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Stability of Partial Differential Equations by Mahgoub Transform Method

Harun BİÇER*[1]

Abstract

The stability theory is an important research area in the qualitative analysis of partial differential equations. The Hyers-Ulam stability for a partial differential equation has a very close exact solution to the approximate solution of the differential equation and the error is very small which can be estimated. This study examines Hyers-Ulam and Hyers-Ulam Rassias stability of second order partial differential equations. We present a new method for research of the Hyers-Ulam stability of partial differential equations with the help of the Mahgoub transform. The Mahgoub transform method is practical as a fundamental tool to demonstrate the original result on this study. Finally, we give an example to illustrate main results. Our findings make a contribution to the topic and complete those in the relevant literature.

Keywords: Hyers-Ulam stability, Mahgoub transform, partial differential equation

1. INTRODUCTION

In the domains of chemistry, biology, engineering, physics, economics etc., partial differential equations can be used as very effective tools for mathematical modeling of systems and processes. Therefore, several real-world events related to the sciences and engineering method disciplines depend greatly on the qualitative characteristics of partial differential equation solutions. However, we do not want to go into the specifics of the partial differential equation applications here. This material highlights the significance of researching qualitative characteristics, Hyers-Ulam stability (HUS) and Hyers-Ulam Rassias stability (HURS) of partial differential equations.

The classic Fourier integral is where the Mahgoub transform gets its name. Mahgoub improved the Mahgoub transform to make it easier to solve ordinary and partial differential equations in the time domain. The most practical mathematical methods for solving differential equations are often Fourier, Laplace, Elzaki, Aboodh, Sumudu and Mahgoub transforms. In order to solve

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differential equations, the Mahgoub transform and some of its essential features are also used.

Ulam discussed the stability of homomorphism in 1940 (see [1]). Hyers [2] in the Banach spaces provided a partial solution to this puzzle. Numerous academics have since studied the stability problems of functional differential equations (see [1-18]).

In 2009, Jung [7] studied the (HUS) of first-order linear partial differential equations

$$au_x(x,y) + bu_y(x,y) + g(y)u(x,y) + h(y) = 0$$

and

$$au_x(x,y) + bu_y(x,y) + g(x)u(x,y) + h(x) = 0$$

in the cases of $a \leq 0, b > 0$ and $a > 0, b \leq 0, (a, b \in \mathbb{R})$ respectively.

Thereafter, Lungu and Popa [9] proved the (HUS) of first order partial differential equation of the form

$$p(x,t)u_x + q(x,t)u_t = p(x,t)r(x)u + f(x,t).$$

Huang and Li [10] proved the (HUS) of the first order linear partial differential equations in n-dimensional space.

See, in particular, the publications of Biçer and Tunç [3-4], Jung and Brzdek [8], Huang and Li [10], Otrocal and Ilea [12], and Tunç and Biçer [16] and the references therein for additional results on the (HUS) of ordinary or partial differential equations.

In this study, using Mahgoub transform method, we investigate the (HUS) of the homogeneous and nonhomogeneous second order partial differential equations

$$u_{\varkappa\varkappa}(\gamma,x) - \lambda^2 u(\gamma,\varkappa) = 0, \quad (1)$$

$$u_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 u(\gamma,\varkappa) = g(\gamma,\varkappa) \quad (2)$$

for $(\gamma,\varkappa) \in D, D = I \times I, I = [a,b], f \in C(I \times I) = \{g: I \times I \to \mathbb{R}: g \text{ is continuous}\}, -\infty < a < b < \infty$.

2. PRELIMINARIES

This section introduces several common terminologies and notations that will help to support our primary findings.

If there exist constants $X, Y \in \mathbb{R}$ such that

$$|h_1(z)| \leq Xe^{Yt}$$

for all $z \geq 0$, then the function $h_1: [0,\infty) \to \mathbb{R}$ is of exponential order.
Similarly, if there exist constants $X, Y \in \mathbb{R}$ such that
$$|h_2(z)| \leq Xe^{Yt}$$

for all $z \geq 0$, then the function $h_2: (-\infty, 0] \to \mathbb{R}$ is of exponential order.

Definition 2.1. ([1]) The definition of Mahgoub integral transform for the function $u: [0,\infty) \to \mathbb{R}$ is

$$\mathcal{M}\{u(\varkappa)\} = v\int_0^\infty u(s)e^{-vs}ds = U(v),$$

in which $\mathcal{M}$ is the Mahgoub integral transform operator.

If $u(\varkappa)$ is piecewise continuous and of exponential order, the Mahgoub integral transform for the function $u: [0,\infty) \to \mathbb{R}$ exists. These are the only prerequisites that are sufficient conditions for the existence the Mahgoub transform of the function $u(\varkappa)$.
For partial differential equation, the Mahgoub transform of the function $u(\gamma,\varkappa)$ is

$$\mathcal{M}\{u(\gamma,\varkappa)\} = v\int_0^\infty u(\gamma,s)e^{-vs}ds = U(\gamma,v).$$

Using integration by parts we obtain the Mahgoub transform for partial derivatives as folllows:

$$\mathcal{M}\{u_\gamma(\gamma,\varkappa)\} = v\int_0^\infty u_\gamma(\gamma,s)e^{-vs}ds = \frac{d}{d\gamma}U(\gamma,v),$$

$$\mathcal{M}\{u_{\gamma\gamma}(\gamma,\varkappa)\} = v\int_0^\infty u_{\gamma\gamma}(\gamma,s)e^{-vs}ds = \frac{d^2}{d\gamma^2}U(\gamma,v),$$

$$\mathcal{M}\{u_\varkappa(\gamma,\varkappa)\} = v\int_0^\infty u_\varkappa(\gamma,s)e^{-vs}ds = vU(\gamma,v) - vu(\gamma,0),$$

$$\mathcal{M}\{u_{\varkappa\varkappa}(\gamma,\varkappa)\} = v\int_0^\infty u_{\varkappa\varkappa}(\gamma,s)e^{-vs}ds = v^2U(\gamma,v) - v^2u(\gamma,0) - vu_\varkappa(\gamma,0).$$

Definition 2.2 ([1]) Let u and w be Lebesgue-integrable functions on $(-\infty,\infty)$. The convolution of the functions $u(\varkappa)$ and $w(\varkappa)$ is denoted by $u(\varkappa) * w(\varkappa)$ and is defined by

$$u(\varkappa) * w(\varkappa) = (u * w)(\varkappa) = \int_{-\infty}^{\infty} u(s)w(\varkappa - s)ds.$$

Theorem 2.3. (Convolution theorem for Mahgoub transform) ([1]). Suppose that $u(\varkappa)$ and $w(\varkappa)$ are given functions defined for $\varkappa \geq 0$. If $\mathcal{M}\{u(\varkappa)\} = U(v)$ and $\mathcal{M}\{w(\varkappa)\} = W(v)$, then

$$\mathcal{M}\{u(\varkappa) * w(\varkappa)\} = \frac{1}{v}U(v)W(v).$$

Definition 2.4. If there is a constant $K > 0$ such that if for $\varepsilon > 0$, there exists $u: I \times I \to \mathbb{R}$, fulfilling the inequality

$$|u_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 u(\gamma,\varkappa)| \leq \varepsilon$$

for all $(\gamma,\varkappa) \in D$, then there exists a solution $w: I \times I \to \mathbb{R}$ satisfying the differential equation

$$w_{\varkappa\varkappa}(\gamma,\varkappa) = \lambda^2 w(\gamma,\varkappa)$$

such that

$$|u(\gamma,\varkappa) - w(\gamma,\varkappa)| \leq K\varepsilon$$

for all $(\gamma,\varkappa) \in D$, where K is the (HUS) constant for (1). In this case, we say that the differential equation (1) has the (HUS).

Definition 2.5. If there is a constant $K_\varphi > 0$ such that if for every $\varepsilon > 0$, there exists $u: I \times I \to \mathbb{R}$, fulfilling the inequality

$$|u_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 u(\gamma,\varkappa) - g(\gamma,\varkappa)| \leq \varepsilon\varphi(\gamma,\varkappa)$$

for all $(\gamma,\varkappa) \in D$, then there exists a solution $w: I \times I \to \mathbb{R}$ satisfying the differential equation

$$w_{\varkappa\varkappa}(\gamma,\varkappa) = \lambda^2 w(\gamma,\varkappa) + g(\gamma,\varkappa)$$

such that

$$|u(\gamma,\varkappa) - w(\gamma,\varkappa)| \leq K_\varphi\varepsilon\varphi(\gamma,\varkappa)$$

for all $(\gamma,\varkappa) \in D$. We call such K_φ the (HURS) constant for (2). In this case, we say that the differential equation (1) has the (HURS).

3. MAIN RESULTS

Theorem 3.1. The differential equation (1) has the (HUS).

Proof. Let $\varepsilon > 0$. Assume that $u: I \times I \to \mathbb{R}$ satisfies

$$|u_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 u(\gamma,\varkappa)| \leq \varepsilon \qquad (3)$$

for all $(\gamma,\varkappa) \in D$. We will show that there exists a $K > 0$ such that

$$|u(\gamma,\varkappa) - w(\gamma,\varkappa)| \leq K\varepsilon$$

for some $w: I \times I \to \mathbb{R}$ satisfying

$$w_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 w(\gamma,\varkappa) = 0$$

for all $(\gamma,\varkappa) \in D$. Define the function $p: I \times I \to \mathbb{R}$

by
$p(\gamma,\varkappa) =: u_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 u(\gamma,\varkappa)$

for all $(\gamma,\varkappa) \in D$. By (3), we have

$|p(\gamma,\varkappa)| \leq \varepsilon.$

Applying the Mahgoub transform to p, we obtain

$$\mathcal{M}\{p(\gamma,\varkappa)\} = (v^2-\lambda^2)\mathcal{M}\{u(\gamma,\varkappa)\} - v^2u(\gamma,0) - vu_\varkappa(\gamma,0), \quad (4)$$

and so

$$\mathcal{M}\{u(\gamma,\varkappa)\} = \frac{\mathcal{M}\{p(\gamma,\varkappa)\} + v^2u(\gamma,0) + vu_\varkappa(\gamma,0)}{\boldsymbol{v^2}-\lambda^2}. \quad (5)$$

From (4), a function $u_0: I \times I \to \mathbb{R}$ is a solution of (1) if and only if

$$(v^2-\lambda^2)\mathcal{M}\{u_0\} - v^2u_0(\gamma,0) - v(u_0)_\varkappa(\gamma,0) = 0.$$

Set
$$w(\gamma,\varkappa) = u(\gamma,0)\left(\frac{e^{\lambda\varkappa}+e^{-\lambda\varkappa}}{2}\right) + u_\varkappa(\gamma,0)\left(\frac{e^{\lambda\varkappa}-e^{-\lambda\varkappa}}{2\lambda}\right).$$

Then, we obtain $w(\gamma,0) = u(\gamma,0)$ and $w_\varkappa(\gamma,0) = u_\varkappa(\gamma,0)$. Applying the Mahgoub transform of w, we get

$$\mathcal{M}\{w(\gamma,\varkappa)\} = \frac{v^2u(\gamma,0)+vu_\varkappa(\gamma,0)}{v^2-\lambda^2}, \quad (6)$$

for $0 < \lambda < v$. Moreover,

$$\mathcal{M}\{w_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 w(\gamma,\varkappa)\} = (v^2-\lambda^2)\mathcal{M}\{w(\gamma,\varkappa)\} - v^2w(\gamma,0) - vw_\varkappa(\gamma,0).$$

Using (6), we get

$$\mathcal{M}\{w_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 w(\gamma,\varkappa)\} = 0.$$

Since $\mathcal{M}$ is a one-to-one linear operator, we have

$$w_{\varkappa\varkappa}(\gamma,\varkappa) - \lambda^2 w(\gamma,\varkappa) = 0.$$

As a result of above equality, we say that w is a solution of (1). It follows from (5) and (6) that

$$\mathcal{M}\{u(\gamma,\varkappa)\} - \mathcal{M}\{w(\gamma,\varkappa)\} = \frac{\mathcal{M}\{p(\gamma,\varkappa)\} + v^2u(\gamma,0) + vu_\varkappa(\gamma,0)}{\boldsymbol{v^2}-\lambda^2} - \frac{v^2u(\gamma,0)+vu_\varkappa(\gamma,0)}{v^2-\lambda^2} = \frac{\mathcal{M}\{p(\gamma,\varkappa)\}}{v^2-\lambda^2},$$

$$\mathcal{M}\{u(\gamma,\varkappa)\} - \mathcal{M}\{w(\gamma,\varkappa)\} = \mathcal{M}\left\{p(\gamma,\varkappa) * \left(\frac{e^{\lambda\varkappa}-e^{-\lambda\varkappa}}{2\lambda}\right)\right\}.$$

From this equalities, we conclude that

$$u(\gamma,\varkappa) - w(\gamma,\varkappa) = p(\gamma,\varkappa) * \left(\frac{e^{\lambda\varkappa}-e^{-\lambda\varkappa}}{2\lambda}\right).$$

Using $|p(\gamma,\varkappa)| \leq \varepsilon$ and taking the modulus on both sides of above equality, we obtain

$$|u(\gamma,\varkappa) - w(\gamma,\varkappa)| = \left|p(\gamma,\varkappa) * \left(\frac{e^{\lambda\varkappa}-e^{-\lambda\varkappa}}{2\lambda}\right)\right| \leq \left|\int_{-\infty}^{\infty} p(\gamma,s)\left(\frac{e^{\lambda(\varkappa-s)}-e^{-\lambda(\varkappa-s)}}{2\lambda}\right)ds\right| \leq \varepsilon\left|\int_{-\infty}^{\infty}\left(\frac{e^{\lambda(\varkappa-s)}-e^{-\lambda(\varkappa-s)}}{2\lambda}\right)ds\right|$$

for all $(\gamma,\varkappa) \in D$, where

$$K = \left|\int_{-\infty}^{\infty}\left(\frac{e^{\lambda(\varkappa-s)}-e^{-\lambda(\varkappa-s)}}{2\lambda}\right)ds\right|.$$

Hence $|u(\gamma,\varkappa) - w(\gamma,\varkappa)| \leq K\varepsilon$.

Theorem 3.2. The differential equation (2) has the (HURS).

Proof. Let $\varepsilon > 0$ and $\varphi \in C(I \times I)$. Assume that $u: I \times I \to \mathbb{R}$ satisfies

$$|u_{\varkappa\varkappa}(\gamma,\varkappa)-\lambda^2u(\gamma,\varkappa)-g(\gamma,\varkappa)|\le\varepsilon\varphi(\gamma,\varkappa) \quad (7)$$

for all $(\gamma,\varkappa)\in D$. We will show that there exists a $K_\varphi>0$ such that

$$|u(\gamma,\varkappa)-w(\gamma,\varkappa)|\le K_\varphi\varepsilon\varphi(\gamma,\varkappa)$$

for some $w: I\times I\to\mathbb{R}$ satisfying

$$w_{\varkappa\varkappa}(\gamma,\varkappa)-\lambda^2w(\gamma,\varkappa)=g(\gamma,\varkappa)$$

for all $(\gamma,\varkappa)\in D$. Define the function $p: I\times I\to\mathbb{R}$ by
$p(\gamma,\varkappa)=: u_{\varkappa\varkappa}(\gamma,\varkappa)-\lambda^2u(\gamma,\varkappa)-\ g(\gamma,\varkappa)$

for all $(\gamma,\varkappa)\in D$. By (7), we have

$$|p(\gamma,\varkappa)|\le\varepsilon\varphi(\gamma,\varkappa).$$

Applying the Mahgoub transform to p, we obtain

$$\mathcal{M}\{p(\gamma,\varkappa)\}=(v^2-\lambda^2)\mathcal{M}\{u(\gamma,\varkappa)\}\text{-}v^2u(\gamma,0) \\ -vu_\varkappa(\gamma,0)-\mathcal{M}\{g(\gamma,\varkappa)\},$$

and so

$$\mathcal{M}\{u(\gamma,\varkappa)\} \\ =\frac{\mathcal{M}\{p(\gamma,\varkappa)\}+v^2u(\gamma,0)+vu_\varkappa(\gamma,0)+\mathcal{M}\{g(\gamma,\varkappa)\}}{v^2-\lambda^2}. \quad (8)$$

From (8), a function $u_0: I\times I\to\mathbb{R}$ is a solution of (2) if and only if

$$\mathcal{M}\{g(\gamma,\varkappa)\}=(v^2-\lambda^2)\mathcal{M}\{u_0\}-v^2u_0 \\ -v(u_0)_\varkappa(\gamma,0).$$

Putting $r(\varkappa)=\frac{e^{\lambda\varkappa}-e^{-\lambda\varkappa}}{2\lambda}$, we obtain

$$w(\gamma,\varkappa)=u(\gamma,0)\left(\frac{e^{\lambda\varkappa}+e^{-\lambda\varkappa}}{2}\right)+u_\varkappa(\gamma,0)r(\varkappa) \\ +[r(\varkappa)*g(\gamma,\varkappa)].$$

Then, we have $w(\gamma,0)=u(\gamma,0)$ and $w_\varkappa(\gamma,0)=u_\varkappa(\gamma,0)$. Applying the Mahgoub transform of w, we get

$$\mathcal{M}\{w(\gamma,\varkappa)\}=\frac{v^2u(\gamma,0)+vu_\varkappa(\gamma,0)+\mathcal{M}\{g(\gamma,\varkappa)\}}{v^2-\lambda^2} \quad (9)$$

for $0<\lambda<v$. As opposed to that,

$$\mathcal{M}\{w_{\varkappa\varkappa}(\gamma,\varkappa)-\lambda^2w(\gamma,\varkappa)\} \\ =(v^2-\lambda^2)\mathcal{M}\{w(\gamma,\varkappa)\} \\ -v^2w(\gamma,0)-vw_\varkappa(\gamma,0).$$

Using (9), we have

$$\mathcal{M}\{w_{\varkappa\varkappa}(\gamma,\varkappa)-\lambda^2w(\gamma,\varkappa)\}=\mathcal{M}\{g(\gamma,\varkappa)\}.$$

The last equality implies that

$$w_{\varkappa\varkappa}(\gamma,\varkappa)-\lambda^2w(\gamma,\varkappa)=g(\gamma,\varkappa).$$

This means that w is a solution of (2). It follows from (8) and (9) that

$$\mathcal{M}\{u(\gamma,\varkappa)\}-\mathcal{M}\{w(\gamma,\varkappa)\} \\ =\frac{\mathcal{M}\{p(\gamma,\varkappa)\}}{v^2-\lambda^2} \\ =\mathcal{M}\{p(\gamma,\varkappa)*r(\varkappa)\}.$$

Thus $u-w=p*r$. Using $|p(\gamma,\varkappa)|\le\varepsilon\varphi(\gamma,\varkappa)$ and taking the modulus on both sides of above equality, we obtain

$$|u(\gamma,\varkappa)-w(\gamma,\varkappa)| \\ =\left|p(\gamma,\varkappa)*\left(\frac{e^{\lambda\varkappa}-e^{-\lambda\varkappa}}{2\lambda}\right)\right| \\ \le\left|\int_{-\infty}^{\infty}p(\gamma,s)r(\varkappa-s)ds\right| \\ \le\varepsilon\varphi(\gamma,\varkappa)\left|\int_{-\infty}^{\infty}\left(\frac{e^{\lambda(\varkappa-s)}-e^{-\lambda(\varkappa-s)}}{2\lambda}\right)ds\right| \\ \le K_\varphi\varepsilon\varphi(\gamma,\varkappa)$$

where

$$K_\varphi=\left|\int_{-\infty}^{\infty}\left(\frac{e^{\lambda(\varkappa-s)}-e^{-\lambda(\varkappa-s)}}{2\lambda}\right)ds\right|.$$

The proof is now complete.

Example 3.3. Consider

$$u_{\varkappa\varkappa}(\gamma,\varkappa) - 4u(\gamma,\varkappa) = 2sin\varkappa, \tag{10}$$

in which $g(\gamma,\varkappa) = 2sin\varkappa$ is a function of exponential order and $\lambda = 2$.

If a continuously differentiable function $y: I \times I \to \mathbb{R}$ satisfies

$$|y_{\varkappa\varkappa}(\gamma,\varkappa) - 4y(\gamma,\varkappa) - 2sin\varkappa| \le \varepsilon$$

for some $\varepsilon > 0$ and all $\varkappa \ge 0$, then there exists a solution $w: I \times I \to \mathbb{R}$ of differential equation (10) such that

$$|y(\gamma,\varkappa) - w(\gamma,\varkappa)| \le K_\varphi \varepsilon,$$

for all $x \ge 0$. Here

$$K_\varphi = \left| \int_{-\infty}^{\infty} \left(\frac{e^{2(\varkappa-s)} - e^{-2(\varkappa-s)}}{4} \right) ds \right|.$$

4. CONCLUSION

In this study, we used the Mahgoub transform to examine the (HUS) of second order homogeneous partial differential equation and (HURS) of second order nonhomogeneous partial differential equation. This study also demonstrates that the Mahgoub transform method is more practical for looking at stability issues with partial differential equations.

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The Declaration of Conflict of Interest/ Common Interest

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The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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