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Research Article

On the optical soliton solutions of (2+1)-Biswas-Milovic equation via modified new Kudryashov method

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Abstract

This study includes the examination of optical soliton solutions of the (2+1)-dimensional Biswas-Milovic equation, which is an important equation modeling the soliton behavior in optical fibers, which has been introduced to the literature recently. Since the investigated equation and method are recently introduced and not much works has been done, the bell shape and periodic bright optical soliton solutions have been obtained and interpreted by supporting the 3D, 2D and contour graphics by utilizing the modified new Kudryashov method.

Keywords: Biswas-Milovic equation; Optical soliton; Optic fiber; Modified new Kudryashov approach.

(2+1)-boyutlu Biswas-Milovic denkleminin modifiye edilmiş yeni Kudryashov yöntemi ile optik soliton çözümleri üzerine

Öz

Bu çalışma, optik fiberlerdeki soliton davranışını modelleyen önemli bir denklem olan, literature son zamanlarda sunulmuş olan (2+1) boyutlu Biswas-Milovic denkleminin optik soliton çözümlerinin incelenmesini içermektedir. İncelenen denklem ve yöntemin yakın zamanda tanıtılması ve çok fazla çalışma yapılmaması nedeniyle, modifiye edilmiş yeni Kudryashov yöntemi kullanılarak 3D, 2D ve kontur grafikleri desteklenerek çan şekli ve periyodik parlak optik soliton çözümleri elde edilmiş ve yorumlanmıştır.

Anahtar Kelimeler: Biswas-Milovic denklemi; Optik soliton; Optik fiber; Modifiye edilmiş yeni Kudryashov yaklaşımı.

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1. Introduction

After the discovery that the Biswas-Milovic equation (BME) can be used effectively in modeling of many physical phenomena, a variety of models and solution techniques have been developed, and many researchers have started to work in many different fields in this manner. Such as, quantum mechanics, genetics, ocean waves, telecommunication, heat transfer, nonlinear waves, solid state physics, statistics, fuzzy logic and many more [1-8]. In addition to these areas, depending on the development of technology and especially telecommunication, subjects such as optical fibers, data transfer in optical fibers, intercontinental transfer of data have also been one of the important subjects studied [9-14]. As a result of all these developments, some models have been made, especially about optical fibers [15-18].

The (2+1)-BME addressed in the study is defined as follows [19].

$$iM_t-\frac{1}{2}M_{xx}+i\left(\beta-|M|^2\right)M=0\quad,\quad i=\sqrt{-1} \tag{1}$$

$$M(x,y,t)=e^{i\theta}M(\zeta),\ \zeta=x+y+at,\ \theta=ax+by+\omega t, \tag{2}$$

where $M(x,y,t)$, $M(\zeta)$ are the complex and real functions, respectively. x, y denotes the spatial variables, t is the time, θ represents the phase component and b,a,β,ω are non-zero real values. Due to the importance of the Biswas-Milovic equation, many researchers have studied the various forms [20-22].

The rest of the article is constructed as follows: Section 2 contains the mathematical analysis of the considered problem, proposed method, and its implementation. Result and Discussion is given in Section 3 which involves some graphics and some comments. The last section is the Conclusion.

2. Mathematical Analysis of the Problem, the Proposed Method and Its Implementation:

Inserting the wave equation in eq. (2) into eq. (1), produces the following equation which is called nonlinear ordinary differential (NODE) form of eq. (1):

$$2\left(M(\zeta)\right)^3-\left(a^2+2\beta-2\omega\right)M(\zeta)+M''(\zeta)=0. \tag{3}$$

Considering the terms $M^3(\zeta)$ and $M''(\zeta)$, we compute the balance number as z=1.

The modified new Kudryashov method [23,24] which is the modified version of the new Kudryashov method [23,25,26], proposes that the following formula is the solution of eq. (3):

$$M(\zeta)=\sum_{i=0}^{z}a_iW^i(\zeta)\quad,\quad a_z\neq 0 \tag{4}$$

where $a_0,...,a_z$ are unknown real parameters to be calculated, z is the balance number and $W(\zeta)$ provides the following Riccati equation:

$$W'(\zeta)=\sqrt{ln(A)^2\delta^2\left[W^2(\zeta))-\lambda W^4(\zeta)\right]},\ 0<A\neq 1, \tag{5}$$

where δ,λ are non-zero real numbers. Eq. (5) has the following solution:

$$W(\zeta)=\frac{4L}{4L^2A^{\delta\zeta}+\lambda A^{-\delta\zeta}}, \tag{6}$$

where L is a real non-zero free parameter.

Considering the balance number is z=1, we rewrite the eq. (4) in the following form:

$$M(\zeta)=a_0+a_1W(\zeta)\quad,\quad a_1\neq 0. \tag{7}$$

Plugging eq. (7) and its derivatives considering the eq. (5) into eq. (3), we derive a polynomial structure in various powers of $W(\zeta)$. Collecting and then taking the coefficients of $W(\zeta)$ to zero, the following system is gained:

$$W^0(\zeta):a_0\left(a^2-2a_0{}^2+2\beta-2\omega\right)=0,$$

$$W^1(\zeta): a_1\left(-\left(\ln(A)\right)^2\delta^2 + a^2 - 6a_0{}^2 + 2\beta - 2\omega\right) = 0, \tag{8}$$

$$W^2(\zeta): a_0 a_1{}^2 = 0,$$

$$W^3(\zeta): 2a_1\left(\ln(A)\right)^2\delta^2\lambda - 2a_1{}^3 = 0.$$

This system generates the some of the following sets:

$$Set_1 = \left\{\omega = -\frac{\left(ln(A)\right)^2\delta^2}{2} + \frac{a^2}{2} + \beta, a_0 = 0, a_1 = \sqrt{\lambda}\delta ln(A)\right\}, \tag{9}$$

$$Set_2 = \left\{\delta = \frac{1}{ln(A)}\sqrt{a^2 + 2\beta - 2\omega}, a_0 = 0, a_1 = \sqrt{\lambda\left(a^2 + 2\beta - 2\omega\right)}\right\}. \tag{10}$$

Combining the eq. (7) with the eq. (2), eq. (6) and eq. (9), together, we derive the following optical soliton solutions:

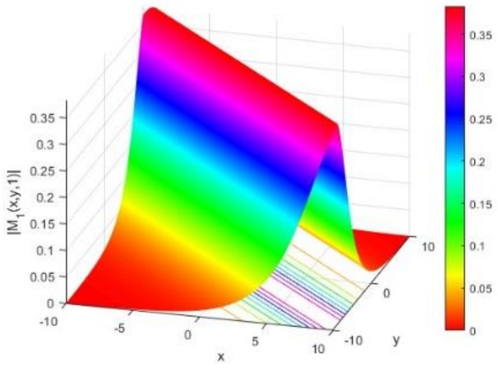
$$M_1(x,y,t) = 4\frac{e^{i\left(ax+by+\left(-1/2\left(ln(A)\right)^2\delta^2+1/2a^2+\beta\right)t\right)}\sqrt{\lambda}\delta ln(A)L}{4L^2A^{\delta(at+x+y)} + \lambda A^{-\delta(at+x+y)}}. \tag{11}$$

Similarly, combination of eq. (7) with the eq. (2), eq. (6) and eq. (10), gives the following solution:

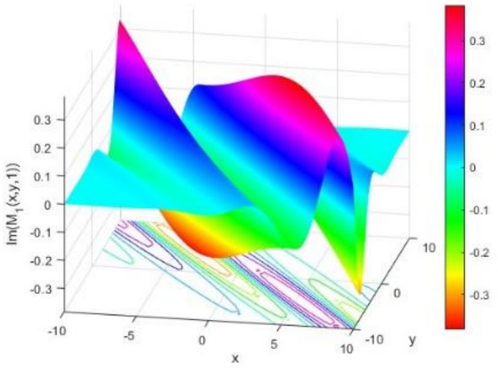
$$M_2(x,y,t) = \frac{4e^{i(ax+by+\omega t)}\sqrt{\lambda\left(a^2+2\beta-2\omega\right)}L}{4L^2A^{\frac{\sqrt{a^2+2\beta-2\omega}(at+x+y)}{ln(A)}} + \lambda A^{-\frac{\sqrt{a^2+2\beta-2\omega}(at+x+y)}{ln(A)}}}. \tag{12}$$

3. Results and Graphical Representation

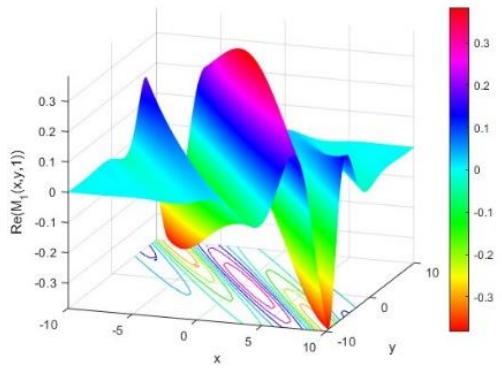
In this section, we presented some graphical simulations. Using the eq. (11) with the Set_1 set in eq. (9), by selecting the parameters as $L=1$, $A=0.6$, $\beta=2$, $\delta=0.75$, $a=1$, $b=0.65$, $\lambda=-0.5$, $y=1$, the fig. 1 is plotted. The fig. 2 belongs to eq. (12), which contains the selected special values as, $L=0.5$, $A=1.2$, $\beta=0.2$, $\delta=-2.5$, $a=1$, $b=0.65$, $\lambda=-0.5$, $y=1$ with Set_2 in eq. (10). As we examine the simulated graphics, each figure has nine subsections. They are 3D modulus in (a), 3D imaginary in (b), 3D real in (c), contour modulus in (d), contour imaginary in (e), contour real in (f), 2D modulus in (g), 2D imaginary in (h) and 2D real in (i), respectively.



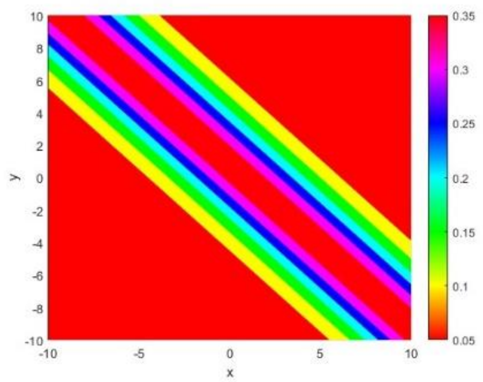
(a) 3D of $|M_1(x,y,1)|$



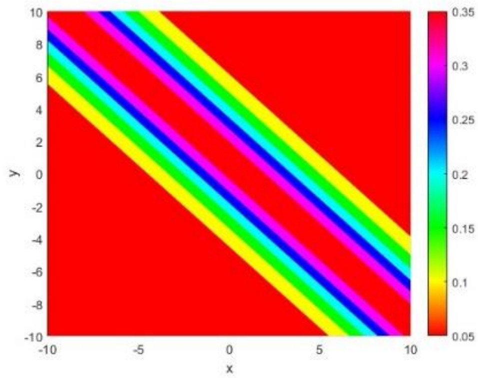
(b) 3D of $Im(M_1(x,y,1))$



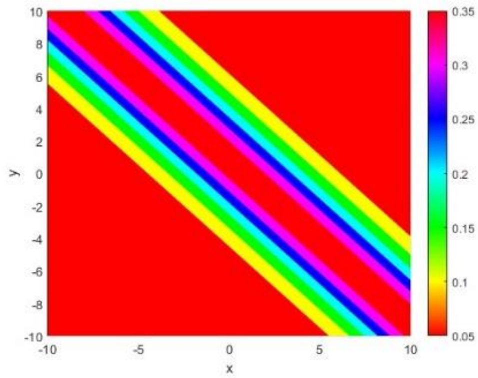
(c) 3D of $Re(M_1(x,y,1))$



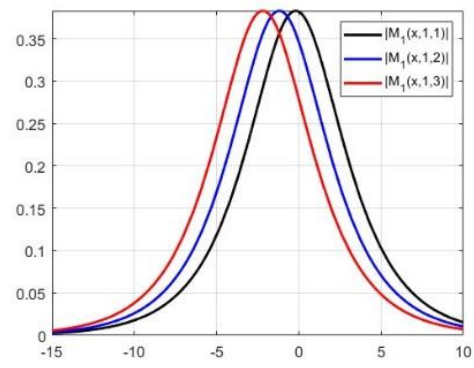
(d) Contour of $|M_1(x,y,1)|$



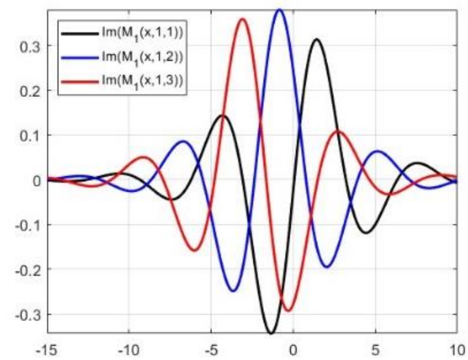
(e) Contour of $Im(M_1(x,y,1))$



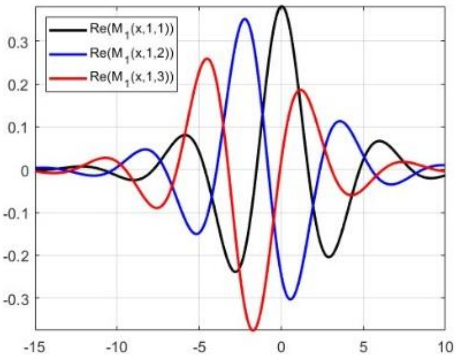
(f) Contour of $Re(M_1(x,y,1))$



(g) 2D of $|M_1(x,y,1)|$

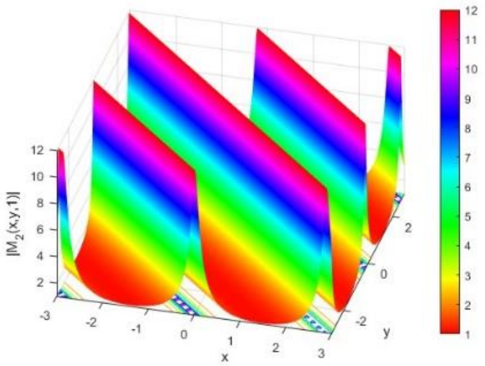


(h) 2D of $Im(M_1(x,y,1))$

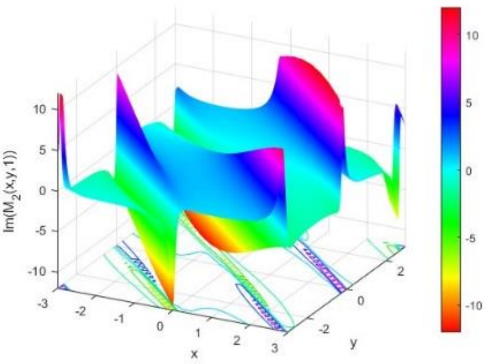


(i) 2D of $Re(M_1(x,y,1))$

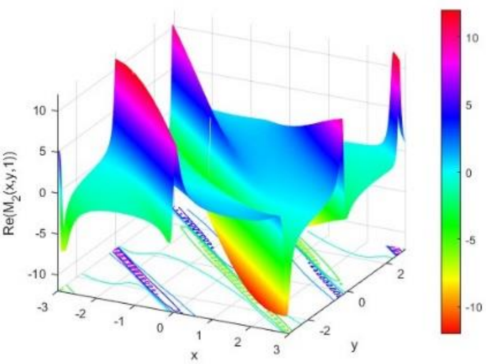
Figure 1 The simulations for $M_1(x,y,t)$ given by eq. (11) with Set_1 in eq. (9).



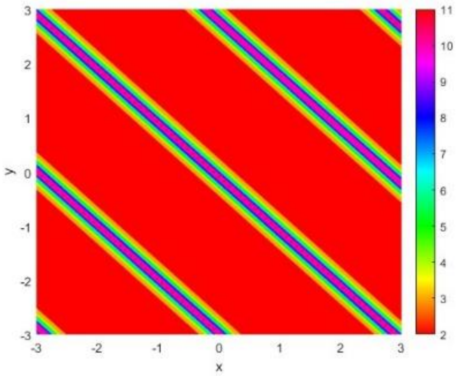
(a) 3D of $|M_2(x,y,1)|$



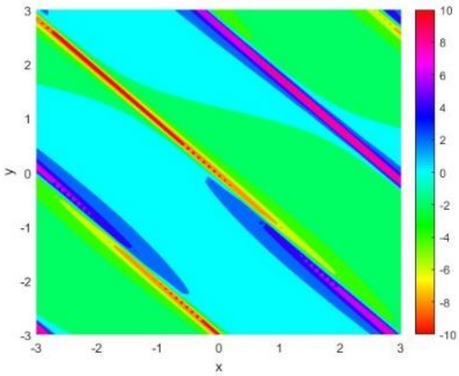
(b) 3D of $Im(M_2(x,y,1))$



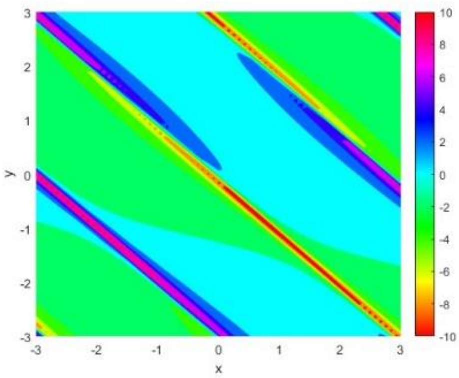
(c) 3D of $Re(M_2(x,y,1))$



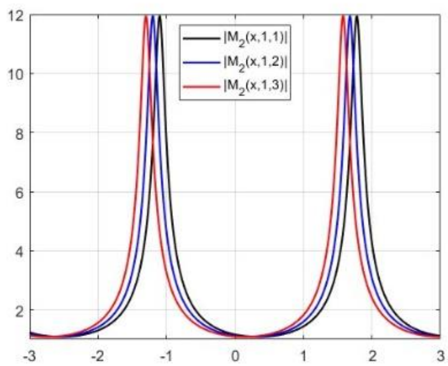
(d) Contour of $|M_2(x,y,1)|$



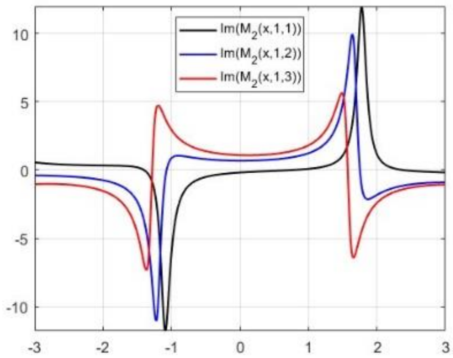
(e) Contour of $Im(M_2(x,y,1))$



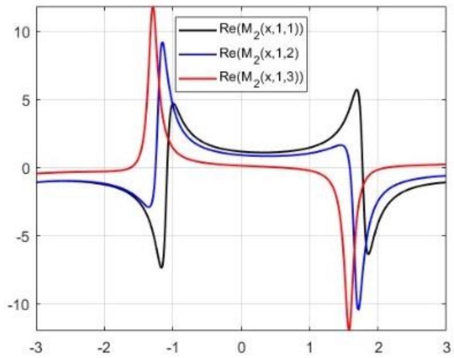
(f) Contour of $Re(M_2(x,y,1))$



(g) 2D of $|M_2(x,y,1)|$



(h) 2D of $Im(M_2(x,y,1))$



(i) 2D of $Re(M_2(x,y,1))$

Figure 2 The simulations for $M_2(x,y,t)$ given by eq. (12) with Set_2 in eq. (10).

4. Conclusion

In this study, the (2+1)-BME, which has been introduced to the literature recently and models the soliton behavior in optic fibers, was investigated. For the analysis, the modified new Kudryashov method, which is new in the literature, was utilized. The obtained optical soliton solutions are basic optical solitons and 3D, contour and 2D graphic presentations of these solutions are also made. Both the model itself and the studies on stochastic and fractional forms of different self-phase modulation forms are some of the topics that may be in the focus of researchers in the future project. The results obtained show that the method is effective, reliable, and easily applicable, and that the (2+1)-BME also represents the soliton behavior in (2+1) dimension.

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