Advances in the Theory of Nonlinear Analysis and its Applications 7 (2023) No. 1, 178–188.
https://doi.org/10.31197/atnaa.1145837
Available online at www.atnaa.org
Research Article

Advances in the Theory of Nonlinear Analysis and its Applications

ISSN: 2587-2648

Peer-Reviewed Scientific Journal

Convergence of Neutrosophic Random Variables

Carlos Granados[a]

[a] *Estudiante de Doctorado en Matemáticas, Magister en Ciencias Matemáticas, Universidad de Antioquia, Medellín, Colombia.*

Abstract

In this paper, we propose and study convergence of neutrosophic random variables. Besides, some relations among these convergences are proved. Besides, we define the notion of neutrosophic weak law of large number and neutrosophic central limit theorem, also some applications examples are shown.

Keywords: Neutrosophic logic neutrosophic random variable convergence of neutrosophic random variables.
2010 MSC: 03B52.

1. Introduction and Preliminaries

The notion of neutrosophic logic is an extension of intuitionistic fuzzy logic by putting indeterminacy item ($\mathfrak{I}$) where $\mathfrak{I}^2 = \mathfrak{I}, ..., \mathfrak{I}^n = \mathfrak{I}, 0.\mathfrak{I} = 0; n \in \mathbb{N}$ and $\mathfrak{I}^{-1}$ is undefined (see [23], [36]). Neutrosophic logic has plenty of applications in many areas of sciences including multicriteria decision making [33], [22], [29], machine learning [6], [31], intelligent disease diagnosis [34], [11], communication services [8], pattern recognition [32], social network analysis and e-learning systems [24], physics [38], sequences spaces [15] and many others. Neutrosophic logic has helped many multicretia decision-making problems efficiently like finding credit rating, personal selection, among other. [26], [27], [28], [1]. For more concepts associated to neutrosophic theory, we refer the reader to [15, 17, 9, 16, 18, 10].

On the other hand, the idea of neutrosophic probability measure as a function $\mathfrak{NP} : \mathfrak{Y} \to [0, 1]^3$ was originally defined by F. Smarandache where $\mathfrak{U}$ is a neutrosophic sample space, and defined the probability function to take the form $\mathfrak{NP}(\mathfrak{S}) = (ch(\mathfrak{S}), ch(neut\mathfrak{S}), ch(anti\mathfrak{S})) = (\mu, \varpi, \varrho)$ where $0 \leq \mu, \varpi, \varrho \leq 1$ and $0 \leq \mu + \varpi + \varrho \leq 3$ [37].

Recently, Bisher and Hatip [41] defined the concept of neutrosophic random variables in which they showed some basics properties. later on, Granados [13] showed new notions on neutrosophic random variables.

Email address: carlosgranadosortiz@outlook.es (Carlos Granados)

Received July 20, 2022; Accepted: January 20, 2023; Online: January 21, 2023.

Also, Granados and Sanabria [14] studied independence neutrosophic random variables. Taking into account these notions, Granados et al. [19] studied some neutrosophic probability distribution based on neutrosophic random variables parameters. Also, Granados [20] applied discrete random distribution such as the uniform discrete distribution, Bernoulli distribution, binomial distribution, geometric distribution, negative binomial distribution, hypergeometric distribution and Poisson distribution by using neutrosophic random variables. Additionally, Mustafa [25] introduced the concept of a neutrosophic stable random variable. They presented both the neutrosophic probability distribution function and the neutrosophic probability density function, and the convolution with the neutrosophic concept.

In this paper, we procure formulas for convergence neutrosophic random variable $\mathfrak{X}_{\mathfrak{N}}$ and prove some relations among them.

Throughout this paper, the set of real number will be denoted by $\mathbb{R}$ or R, Ω denotes the set of sample space and ω denotes an event of the sample space, $\mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{Y}_{\mathfrak{N}}$ denote neutrosophic random variables.

2. Preliminaries

In this section, we show some definitions which will be useful for the development of this paper.

Definition 2.1. *(see [35]) Let $\mathfrak{X}$ be a non-empty fixed set. A neutrosophic set $\mathfrak{A}$ is an object having the form $\{x, (\nu\mathfrak{A}(x), \lambda\mathfrak{A}(x), \varphi\mathfrak{A}(x)) : x \in \mathfrak{X}\}$, where $\nu\mathfrak{A}(x), \lambda\mathfrak{A}(x)$ and $\varphi\mathfrak{A}(x)$ denote the degree of membership, the degree of indeterminacy , and the degree of non-membership respectively of each element $x \in \mathfrak{X}$to the set $\mathfrak{A}$.*

Definition 2.2. *(see [5]) Let $\mathfrak{K}$ be a field, the neutrosophic filed generated by $\mathfrak{K}$ and $\mathfrak{I}$ is denoted by $\langle \mathfrak{K} \cup \mathfrak{I} \rangle$ under the operations of $\mathfrak{K}$, where $\mathfrak{I}$ is the neutrosophic element with the property $\mathfrak{I}^2 = \mathfrak{I}$.*

Definition 2.3. *(see [36]) Classical neutrosophic number has the form $q + g\mathfrak{I}$ where q, g are real or complex numbers and $\mathfrak{I}$ is the indeterminacy such that $0.\mathfrak{I} = 0$ and $\mathfrak{I}^2 = \mathfrak{I}$ which results that $\mathfrak{I}^n = \mathfrak{I}$ for all positive integers n.*

Definition 2.4. *(see [37]) The neutrosophic probability of event $\mathfrak{A}$ occurrence is $\mathfrak{NP}(\mathfrak{A}) = (ch(\mathfrak{A}), ch(neut\mathfrak{A}), ch(anti\mathfrak{A}$ $(\mathfrak{T}, \mathfrak{I}, \mathfrak{F})$ where $\mathfrak{T}, \mathfrak{I}, \mathfrak{F}$ are standard or non-standard subsets of the non-standard unitary interval $]^{-}0, 1^{+}[$.*

The following results were introduced by [41].

Definition 2.5. *Let us assume a real valued crisp random variable $\mathfrak{X}$ which is given by:*

$$\mathfrak{X} : \Omega \to \mathbb{R}$$

where Ω is the events space. By this, [41] gave a neutrosophic random variable $\mathfrak{X}_{\mathfrak{N}}$ as:

$$\mathfrak{X}_{\mathfrak{N}} : \Omega \to \mathbb{R}(\mathfrak{I})$$

and

$$\mathfrak{X}_{\mathfrak{N}} = \mathfrak{X} + \mathfrak{I}$$

where $\mathfrak{I}$ represents the indeterminacy.

Theorem 2.6. *Let us assume the neutrosophic random variable $\mathfrak{X}_{\mathfrak{N}} = \mathfrak{X} + \mathfrak{I}$ where cumulative distribution function of $\mathfrak{X}$ is given by $F_{\mathfrak{X}}(x) = \mathfrak{P}(\mathfrak{X} \leq x)$. Then, the following conditions are satisfied:*

1. $F_{\mathfrak{X}_{\mathfrak{N}}}(x) = F_{\mathfrak{X}}(x - \mathfrak{I})$,
2. $f_{\mathfrak{X}_{\mathfrak{N}}}(x) = f_{\mathfrak{X}}(x - \mathfrak{I})$.

Where $F_{\mathfrak{X}_{\mathfrak{N}}}$ and $f_{\mathfrak{X}_{\mathfrak{N}}}$ are cumulative distribution function and probability density function of $\mathfrak{X}_{\mathfrak{N}}$, respectively.

Theorem 2.7. *Let us assume the neutrosophic random variable $\mathfrak{X}_{\mathfrak{N}} = \mathfrak{X} + \mathfrak{I}$, expected value can be determined by:*

$$E(\mathfrak{X}_{\mathfrak{N}}) = E(\mathfrak{X}) + \mathfrak{I}.$$

3. Main Results

In this section, we study results that provide formulas for convergence neutrosophic random variables $\mathfrak{X}_{\mathfrak{N}_n}$. Besides, we find relations among them.

Definition 3.1. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ be a sequence of neutrosophic random variables, $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ converges point-wise to $\mathfrak{X}_{\mathfrak{N}}$ if for each $\omega \in \Omega$,*

$$\lim_{n\to\infty} \mathfrak{X}_{\mathfrak{N}_n}(\omega) = \mathfrak{X}_{\mathfrak{N}}(\omega).$$

We denote this convergence as $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p.s} \mathfrak{X}_{\mathfrak{N}}$.

Definition 3.2. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ be a sequence of neutrosophic random variables, $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ converges almost sure to $\mathfrak{X}_{\mathfrak{N}}$ if,*

$$\mathfrak{P}\left\{\omega \in \Omega : \lim_{n\to\infty} \mathfrak{X}_{\mathfrak{N}_n}(\omega) = \mathfrak{X}_{\mathfrak{N}}(\omega)\right\} = 1.$$

We denote this convergence as $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}$.

Following results follow from Definition 3.2, therefore their proofs are omitted.

Proposition 3.3. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}$, then $a\mathfrak{X}_{\mathfrak{N}_n} + b \xrightarrow{N.a.s} a\mathfrak{X}_{\mathfrak{N}} + b$ where $a, b \in \mathbb{R}$.*

Proposition 3.4. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.a.s} Y_N$, then*

1. $\mathfrak{X}_{\mathfrak{N}_n} + \mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}} + \mathfrak{Y}_{\mathfrak{N}}$,
2. $\mathfrak{X}_{\mathfrak{N}_n}\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}\mathfrak{Y}_{\mathfrak{N}}$.

Proposition 3.5. *$\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}$ if and only if for any $\varepsilon > 0$,*

$$\mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) = 0,$$

for any valued of n.

Proposition 3.6. *If for any $\varepsilon > 0$, $\sum_{n=1}^{\infty} \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) < \infty$, then $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}$.*

Definition 3.7. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ be a sequence of neutrosophic random variables, $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ converges in probability to $\mathfrak{X}_{\mathfrak{N}}$ if,*

$$\lim_{n\to\infty} \mathfrak{P}\left\{\omega \in \Omega : |\mathfrak{X}_{\mathfrak{N}_n}(\omega) - \mathfrak{X}_{\mathfrak{N}}(\omega)| > \varepsilon\right\} = 0.$$

We denote this convergence as $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$.

Following results follow from Definition 3.7, therefore their proofs are omitted.

Proposition 3.8. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$, then $a\mathfrak{X}_{\mathfrak{N}_n} + b \xrightarrow{N.p} a\mathfrak{X}_{\mathfrak{N}} + b$ where $a, b \in \mathbb{R}$.*

Proposition 3.9. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{Y}_{\mathfrak{N}}$, then*

1. $\mathfrak{X}_{\mathfrak{N}_n} + \mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}} + \mathfrak{Y}_{\mathfrak{N}}$,
2. $\mathfrak{X}_{\mathfrak{N}_n}\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}\mathfrak{Y}_{\mathfrak{N}}$.

Proposition 3.10. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$, then ${\mathfrak{X}_{\mathfrak{N}_n}}^2 \xrightarrow{N.p} {\mathfrak{X}_{\mathfrak{N}}}^2$.*

Definition 3.11. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, ...$ be a sequence of neutrosophic random variables, $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, ...$ converges in mean to $\mathfrak{X}_{\mathfrak{N}}$ if,*

$$\lim_{n\to\infty} E|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| = 0.$$

We denote this convergence as $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}}$ or $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.E} \mathfrak{X}_{\mathfrak{N}}$.

Following results follow from Definition 3.11, therefore their proofs are omitted.

Proposition 3.12. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}}$, then $a\mathfrak{X}_{\mathfrak{N}_n} + b \xrightarrow{N.m} a\mathfrak{X}_{\mathfrak{N}} + b$ where $a, b \in \mathbb{R}$.*

Proposition 3.13. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{Y}_{\mathfrak{N}}$, then $\mathfrak{X}_{\mathfrak{N}_n} + \mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}} + \mathfrak{Y}_{\mathfrak{N}}$.*

Definition 3.14. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, ...$ be a sequence of neutrosophic random variables, $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, ...$ converges in mean-square to $\mathfrak{X}_{\mathfrak{N}}$ if,*

$$\lim_{n\to\infty} E|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| = 0.$$

We denote this convergence as $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m.s} \mathfrak{X}_{\mathfrak{N}}$.

Following results follow from Definition 3.14, therefore their proofs are omitted.

Proposition 3.15. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m.s} \mathfrak{X}_{\mathfrak{N}}$, then $a\mathfrak{X}_{\mathfrak{N}_n} + b \xrightarrow{N.m.s} a\mathfrak{X}_{\mathfrak{N}} + b$ where $a, b \in \mathbb{R}$.*

Proposition 3.16. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.m.s} \mathfrak{Y}_{\mathfrak{N}}$, then $\mathfrak{X}_{\mathfrak{N}_n} + \mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.m.s} \mathfrak{X}_{\mathfrak{N}} + \mathfrak{Y}_{\mathfrak{N}}$.*

Definition 3.17. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, ...$ be a sequence of neutrosophic random variables, $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, ...$ converges in distribution to $\mathfrak{X}_{\mathfrak{N}}$ if for every x in which $F_{\mathfrak{X}_{\mathfrak{N}}}(x)$ is continuous,*

$$\lim_{n\to\infty} F_{\mathfrak{X}_{\mathfrak{N}_n}}(x) = F_{\mathfrak{X}_{\mathfrak{N}}}(x).$$

We denote this convergence as $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$

The following result follows from Definition 3.17, therefore its proof is omitted.

Proposition 3.18. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{Y}_{\mathfrak{N}}$, then*

1. *If $c\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} c\mathfrak{X}_{\mathfrak{N}}$, where $c \in \mathbb{R}$,*
2. *If $\mathfrak{X}_{\mathfrak{N}_n} + c \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}} + c$, where $c \in \mathbb{R}$,*
3. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} 0$, then $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} 0$,*
4. *$\mathfrak{X}_{\mathfrak{N}_n} + \mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}} + \mathfrak{Y}_{\mathfrak{N}}$,*
5. *$\mathfrak{X}_{\mathfrak{N}_n}\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}\mathfrak{Y}_{\mathfrak{N}}$.*

Next, we show some relations among convergences defined above.

Proposition 3.19. *Let $d \in \mathbb{R}$. If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} d$, then $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} d$.*

Proof. Neutrosophic distribution function of c is defined as follows

$$F_{\mathfrak{X}_{\mathfrak{N}}}(x) = \begin{cases} 0 & if\ x < c + \mathfrak{I}, \\ & \\ 1 & if\ x \geq c + \mathfrak{I}, \end{cases}$$

which has one point of discontinuity in $x = d$ where $d = c + I \in \mathbb{R}$. Consider $F_{\mathfrak{X}_{\mathfrak{N}_n}}(x) \to F_N(x)$ for $x \neq d$. For any $\varepsilon > 0$ we have

$$\begin{aligned} P(|\mathfrak{X}_{\mathfrak{N}_n} - d| \geq \varepsilon) &= P(\mathfrak{X}_{\mathfrak{N}_n} \leq d - \varepsilon) + P(\mathfrak{X}_{\mathfrak{N}_n} \geq d + \varepsilon) \\ &\leq P(\mathfrak{X}_{\mathfrak{N}_n} \leq d - \varepsilon) + P(\mathfrak{X}_{\mathfrak{N}_n} > d + \frac{\varepsilon}{2}) \\ &= F_{\mathfrak{X}_{\mathfrak{N}_n}}(d - \varepsilon) + 1 - F_{\mathfrak{X}_{\mathfrak{N}_n}}(c + \frac{\varepsilon}{2}). \end{aligned}$$

Therefore, $\lim_{n\to\infty} \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - d| \geq \varepsilon) = 0$. □

Theorem 3.20. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}}$, then $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$.*

Proof. Let ε and for any $n \in \mathbb{N}$, define

$$\mathcal{A}_n = \bigcup_{i=n}^{\infty} (|\mathfrak{X}_{\mathfrak{N}_i} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon).$$

Since $(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) \subset \mathcal{A}_n$, then $\mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) \leq \mathfrak{P}(\mathcal{A}_n)$. Hence,

$$\begin{aligned} \lim_{n\to\infty} \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) &\leq \lim_{n\to\infty} \mathfrak{P}(\mathcal{A}_n) \\ &= \mathfrak{P}\left(\lim_{n\to\infty} \mathcal{A}_n\right) \\ &= \mathfrak{P}\left(\bigcap_{n=1}^{\infty} \mathcal{A}_n\right) \\ &= \mathfrak{P}(|X_{N_n} - X_N| > \varepsilon),\ for\ each\ n \geq 1 \\ &= \mathfrak{P}\left(\lim_{n\to\infty} X_{N_n} \neq X_N\right) \\ &= 0. \end{aligned}$$

□

Remark 3.21. *It is easy to check that converse of Theorem 3.20 need not be true.*

Proposition 3.22. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m.s} \mathfrak{X}_{\mathfrak{N}}$, then $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}}$.*

Proof. It follows from $E^2|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| \leq E|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}|^2$ by using Jensen's inequality and Cauchy-Schwarz's inequality. □

Remark 3.23. *It is easy to check that converse of Proposition 3.22 need not be true.*

Theorem 3.24. *If $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}}$, then $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$.*

Proof. For each $\varepsilon > 0$, define $\mathcal{A}_n = (|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon)$. Then,

$$\begin{aligned} E|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| &= E(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}|1_{\mathcal{A}_n}) + E(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}|1^c_{\mathcal{A}_n}) \\ &\geq E(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}|1_{\mathcal{A}_n}) \\ &\geq \varepsilon P(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon). \end{aligned}$$

By hypothesis, $\lim_{n\to\infty} E|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| = 0$, therefore $\mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) \to \varepsilon$. □

Remark 3.25. *It is easy to check that converse of Theorem 3.24 need not be true.*

Theorem 3.26. *If* $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$*, then* $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$*.*

Proof. Consider $\mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}}$ and let x one point of discontinuity of $F_{\mathfrak{X}_{\mathfrak{N}}}(x)$. For any $\varepsilon > 0$,

$$\begin{aligned} F_{\mathfrak{X}_{\mathfrak{N}_n}}(x) &= P(\mathfrak{X}_{\mathfrak{N}_n} \leq x) \\ &= \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}_n} \leq x, |X_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}} \leq \varepsilon) + \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}_n} \leq x, |\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) \\ &\leq \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}} \leq x + \varepsilon) + \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon). \end{aligned}$$

Then, for any $\varepsilon > 0$,

$$\limsup_{n\to\infty} F_{\mathfrak{X}_n}(x) \leq F_{\mathfrak{X}_{\mathfrak{N}}}(x + \varepsilon).$$

This implies,

$$\limsup_{n\to\infty} F_{\mathfrak{X}_n}(x) \leq F_{\mathfrak{X}_{\mathfrak{N}}}(x). \tag{1}$$

Now, we will show the another implication. For any $\varepsilon > 0$,

$$\begin{aligned} F_{\mathfrak{X}_{\mathfrak{N}}}(x - \varepsilon) &= \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}} \leq x - \varepsilon) \\ &= \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}} \leq x - \varepsilon, |\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| \leq \varepsilon) + \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}} \leq x - \varepsilon, |\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon) \\ &\leq \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}_n} \leq x) + \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mathfrak{X}_{\mathfrak{N}}| > \varepsilon). \end{aligned}$$

Then,

$$F_{\mathfrak{X}_{\mathfrak{N}}}(x - \varepsilon) \leq \liminf_{n\to\infty} F_{\mathfrak{X}_{\mathfrak{N}_n}}(x).$$

This implies,

$$F_{\mathfrak{X}_{\mathfrak{N}}}(x) \leq \liminf_{n\to\infty} F_{\mathfrak{X}_{\mathfrak{N}_n}}(x). \tag{2}$$

By (1) and (2), we have

$$F_{\mathfrak{X}_{\mathfrak{N}}}(x) \leq \liminf_{n\to\infty} F_{\mathfrak{X}_{\mathfrak{N}_n}}(x) \leq \limsup_{n\to\infty} F_{\mathfrak{X}_n}(x) \leq F_{\mathfrak{X}_{\mathfrak{N}}}(x).$$

□

Remark 3.27. *It is easy to check that converse of Theorem 3.26 need not be true.*

The following diagram shows relations of convergence of neutrosophic random variables proved above:

$$\begin{array}{c} \mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m.c} \mathfrak{X}_{\mathfrak{N}} \Rightarrow \mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.m} \mathfrak{X}_{\mathfrak{N}} \Rightarrow \mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.p} \mathfrak{X}_{\mathfrak{N}} \Rightarrow \mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}} \\ \Uparrow \\ \mathfrak{X}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{X}_{\mathfrak{N}} \end{array}$$

Theorem 3.28 (Skorokhod's Neutrosophic Representation Theorem)**.** *Let* $\{\mathfrak{X}_{\mathfrak{N}_n}, n \geq 1\}$ *and* $\mathfrak{X}_{\mathfrak{N}}$ *be neutrosophic random variables on* $(\Omega, \mathcal{F}, \mathfrak{P})$ *such that* $\mathfrak{X}_{\mathfrak{N}}$ *in neutrosophic distribution. Then, there exists a probability space* $(\bar{\Omega}, \bar{\mathcal{F}}\bar{\mathfrak{P}})$*, and neutrosophic random variables* $\{\mathfrak{Y}_{\mathfrak{N}_n}, n \geq 1\}$ *and* $\mathfrak{Y}_{\mathfrak{N}}$ *on* $(\bar{\Omega}, \bar{\mathcal{F}}\bar{\mathfrak{P}})$ *such that,*

1. $\{\mathfrak{Y}_{\mathfrak{N}_n}, n \geq 1\}$ *and* $\mathfrak{Y}_{\mathfrak{N}}$ *has the same neutrosophic distributions as* $\{\mathfrak{X}_{\mathfrak{N}_n}, n \geq 1\}$ *and* $\mathfrak{X}_{\mathfrak{N}}$*, respectively,*
2. $\mathfrak{Y}_{\mathfrak{N}_n} \xrightarrow{N.a.s} \mathfrak{Y}_{\mathfrak{N}}$ *as* $n \to \infty$*.*

Proof. It proves similarly to random variables $\{\mathfrak{X}_{\mathfrak{n}}, n \geq 1\}$ and $\mathfrak{X}$. □

Theorem 3.29. *If $\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$ and $g : \mathbb{R} \to \mathbb{R}$ is continuous, then $g(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}) \xrightarrow{N.d} g(\mathfrak{X}_{\mathfrak{N}})$.*

Proof. By Skorokhod's Neutrosophic Representation Theorem, there exists a probability space $(\bar{\Omega}, \bar{\mathcal{F}}\bar{\mathfrak{P}})$, and $\{\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}, n \geq 1\}$, $\mathfrak{Y}_{\mathfrak{N}}$ on $(\bar{\Omega}, \bar{\mathcal{F}}\bar{\mathfrak{P}})$ such that $\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.a.s} \mathfrak{Y}_{\mathfrak{N}}$. Besides, form continuity of g, we have $\{\omega \in \bar{\Omega} : |g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}(\omega) \to g(\mathfrak{Y}_{\mathfrak{N}}(\omega))\} \supseteq \{\omega \in \bar{\Omega} : |\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}(\omega) \to \mathfrak{Y}_{\mathfrak{N}}(\omega)\}$, thus

$$\begin{aligned}
&\mathfrak{P}(\{\omega \in \bar{\Omega} : |g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}(\omega) \to g(\mathfrak{Y}_{\mathfrak{N}}(\omega))\}) \geq \mathfrak{P}(\{\omega \in \bar{\Omega} : |\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}(\omega) \to \mathfrak{Y}_{\mathfrak{N}}(\omega)\}),\\
&\Rightarrow \mathfrak{P}(\{\omega \in \bar{\Omega} : |g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}(\omega) \to g(\mathfrak{Y}_{\mathfrak{N}}(\omega))\}) \geq 1,\\
&\Rightarrow g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}) \xrightarrow{N.a.s} g(\mathfrak{Y}_{\mathfrak{N}}),\\
&\Rightarrow g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}) \xrightarrow{N.d} g(\mathfrak{Y}_{\mathfrak{N}}).
\end{aligned}$$

□

Theorem 3.30. *$\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$ if and only if every bounded continuous function $g : \mathbb{R} \to \mathbb{R}$, $E(g(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}) \to E(g(\mathfrak{X}_{\mathfrak{N}}))$.*

Proof. Consider $\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$. From Skorokhod's Neutrosophic Representation Theorem, there exist neutrosophic random variables $\{\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}, n \geq 1\}$ and $\mathfrak{Y}_{\mathfrak{N}}$ such that $\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.a.s} \mathfrak{Y}_{\mathfrak{N}}$. From continuous mapping theorem, it follows that $g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}) \xrightarrow{N.a.s} g(\mathfrak{Y}_{\mathfrak{N}})$, since g is given to be continuous. Since g is bounded, $E(g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}) \to E(g(\mathfrak{Y}_{\mathfrak{N}}))$. Since, $g(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}})$ has the same distribution as $g(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}})$, and $g(\mathfrak{Y}_{\mathfrak{N}}$ has the same distribution as $g(\mathfrak{X}_{\mathfrak{N}})$, this implies that $E(g(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}) \to E(g(\mathfrak{X}_{\mathfrak{N}}))$. □

Theorem 3.31. *If $\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$, then $C_{\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}}(t) \to C_{\mathfrak{X}_{\mathfrak{N}}}(t)$.*

Proof. If $\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.d} \mathfrak{X}_{\mathfrak{N}}$, From Skorokhod's Neutrosophic Representation Theorem, there exist neutrosophic random variables $\{\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}, n \geq 1\}$ and $\mathfrak{Y}_{\mathfrak{N}}$ such that $\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}} \xrightarrow{N.a.s} \mathfrak{Y}_{\mathfrak{N}}$. Thus,

$$\cos(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}} t) \to \cos(\mathfrak{Y}_{\mathfrak{N}} t),$$

and

$$\cos(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} t) \to \cos(\mathfrak{X}_{\mathfrak{N}} t).$$

As $\cos(\cdot)$ and $\sin(\cdot)$ are bounded functions, we have

$$E(\cos(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}} t)) + iE(\sin(\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}} t)) \to E(\cos(\mathfrak{Y}_{\mathfrak{N}} t)) + iE(\sin(\mathfrak{Y}_{\mathfrak{N}} t)),$$

then $C_{\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}}(t) \to C_{\mathfrak{Y}_{\mathfrak{N}}}(t)$. So, we obtain $C_{\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}}(t) \to C_{\mathfrak{X}_{\mathfrak{N}}}(t)$. This follows from Skorokhod's Neutrosophic Representation Theorem since $\{\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}\}$ and $\mathfrak{X}_{\mathfrak{N}}$ have the same distribution as $\{\mathfrak{Y}_{\mathfrak{N}_{\mathfrak{n}}}\}$ and $\mathfrak{Y}_{\mathfrak{N}}$, respectively. □

Next, we show a couple of theorems which lie from convergence of neutrosophic random variables, therefore their proofs are omitted.

Theorem 3.32. *Let $0 \leq \mathfrak{X}_{\mathfrak{N}_1} \leq \mathfrak{X}_{\mathfrak{N}_2} \leq \ldots$ be a sequence of neutrosophic random variables which converges almost sure to $\mathfrak{X}_{\mathfrak{N}}$. Then,*

$$\lim_{n\to\infty} E(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}) = E(\mathfrak{X}_{\mathfrak{N}}).$$

Theorem 3.33. *Let $\mathfrak{X}_{\mathfrak{N}_1}, \mathfrak{X}_{\mathfrak{N}_2}, \ldots$ be a sequence of neutrosophic random variables in which exists another neutrosophic random variable $\mathfrak{Y}_{\mathfrak{N}}$ which is integrable such that $|\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}| \leq \mathfrak{Y}_{\mathfrak{N}}$, for $n \geq 1$. If $\lim_{n\to\infty} \mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}} = \mathfrak{X}_{\mathfrak{N}}$ a.s., then $\mathfrak{X}_{\mathfrak{N}}$ and $\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}$ are integrable and*

$$\lim_{n\to\infty} E(\mathfrak{X}_{\mathfrak{N}_{\mathfrak{n}}}) = E(\mathfrak{X}_{\mathfrak{N}}).$$

4. Applications

The first application comes to a crowning achievement in neutrosphic probability theory, the neutrosophic weak law of large numbers. This theorem says that, in some sense, the neutrosophic mean of a large sample is close to the neutrosophic mean of the neutrosophic distribution. Before to present the theorem, we first mention that Chebyshev's neutrosophic inequality is satisfied, i.e. $\mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}} - \mu_{\mathfrak{N}}| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$, where $\mu_{\mathfrak{N}}=\mu + \mathfrak{I}$. This can be verified as follows:

$$\mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}} - \mu_{\mathfrak{N}}|) = \mathfrak{P}(|\mathfrak{X} + \mathfrak{I} - \mu - \mathfrak{I}|) = \mathfrak{P}(|\mathfrak{X} - \mu|),$$

by Chebyshev's inequality, we have

$$\mathfrak{P}(|\mathfrak{X} - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}.$$

Theorem 4.1 (Neutrosophic Weak Law of Large Numbers)**.** *If $\mathfrak{X}_{\mathfrak{N}_1}, ..., \mathfrak{X}_{\mathfrak{N}_n}$ are i.i.d, then $\bar{\mathfrak{X}}_{\mathfrak{N}_n} \xrightarrow{N.p} \mu_{\mathfrak{N}}$, where $\mu_{\mathfrak{N}} = \mu + \mathfrak{I}$.*

Proof. Assume that $\sigma < \infty$. This is not necessary but it simplies the proof. Using Chebyshev's neutrosophic inequality,

$$\mathfrak{P}(|\bar{\mathfrak{X}}_{\mathfrak{N}_n} - \mu_{\mathfrak{N}}| > \varepsilon) \leq \frac{Var(\bar{\mathfrak{X}}_{\mathfrak{N}_n})}{\varepsilon^2} = \frac{\sigma^2}{n\varepsilon^2}$$

which tends to 0 as $n \to \infty$. □

Example 4.2. *Consider flipping a coin for which the probability of heads is p. Let $\mathfrak{X}_{\mathfrak{N}_i}$ denote the outcome of a single toss (0 or 1) with an indeterminacy of [0.1,0.3]. Hence, $p = \mathfrak{P}(\mathfrak{X}_{\mathfrak{N}_i} = 1) = E(\mathfrak{X}_{\mathfrak{N}_i})$. The fraction of heads after n tosses is $\mathfrak{X}_{\mathfrak{N}_n}$. According to the neutrosophic law of large numbers, $\mathfrak{X}_{\mathfrak{N}_n}$ converges to p in neutrosophic probability. This does not mean that $\mathfrak{X}_{\mathfrak{N}_n}$ will numerically equal p. It means that, when n is large, the neutrosophic distribution of $\mathfrak{X}_{\mathfrak{N}_n}$ is tightly concentrated around p. Let us try to quantify this more. Suppose the coin is fair, i.e. $p = 1/2$. How large should n be so that $\mathfrak{P}(0.4 \leq \mathfrak{X}_{\mathfrak{N}_n} \leq 0.6) \geq 0.7$? First, $E(\mathfrak{X}_{\mathfrak{N}_n}) = p + \mathfrak{I} = 1/2 + [0.1, 0.3] = [0.6, 0.8]$ and $Var(\mathfrak{X}_{\mathfrak{N}_n}) = [0.12, 0.32]/n$. Now, by using Chebyshev's neutrosophic inequality:*

$$\begin{aligned}\mathfrak{P}(0.4 \leq \mathfrak{X}_{\mathfrak{N}_n} \leq 0.6) &= \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mu| \leq 0.1)\\ &= 1 - \mathfrak{P}(|\mathfrak{X}_{\mathfrak{N}_n} - \mu| > 0.1)\\ &\geq 1 - \frac{[0.12, 0.32]}{n(0.1)^2} = 1 - \frac{[12, 32]}{n}.\end{aligned}$$

The last expression will be larger than 0.7, if $1-(12/n)$; $n > 40$ but, if $1-(32/n)$; $n > 106.7$.

If we make this exercise in a classical way, we will obtain that $n = 84$ and $84 \in (106.7, \infty) \in (40, \infty)$ which was the results obtained in neutrosophic way.

For the following application, we shall show that the sum (or average) of neutrosophic random variables has a neutrosophic distribution which is approximately Normal.

Theorem 4.3 (Neutrosophic Central Limit Theorem)**.** *Let $\mathfrak{X}_{\mathfrak{N}_n}$ for $n \in \mathbb{N}$ be i.i.d with mean $\mu_{\mathfrak{N}}$ ($\mu_{\mathfrak{N}} = \mu + \mathfrak{I}$) and variance σ^2. Then,*

$$\mathfrak{Z}_{\mathfrak{N}_n} \equiv \frac{\sqrt{n}(\bar{\mathfrak{X}}_{\mathfrak{N}} - \mu_{\mathfrak{N}})}{\sigma} \xrightarrow{N.d} \mathfrak{Z}_{\mathfrak{N}}$$

where $\mathfrak{Z}_{\mathfrak{N}} \sim N(0, 1)$.

Proof. There are several ways to denote the fact that the neutrosophic distribution of $\mathfrak{Z}_{\mathfrak{N}_n}$ can be approximated be a normal. They all mean the same thing. Here they are:

$$\mathfrak{Z}_{\mathfrak{N}_n} \approx N(\mathfrak{I}, 1)$$
$$\bar{\mathfrak{X}}_{\mathfrak{N}_n} \approx N(\mu_{\mathfrak{N}}, \frac{\sigma^2}{n})$$
$$\bar{\mathfrak{X}}_{\mathfrak{N}_n} - \mu_{\mathfrak{N}} \approx N(0, \frac{\sigma^2}{n})$$
$$\sqrt{n}(\bar{\mathfrak{X}}_{\mathfrak{N}_n} - \mu_{\mathfrak{N}}) \approx N(0, \sigma^2)$$
$$\frac{\sqrt{n}(\bar{\mathfrak{X}}_{\mathfrak{N}} - \mu_{\mathfrak{N}})}{\sigma} \approx N(0, 1).$$

□

Example 4.4. *Suppose that the number of errors per computer program has a neutrosophic Poisson distribution with mean* $[5.5, 5.8]$*. We get 125 programs. Let* $\mathfrak{X}_{\mathfrak{N}_1}, ..., \mathfrak{X}_{\mathfrak{N}_{125}}$ *be the neutrosophic number of errors in the programs. Let* $\bar{\mathfrak{X}}_{\mathfrak{N}}$ *be the neutrosophic average number of errors. We want to approximate* $P(\bar{\mathfrak{X}}_{\mathfrak{N}} < [6.5, 6.8])$*. Let* $\mu = E(\mathfrak{X}_1) = [5.5, 5.8]$ *and* $\sigma^2 = 5$*. Thus,*

$$\mathfrak{Z}_{\mathfrak{N}_n} = \frac{\sqrt{125}(\bar{\mathfrak{X}}_{\mathfrak{N}_n} - [5.5, 5.8]}{\sqrt{5}}$$
$$= 5(\bar{\mathfrak{X}}_{\mathfrak{N}_n} - [5.5, 5.8]) \simeq N(0, 1).$$

Where indeterminacy is $[0.5, 0.8]$*. Hence,*

$$\mathfrak{P}(\bar{\mathfrak{X}}_{\mathfrak{N}_n} < [6.5, 6.8]) = \mathfrak{P}(\mathfrak{Z}_{\mathfrak{N}} < [2, 3.5]) = [0.94, 0.98].$$

If we make this exercise in a classical way, we will obtain that $\mathfrak{P}(\mathfrak{Z}) = 0.96 \in \mathfrak{P}(\mathfrak{Z}_{\mathfrak{N}})$*.*

5. Conclusion

In this paper, we have defined and studied convergence neutrosophic random variable, some examples were shown to support the results. For future work, it can be defined more formulas for neutrosophic laws of large numbers (weak) and define the neutrosophic strong law of large numbers and more results for central limit theorem for neutrosophic random variables can be obtained.

Funding

This research received no external funding.

Conflicts of Interest

The authors declare no conflict of interest.

Data availability statement

This manuscript has no associated data.

Acknowledgements

The authors are very grateful to the referees for their careful reading with corrections and useful comments, which improved this work very much.

References

[1] M. Abdel-Basset, N.A. Nabeeh, H.A. El-Ghareeb and A. Aboelfetouh, Utilizing Neutrosophic Theory to Solve Transition Difficulties of IoT-Based Enterprises, Enterprise Information Systems, 14(9-10)(2019), 1304–1324.

[2] R. Alhabib, M.M. Ranna, H. Farah and A. Salama, Some Neutrosophic Probability Distributions, Neutrosophic Sets and Systems, 22(2018), 30–38.

[3] R. Alhabib and A.A. Salama, The Neutrosophic Time Series-Study Its Models (Linear-Logarithmic) and test the Coefficients Significance of Its linear model, Neutrosophic Sets and Systems, 33(2020), 105–115.

[4] R. Alhabib and A.A. Salama, Using Moving Averages To Pave The Neutrosophic Time Series, *International Journal of Neutrosophic Science*, 3(1)(2020), 14–20.

[5] M. Ali, F. Smarandache, M. Shabir and L. Vladareanu, Generalization of Neutrosophic Rings and Neutrosophic Fields, Neutrosophic Sets and Systems, 5(2014), 9–14.

[6] J. Anuradha and V.S., Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka, *Neutrosophic Sets and Systems*, 31(2020), 179–199.

[7] M. Bisher and A. Hatip, Neutrosophic Random variables, *Neutrosophic Sets and Systems*, 39(2021), 45–52.

[8] A. Chakraborty, B. Banik, S.P. Mondal and S. Alam, Arithmetic and Geometric Operators of Pentagonal Neutrosophic Number and its Application in Mobile Communication Service Based MCGDM Problem, *Neutrosophic Sets and Systems*, 32(2020), 61–79.

[9] S. Das, R. Das, C. Granados, A. Mukherjee, Pentapartitioned neutrosophic Q-ideals of Q-algebra, Neutrosophic Sets and Systems 41(2021), 53-63.

[10] S. Das, R. Das, C. Granados, Topology on quadripartitioned neutrosophic sets, Neutrosophic Sets and Systems 45(2021), 54-61.

[11] O. A. Ejaita and P. Asagba, An Improved Framework for Diagnosing Confusable Diseases Using Neutrosophic Based Neural Network, Neutrosophic Sets and Systems, 16(2017), 28–34.

[12] L. Esther Valencia Cruzaty, M. Reyes Tomalá and C. Manuel Castillo Gallo, A Neutrosophic Statistic Method to Predict Tax Time Series in Ecuador, *Neutrosophic Sets and Systems*, 34(2020), 33–39.

[13] C. Granados, New results on neutrosophic random variables. Neutrosophic Sets and Systems, 47(2021), 286-297.

[14] C. Granados and J. Sanabria, On independence neutrosophic random variables, Neutrosophic Sets and Systems, 47(2021), 541-557.

[15] C. Granados and A. Dhital, Statistical Convergence of Double Sequences in Neutrosophic Normed Spaces, *Neutrosophic Sets and Systems*, 42(2021), 333–344.

[16] C. Granados, A. Dhital, New results on Pythagorean neutrosophic open sets in Pythagorean neutrosophic topological spaces, Neutrosophic Sets and Systems 43(2021), 12-23.

[17] C. Granados, Una nueva noción de conjuntos neutrosóficos a través de los conjuntos $^*b$-abiertos en espacios topológicos neutrosóficos, Eco Matemático 12(2)(2021), 1-12.

[18] C. Granados, Un nuevo estudio de los conjuntos supra neutrosophic crisp, Revista Facultad de Ciencias Básicas 16(2)(2020), 65-75.

[19] C. Granados, A.K. Das, B. Das, Some continuous neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables, Advances in the Theory of Nonlinear Analysis and its Applications 6(3)(2022), 380-389.

[20] C. Granados, Some discrete neutrosophic distributions with neutrosophic parameters based on neutrosophic random variables, Hacettepe Journal of Mathematics & Statistics 51(5)(2022), 1442-1457.

[21] K. Hamza Alhasan and F. Smarandache, Neutrosophic Weibull distribution and Neutrosophic Family Weibull Distribution, Neutrosophic Sets and Systems, 28(2019), 191–199.

[22] H. Kamaci, Neutrosophic Cubic Hamacher Aggregation Operators and Their Applications in Decision Making, *Neutrosophic Sets and Systems*, 33(2020), 234–255.

[23] W.B.V. Kandasamy and F. Smarandache, Neutrosophic Rings, Hexis, Phoenix, Arizona: Infinite Study, 2006.

[24] M.M. Lotfy, S. ELhafeez, M. Eisa and A.A. Salama, Review of Recommender Systems Algorithms Utilized in Social Networks based e-Learning Systems & Neutrosophic System,*Neutrosophic Sets and Systems*, 8(2015), 32–41.

[25] A. Mustafa, O. Zeitouny, S. Alabdallah, Neutrosophic stable random variables, Neutrosophic Sets and Systems 50(2022), 420-430.

[26] N.A. Nabeeh, M. Abdel-Basset and G. Soliman, A model for evaluating green credit rating and its impact on sustainability performance, Journal of Cleaner Production, 280(1)(2021), 124–299.

[27] N.A. Nabeeh, F. Smarandache, M. Abdel-Basset, H.A. El-Ghareeb and A. Aboelfetouh, An Integrated Neutrosophic-TOPSIS Approach and its Application to Personnel Selection: A New Trend in Brain Processing and Analysis, IEEE Access, 29734–29744, 2017.

[28] N.A. Nabeeh, M. Abdel-Basset, H.A. El-Ghareeb and A. Aboelfetouh , Neutrosophic Multi-Criteria Decision Making Approach for IoT-Based Enterprises, IEEE Access, 2019.

[29] N. Olgun and A. Hatip, The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications, Belgium, EU, Pons Editions Brussels, 2020, 238–253.

[30] S.K. Patro and F. Smarandache, The neutrosophic statistical distribution, more problems, more solutions, Neutrosophic Sets and Systems, 12(2016), 73–79.

[31] R. Sahin, Neutrosophic Hierarchical Clustering Algoritms, Neutrosophic Sets and Systems, 2(2014), 19–24.

[32] M. Sahin, N. Olgun, V. Uluçay, A. Kargın and F. Smarandache, A New Similarity Measure Based on Falsity Value between Single Valued Neutrosophic Sets Based on the Centroid Points of Transformed Single Valued Neutrosophic Numbers with Applications to Pattern Recognition, Neutrosophic Sets and Systems, 15(2017), 31–48.

[33] A. Salama, A. Sharaf Al-Din, I. Abu Al-Qasim, R. Alhabib and M. Badran, Introduction to Decision Making for Neutrosophic Environment Study on the Suez Canal Port, *Neutrosophic Sets and Systems*, 35(2020), 22–44.

[34] G. Shahzadi, M. Akram and A.B. Saeid, An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis, Neutrosophic Sets and Systems, 18(2017), 80–88.

[35] F. Smarandache, Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets, Inter. J. Pure Appl. Math., 2005, 287–297.

[36] F. Smarandache, Introduction to Neutrosophic Statistics, USA: Sitech & Education Publishing, 2014.

[37] F. Smarandache, Introduction to Neutrosophic Measure, Neutrosophic Integral and Neutrosophic Probability, Craiova, Romania: Sitech - Education, 2013.

[38] F. Yuhua, Neutrosophic Examples in Physics, Neutrosophic Sets and Systems, 1(2013), 26–33.

[39] M.B. Zeina, Neutrosophic Event-Based Queueing Model, International Journal of Neutrosophic Science ,6(1)(2020), 48–55.

[40] M.B. Zeina, Erlang Service Queueing Model with Neutrosophic Parameters, International Journal of Neutrosophic Science, 6(2)(2020), 106–112.

[41] M.B. Zeina and A. Hatip, Neutrosophic random variables. Neutrosophic Sets and Systems, 39(2021), 44-52.