n)] = -\omega D_o \tag{10}$$

Eqs. (11) and (12) are obtained using Eqs. (9) and (10), The solution of these equations gives Eqs. (13) and (14),

$$k_p A(\omega) + k_d\,B(\omega) = X(\omega) \tag{11}$$

$$k_p\,C(\omega) + k_d\,D(\omega) = Y(\omega) \tag{12}$$

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$$k_p = \frac{X(\omega)D(\omega) - Y(\omega)B(\omega)}{A(\omega)D(\omega) - C(\omega)B(\omega)} \tag{13}$$

$$k_d = \frac{Y(\omega)A(\omega) - X(\omega)C(\omega)}{A(\omega)D(\omega) - C(\omega)B(\omega)} \tag{14}$$

Here, the parameters are as follows.

$$\begin{aligned} X(\omega) &= -D_e \\ Y(\omega) &= -\omega D_o \\ A(\omega) &= N_e e^{-m}\cos(n) + \omega N_o e^{-m}\sin(n) \\ B(\omega) &= \omega N_e e^{-m}\sin(n) - \omega^2 N_o e^{-m}\cos(n) \\ C(\omega) &= \omega N_o e^{-m}\cos(n) - N_e e^{-m}\sin(n) \\ D(\omega) &= \omega^2 N_o e^{-m}\sin(n) + \omega N_e e^{-m}\cos(n) \end{aligned} \tag{15}$$

By using these equations into Eqs. (13) and (14), k_p and k_d parameters are obtained as follows, respectively.

$$k_p = \frac{(\omega^2 N_o D_o + N_e D_e)\cos(n) + \omega(N_o D_e - N_e D_o)\sin(n)}{-e^{-m}(N_e^{\,2} + \omega^2 N_o^{\,2})} \tag{16}$$

$$k_d = \frac{\omega^2(N_o D_e - N_e D_o)\cos(n) - \omega(N_e D_e + \omega^2 N_o D_o)\sin(n)}{\omega^2 e^{-m}(N_e^{\,2} + \omega^2 N_o^{\,2})} \tag{17}$$

If Eqs. (6) and (7) are substituted in Eqs. (16) and (17), one obtains Eqs. (18) and (19) .

$$k_p = \frac{(\omega^2 N_o D_o + N_e D_e)\cos(\omega^\alpha(\sin\frac{\pi}{2}\alpha)\tau^\alpha) + \omega(N_o D_e - N_e D_o)\sin(\omega^\alpha(\sin\frac{\pi}{2}\alpha)\tau^\alpha)}{-e^{-(\cos\frac{\pi}{2}\alpha)\omega^\alpha\tau^\alpha}(N_e^{\,2} + \omega^2 N_o^{\,2})} \tag{18}$$

$$k_d = \frac{\omega^2(N_o D_e - N_e D_o)\cos(\omega^\alpha(\sin\frac{\pi}{2}\alpha)\tau^\alpha) - \omega(N_e D_e + \omega^2 N_o D_o)\sin(\omega^\alpha(\sin\frac{\pi}{2}\alpha)\tau^\alpha)}{\omega^2 e^{-(\cos\frac{\pi}{2}\alpha)\omega^\alpha\tau^\alpha}(N_e^{\,2} + \omega^2 N_o^{\,2})} \tag{19}$$

2.2. An Approximation Method for Systems Having Fractional Order Delay

The exponential transcendental representation brings infinitely many isolated roots [19]. Thus, stability analysis of time delay systems is complicated. The analysis becomes much more complex when the time delay is fractional order. An approximation method is proposed here to simplify operations. One of the aims of the approximations is to reduce the difficulties in theoretical or

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numerical analysis by replacing a complex function with a less complex one. Another goal is to find a relatively uncomplicated approximation function that is a valid model for a physical system or device [14]. For this purpose, an approximation method has been proposed to perform stability analysis for systems with FOTD [23]. Necessary details about the method can be found in [23]. The approximations are given in Table 1. Here, α is considered to be in the range of $0 \le \alpha \le 1$.

Table 1. Approximations for FOTD term [23].

Delay	First order approximation	Second order approximation	Third order approximation
$e^{-(s\tau)^{\alpha}}$	$\dfrac{-\dfrac{(s\tau)^{\alpha}}{2}+1}{\dfrac{(s\tau)^{\alpha}}{2}+1}$	$\dfrac{\dfrac{(s\tau)^{2\alpha}}{12}-\dfrac{(s\tau)^{\alpha}}{2}+1}{\dfrac{(s\tau)^{2\alpha}}{12}+\dfrac{(s\tau)^{\alpha}}{2}+1}$	$\dfrac{-\dfrac{(s\tau)^{3\alpha}}{120}+\dfrac{(s\tau)^{2\alpha}}{10}-\dfrac{(s\tau)^{\alpha}}{2}+1}{\dfrac{(s\tau)^{3\alpha}}{120}+\dfrac{(s\tau)^{2\alpha}}{10}+\dfrac{(s\tau)^{\alpha}}{2}+1}$

Using these approximations, the FOTD term is converted into a FOTF. Then, the total TF of the system can be obtained. In this way, stability test can be done easily. If the time response of the system is desired to be obtained, the continued fraction expansion (CFE) method can be used, and the TF is converted to an integer order structure and the results can be obtained. Details on the CFE can be found in [28] and [29].

3. SIMULATION RESULTS

In this section, the effects of the obtained equations and the proposed approximation method are investigated. For a better understanding of the subject, let us consider the example given below.

3.1. Example

For a unity feedback control system shown in Figure 1, the TF is given by Eq. (20),

$$G_p(s) = \frac{1.5}{s(0.4s+1)} e^{-\sqrt{s}} \tag{20}$$

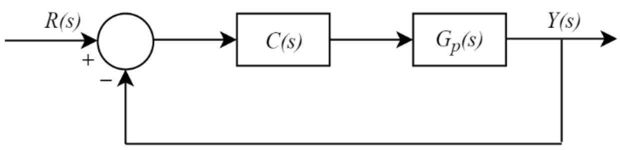

Figure 1. Feedback control system.

By using Eqs. (18) and (19), the stability region of the system is obtained as in Figure 2.

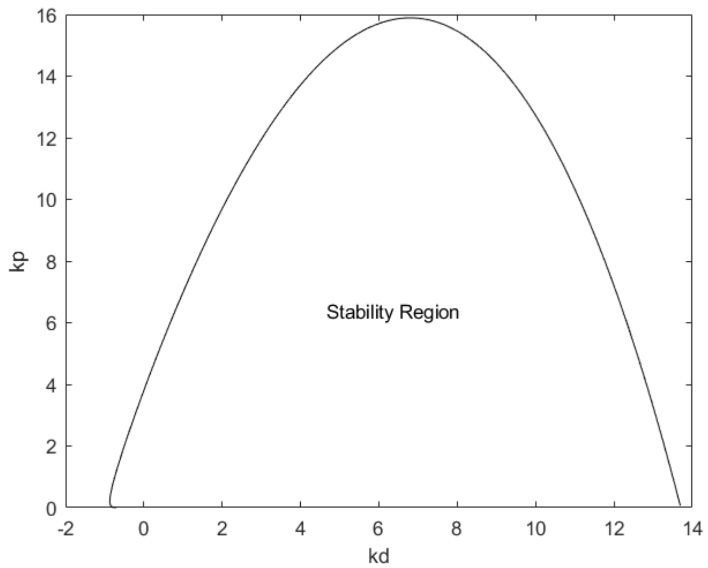

Figure 2. Stability region of the system.

Let $k_p = 4$ and $k_d = 4$ are chosen from the stability region shown in Figure 2. For this case, the CE is found as follows. Here, second order approximation is used.

$$\begin{aligned}\Delta(s) &= 0.4s^{3} + 2.4s^{2.5} + (5.8 + 1.5k_d)s^{2} + (6 - 9k_d)s^{1.5} + (12 + 1.5k_p + 18k_d)s - 9k_p s^{0.5} + 18k_p \\ &= 0.4s^{3} + 2.4s^{2.5} + 11.8s^{2} - 30s^{1.5} + 90s - 36s^{0.5} + 72\end{aligned} \tag{21}$$

If $\sqrt{s} = q$ is taken in this equation, the CE is rearranged as follows.

$$\Delta(q) = 0.4q^{6} + 2.4q^{5} + 11.8q^{4} - 30q^{3} + 90q^{2} - 36q + 72 \tag{22}$$

As seen from Eq. (22) the CE has been converted to integer order form. The roots of this equation are obtained as follows.

$$\begin{aligned}q_{1,2} &= -4.2773 \pm 5.2687j = 6.7863^{\angle \pm 2.2527} \\ q_{3,4} &= 1.2348 \pm 1.6191j = 2.0362^{\angle \pm 0.9193} \\ q_{5,6} &= 0.0425 \pm 0.9700j = 0.9709^{\angle \pm 1.5270}\end{aligned} \tag{23}$$

For FOS, the stability condition for the CE $\Delta(q)$ is defined by Eq. (24) in the $\sqrt{s} = q$ plane.

$$\left|\arg(q_i)\right| > \frac{\pi}{2}\alpha, \ \forall_i = 1, \ldots, n \tag{24}$$

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Where, $0 < \alpha \le 2$. For this example $\alpha = 1/2$. Thus, argument of q must be greater than 0.7854, $|\arg(q)| > \frac{\pi}{4} = 0.7854$. According to Eq. (24), it is seen that the system satisfies the stability condition. Because the arguments for all roots are greater than $\pi/4 = 0.7854$. The roots of the CE are also shown in Figure 3. Here, second order approximation is used. As seen from Figure 3, the roots lie in the desired region. Therefore, the system is stable. This shows that the approximation method provides a satisfactory result for the stability test. To obtain step responses of the system, the TF is transformed into a classical order TF by using the second order approximation given in Table 1 and the CFE method. Here, 4th order CFE is used. The unit step responses of the system are shown in Figures 4 and 5 for $k_p = 4$ and $k_d = 4$, and for $k_p = 2$ and $k_d = 2$, respectively.

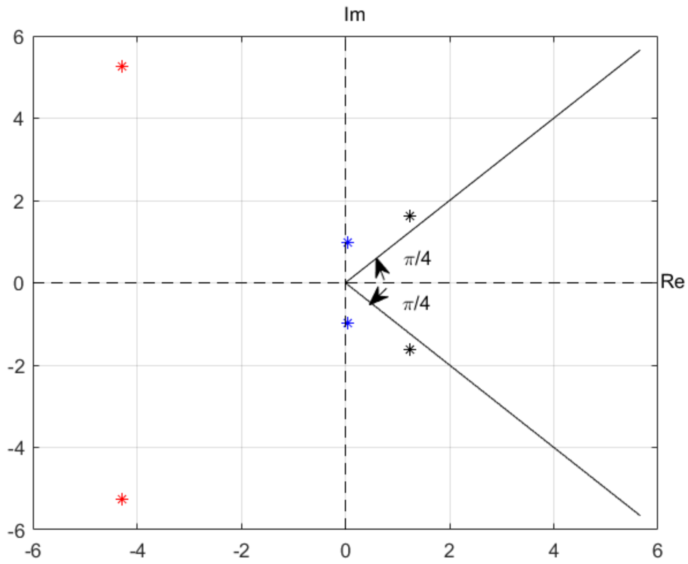

Figure 3. The roots of the CE for the second order approximation.

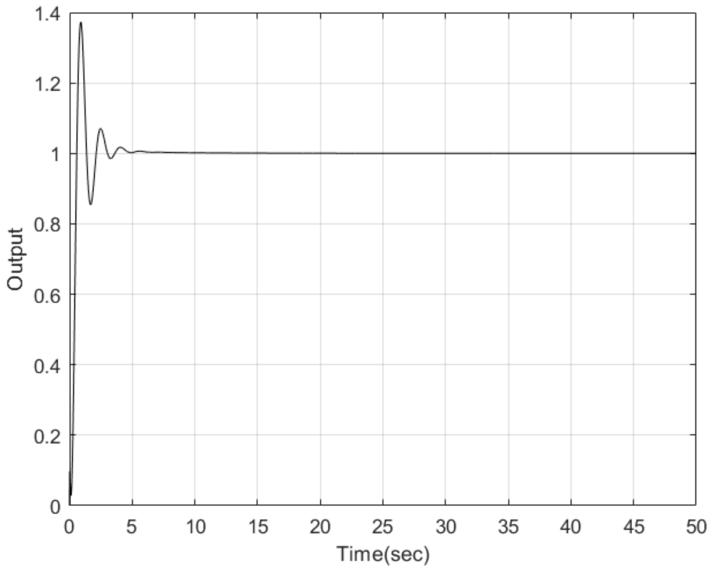

Figure 4. Step response of the system for $k_p = 4$ and $k_d = 4$.

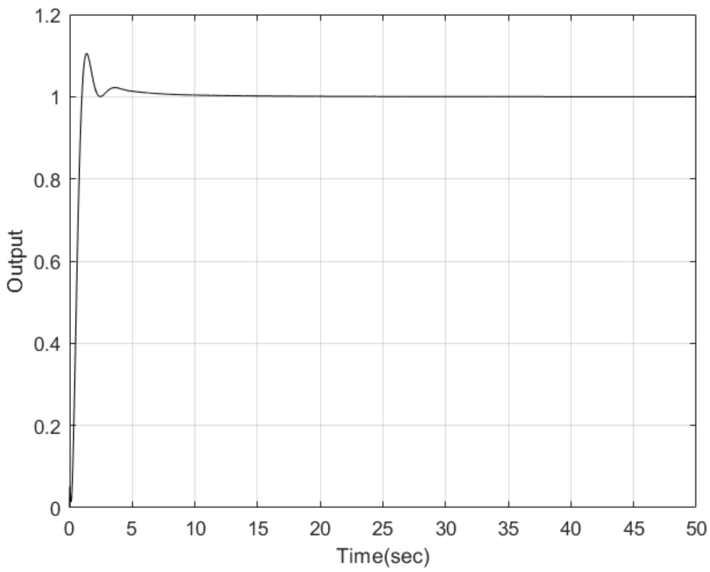

Figure 5. Step response of the system for $k_p = 2$ and $k_d = 2$.

Now let's examine the impact of first and third order approximations for this example. By using the first order approximation given in Table 1, for $k_p = 4$ and $k_d = 4$, the CE is obtained as follows.

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$$\begin{aligned}\Delta(s) &= 0.4s^{2.5} + 0.8s^2 + (1 - 1.5k_d)s^{1.5} + (2 + 3k_d)s - 1.5k_p s^{0.5} + 3k_p \\ &= 0.4s^{2.5} + 0.8s^2 - 5s^{1.5} + 14s - 6s^{0.5} + 12\end{aligned} \tag{25}$$

If $\sqrt{s} = q$ is taken in Eq. (25), the CE is rearranged as follows.

$$\Delta(q) = 0.4q^5 + 0.8q^4 - 5q^3 + 14q^2 - 6q + 12 \tag{26}$$

The roots of the CE are obtained as follows.

$$\begin{aligned}q_1 &= -5.5277 + 0j = 5.5277^{\angle 3.1416} \\ q_{2,3} &= 1.7239 \pm 1.6779j = 2.4057^{\angle \pm 0.7719} \\ q_{4,5} &= 0.0399 \pm 0.9676j = 0.9684^{\angle \pm 1.5296}\end{aligned} \tag{27}$$

The roots of the CE are shown in Figure 6. We know that $|\arg(q)| > \frac{\pi}{4} = 0.7854$. However, for the first order approximation $|\arg(q_{2,3})| = 0.7719$ and this value is less than 0.7854. So, this approximation does not provide a satisfactory result for this example. However, this does not mean that it will not work well for other systems. But, in general it would be wise not to expect very good performance from first order approximation.
By using the third order approximation, the CE is obtained as follows for $k_p = 4$ and $k_d = 4$.

$$\begin{aligned}\Delta(s) &= 0.4s^{3.5} + 4.8s^3 + (25 - 1.5k_d)s^{2.5} + (60 + 18k_d)s^2 + (60 - 1.5k_p - 90k_d)s^{1.5} \\ &+ (120 + 18k_p + 180k_d)s - 90k_p s^{0.5} + 180k_p \\ &= 0.4s^{3.5} + 4.8s^3 + 19s^{2.5} + 132s^2 - 306s^{1.5} + 912s - 360s^{0.5} + 720\end{aligned} \tag{28}$$

If $\sqrt{s} = q$ is taken in Eq. (28), the CE is obtained as follows.

$$\Delta(q) = 0.4q^7 + 4.8q^6 + 19q^5 + 132q^4 - 306q^3 + 912q^2 - 360q + 720 \tag{29}$$

One obtains the roots of the CE as follows.

$$\begin{aligned}q_1 &= -11.1114 + 0j = 11.1114^{\angle 3.1416} \\ q_{2,3} &= -1.7108 \pm 6.1660j = 6.3989^{\angle \pm 1.8414} \\ q_{4,5} &= 1.2240 \pm 1.6426j = 2.0485^{\angle \pm 0.9304} \\ q_{6,7} &= 0.0425 \pm 0.9700j = 0.9709^{\angle \pm 1.5270}\end{aligned} \tag{30}$$

The roots of the CE are shown in Figure 7. It is obvious that the system satisfies stability condition for the third order approximation. However, since this approximation makes solutions more difficult mathematically, it would be better to prefer the second order approximation. Comparison of step

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responses of second order and third order approximations is given in Figure 8. As can be seen in Figure 8, the second order and third order approximations give very close results. This shows that the econd order approximation will be sufficient for the stability test.

It is clear that the second and third approximations used for the stability test of the system give good results as expected. The first-order approximation may be sufficient for the stability test, but this issue should be investigated for the different types of systems as first-order approximations generally may not give very good results. In the given example, the first-order approximation did not provide a satisfactory result. But this cannot be a generalization. This issue needs to be investigated in detail. The unit step responses for various points selected within the stability region are stable as expected. This shows the accuracy of the obtained equations and the used approximations.

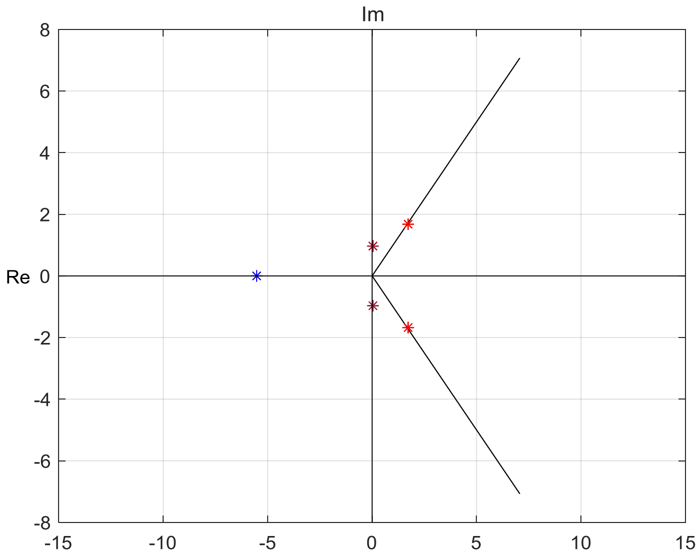

Figure 6. The roots of the CE for the first order approximation.

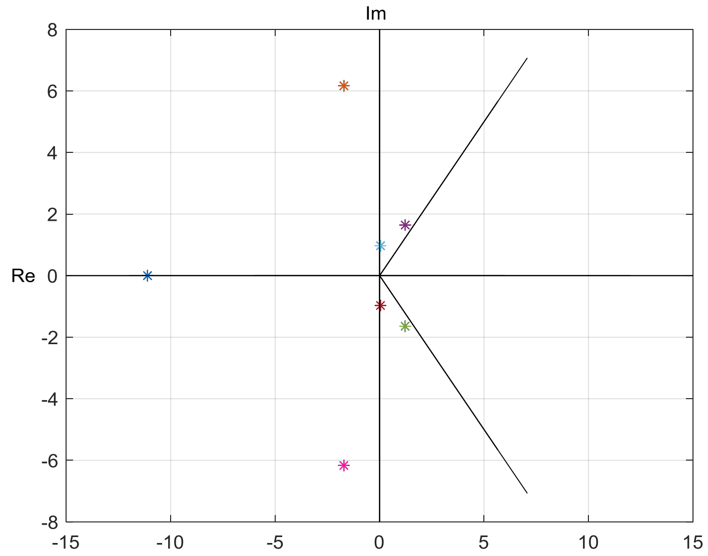

Figure 7. The roots of the CE for the third order approximation.

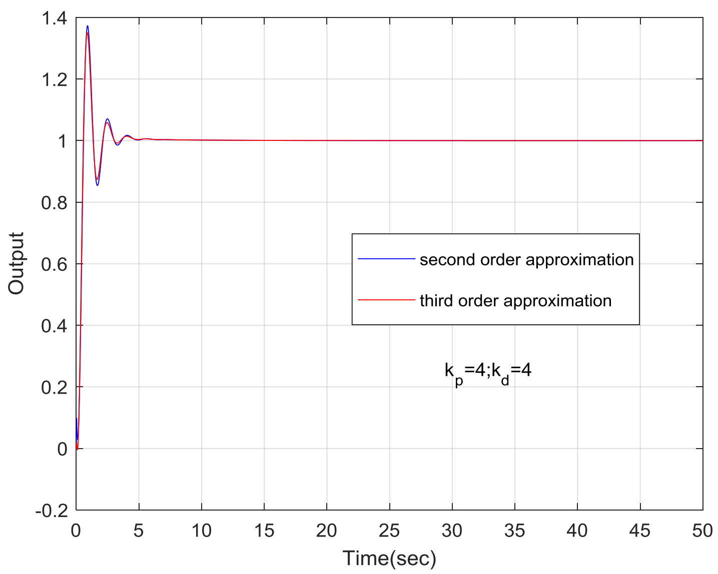

Figure 8. Step responses for the second order and third order approximations

3. CONCLUSION

In this study, all PD controllers providing stability for systems having FOTD are obtained and the approximation method presented in Table 1 is used to test the stability of such systems. The SBL

method is applied to such systems for the first time and the equations are obtained. Here, the results can be summarized as follows.
The first-order approximation may be preferred in the first place for simlicity of the mathematical operation for the stability test, but may not provide the desired performance. This conclusion is obtained depending on one numerical example. Thus, this result cannot be a generalization. This result needs to be investigated in detail for the different types of systems. The second order approximation provides successful results as expected. However, this conclusion should also be reinforced with many numerical examples. The third order approximation also gives good results for the stability test. This result also needs to be supported with many examples. In addition, this approximation makes mathematical operations more complex. For this reason, it will be sufficient to prefer the second order approximation for the stability test.

Using the approximation given in Table 1, FOTD term is transformed into a FOTF. Then the total TF of the system is obtained. This TF is a FOTF. Thus, stability test procedure for FOS can be used. Using the CFE method, FOTF is converted into integer order transfer function to obtain step responses of the system. In the study, it is seen that the second order and third order approximations gave very close results in terms of unit step response.

In the future, studies in which the approximations presented in Table 1 are compared can be conducted to investigate which approximation yields more reasonable and satisfactory results for different control systems. In addition, the necessary equations for different controller types such as classical PI and PID, fractional order PI, PD, and PID can be obtained using the SBL method, and stability analysis can be made. Different techniques can be explored for controller design. In this study, commensurate order systems are considered. It will be important to examine non-commensurate order systems. There is a potential for many different studies on this subject.

4. DISCUSSION

As stated before, studies in which the time delay is fractional order are very limited in the literature. In this study, equations providing all stabilizing PD controller parameters for the given system are obtained using SBL method. To the best of the author's knowledge, stability test of such systems is also an important problem, and satisfactory general solutions are not available in the literature. In this study, an approximation method is used to test the stability of such systems. It is seen that the proposed approximation provides good results for the stability test. The first, second, and third order approximations are compared on the given example. It was seen that the first order approximation did not provide the desired performance in the stability test for the given system. However, this should be investigated in detail for many different systems. Besides, it would be better not to expect first order approximation to perform very well in general. However, extensive studies should be made on this subject. The second and third order approximations yielded successful results as expected. Graphical methods such as Nyquist are also available for stability analysis, but approximation methods are needed to perform stability analysis analytically.The parameters selected from the stability region also give stable responses as expected.

The fractional order structure discussed here is known as commensurate order. However, cases, where the order is non-commensurate, should be investigated in detail, as well. This topic is open to development and needs detailed studies.

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Investigating the superior performance of any controller is not the main issue in this study. The main goal is to be able to analyze the stability of FOTD systems and to design a PD controller, which is a subject that has not been studied much in the literature. However, since there are hardly any studies on controller design on this subject, these studies should be increased and the performance analysis of basic controllers such as PI, PD and PID for such systems should be made and compared. Besides, fractional order controller types such as Fractional order PI (FO-PI), FO-PD, FO-PID can be designed for such systems. For this reason, it would be a more correct approach to seek answers to questions such as how the stability analysis of such systems can be made and how the controller design should be, rather than the superiority of any controller for now. Since there are not enough studies on this subject, any study on controller design for such systems will make a significant contribution to the field. When there is a large amount of data on this subject, the issue of superiority among the controllers can be examined more easily.

ACKNOWLEDGEMENT

I would like to thank the Dean of the Faculty of Engineering at Aydın Adnan Menderes University and the Head of the Department of Electrical and Electronics Engineering for the computer support that enabled me to carry out this study.

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