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Research Article

Some Fixed Point Theorems in Extended Fuzzy Metric Spaces

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Abstract
In this article, we present a newly fuzzy contraction mapping and using it we prove a fixed point theorem. In fact, we transfer this contraction mapping, first defined in metric spaces, and then transferred to fuzzy metric spaces with modification, to extended fuzzy metric spaces. And so we prove some fixed point theorems existing in the literature in the new space.

Keywords: Fixed-point, extended fuzzy metric space, fuzzy contraction.

Genişletilmiş Bulanık Metrik Uzaylarda Bazı Sabit Nokta Teoremleri

Öz

Bu makalede, yeni bir bulanık büzülme dönüşümü sunuyoruz ve bunu kullanarak bir sabit nokta teoremi ispatlıyoruz. Aslında, önce metrik uzaylarda tanımlanan ve daha sonra modifiye edilerek bulanık metrik uzaylara aktarılan bu büzülme dönüşümünü, genişletilmiş bulanık metrik uzaylara aktarıyoruz. Ve böylece yeni uzaylarda literatürde var olan bazı sabit nokta teoremlerini ispatlıyoruz.

Anahtar Kelimeler: Sabit nokta, genişletilmiş bulanık metrik uzay, bulanık büzülme.

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1. Introduction

The idea of fuzzy was first defined by Zadeh [4]. It has used and attracted attention not only in mathematics but also in many fields. Kramosil and Michalek [5] contributed to the literature fuzzy metric spaces generalizing probabilistic metric spaces, and then George and Veeramani [6] made slight modification in this concept and V. Gregori et al. [3] introduced a kind of generalized version this concept called extended fuzzy metric spaces. Recently, it is a paramount development that defining the concept of contractive mapping in some fuzzy spaces. After the remarkable Banach [7] contractivity, a large amount of mathematicians studied some contractive mappings to proof a fixed point exists such as Grabiec [8], Gregori and his coauthors ([9], [10]), Mihet ([11], [12]). And numerous authors studied different versions contractive mappings in the different spaces ([13], [14], [15]). Concepts, properties and especially some contraction mappings defined in metric spaces in the literature have been transferred to fuzzy metric spaces. For example Wardowski [1] manifested a special contraction and using it he demonstrated theorems in metric spaces. And then inspiring by him, H. Huang and coauthors [2] presented the fuzzy version with simplification. They made slight modification on it and then they indicated some theorems via this contraction in fuzzy metric spaces [2].

In this paper, we define a new fuzzy contraction. Using this newly concept, we verify some theorems [2] in the extended fuzzy metric spaces. And so we get "t=0", versions which are exist in the literature.

While proving theorems in extended novel spaces, we considered two cases. First one is "$t>0$", which expresses fuzzy metric spaces. The second is the case of "$t=0$", which is an important point. This situation corresponds to stationary fuzzy metric spaces. This is why we consider the study we obtained by adding the "$t=0$", point to the existing one in the literature more comprehensive.

2. Preliminaries

In this section, we remember some descriptions and results that will be used later.

Definition 2.1: [16] A binary operation $T:[0,1]\times[0,1]\to[0,1]$ is t-norm, if the subsequent circumstances hold:

(TN_1) $T(\rho,\varphi)=T(\varphi,\rho)$;

(TN_2) $T(\rho,\varphi)\le T(\gamma,\delta)$ if $\rho\le\gamma$ and $\varphi\le\delta$;

(TN_3) $T(\rho,T(\varphi,\gamma))=T(T(\rho,\varphi),\gamma)$;

(TN_4) $T(\rho,1)=\rho$.

Now we present definitions of fuzzy metric space (FMS), stationary fuzzy metric space (SFMS) and extended fuzzy metric space (EFMS), each of which is a trio $(Y,A,*)$, where $Y\neq\emptyset$ is a set, $*$ is a continuous t-norm and A is a fuzzy set on YxYx $(0,\infty)$, YxY and YxYx $[0,\infty)$ respectively.

Definition 2.2: [6] It is FMS, ensuring $\forall\, u,v,w\in Y$ and $\forall\, t,s>0$ the next items:

(FMS_1) $A(u,v,t)>0$;

(FMS_2) $A(u,v,t) = 1 \Leftrightarrow u = v$;
(FMS_3) $A(u,v,t) = A(v,u,t)$;
(FMS_4) $A(u,v,t) * A(v,w,s) \leq A(u,w,t+s)$;
(FMS_5) $A(u,v,.):(0,\infty) \to (0,1]$ is continuous.

Definition 2.3: [9] It is SFMS, ensuring $\forall\, u,v,w \in Y$ the next items:
($SFMS_1$) $A(u,v) > 0$;
($SFMS_2$) $A(u,v) = 1 \Leftrightarrow u = v$;
($SFMS_3$) $A(u,v) = A(v,u)$;
($SFMS_4$) $A(u,v) * A(v,w) \leq A(u,w)$.
$\{u_i\}$ named Cauchy, if $\lim_{i,j\to\infty} A(u_i,u_j) = 1$; $u_i \to u$, if $\lim_{i\to\infty} A(u_i,u) = 1$

Definition 2.4: [3] It is EFMS, ensuring $\forall\, u,v,w \in Y$ and $\forall\, t,s \geq 0$, the next items:
($EFMS_1$) $A^0(u,v,t) > 0$;
($EFMS_2$) $A^0(u,v,t) = 1 \Leftrightarrow u = v$;
($EFMS_3$) $A^0(u,v,t) = A^0(v,u,t)$;
($EFMS_4$) $A^0(u,v,t) * A^0(v,w,s) \leq A^0(u,w,t+s)$;
($EFMS_5$) ${A^0}_{u,v}: [0,\infty) \to (0,1]$ is continuous; ${A^0}_{u,v}(t) = A^0(u,v,t)$.

There are different completeness and Cauchy sequence definitions in FMS ([6],[8]). The authors adapted the M-Cauchy in [3] from FMS to EFMS. As follows;

Definition 2.5: [3] A sequence $\{u_n\}$ in Y is named Cauchy if, given $\varepsilon \in (0,1)$, it can be find $n_\varepsilon \in \mathbb{N}$ such that $A^0(u_n,u_m,0) > 1-\varepsilon$ for all $n,m \geq n_\varepsilon$.

$$\{u_n\} \text{ is a Cauchy } \Leftrightarrow \lim_{m,n} A^0(u_n,u_m,0) = 1.$$

An EFMS is called complete if every Cauchy sequence is convergent.

EFMS, defined in [3] and chosen as the study space in our article, is separated from FMS by the "t=0" point. This is the difference between Definition 2.2 and Definition 2.4 given above. For this reason, we examine the proof of theorems in EFMS in two cases; the first is "t>0", which denotes fuzzy metric spaces, the second is "t=0", which represents stationary fuzzy metric spaces.

We continue with theorems and propositions about EFMS.

Theorem 2.1: [3] Let be a fuzzy set on YxYx $(0,\infty)$, and its extension A^0 is on YxYx $[0,\infty)$ given by $\forall\, u,v \in Y$

$$A^0(u,v,t) = A(u,v,t),\;\; t > 0 \;\text{ and } A^0(u,v,0) = \wedge_{t>0} A(u,v,t).$$

Then, $(Y,A^0,*)$ is an EFMS $\Leftrightarrow$ $(Y,A,*)$ is a FMS, A is called extendable ensuring $\forall\, u,v \in Y$, the condition $\wedge_{t>0} A(u,v,t) > 0$.

Proposition 2.1: [3] Let $(Y,A,*)$ be a FMS, given by $N_A(u,v) = \wedge_{t>0} A(u,v,t)$
Then, $(N_A,*)$ is a SFMS on Y $\Leftrightarrow \wedge_{t>0} A(u,v,t) > 0$; $\forall\, u,v \in Y$.
It is clear that;

$$N_A(u,v) = A^0(u,v,0) = \wedge_{t>0} A(u,v,t) \quad (1)$$

Proposition 2.2: [3] Let $(Y, A^0, *)$ is complete $\Leftrightarrow$ $(Y,N_A,*)$ is complete.

H. Huang and coauthors [2] presented a new concept and they verified some fixed point theorems using it in FMS. And so, they modified and generalized some notions in the literature ones [1].

The class of $F_H: [0,1] \to (0,\infty)$ mappings is $\mathcal{F_H}$, ensuring $\forall$ u, v [0,1], $u < v$ implies $F_H(u) < F_H(v)$. That is F_H is strictly increasing.

Definition 2.6: [2] Let $(Y, A,*)$ be a FMS and $F_H \in \mathcal{F_H}$. $\Im: Y \to Y$ is called a fuzzy F_H-contraction if $\exists\ \tau \in (0,1)$ such that

$$\tau. F_H\ (A(\Im u, \Im v, t)) \ \geq\ F_H(A(u, v, t)) \qquad (2)$$

for all $u, v\ \in Y$ $(u \neq v)$ and $t > 0$.

3. Main Theorems and Proofs

We present F_{H^0}-fuzzy contraction. This new notion can be consider as extended version of the contraction which introduced by H. Huang and his coauthors [2]. In addition, we prove their theorems in the extended fuzzy metric space. And so, we obtain new results which are generalizations of ones exist in the literature.

Definition 3.1: Let $(Y, A^0,*)$ be an EFMS, $F_H \in \mathcal{F_H}$ and an injective mapping $\Im: Y \to Y$ is named F_{H^0}-fuzzy contraction, if (2) is ensured for $\forall\ u, v\ \in Y$ and $t \geq 0$.

Theorem 3.1: Let $(Y, A^0,*)$ be a complete EFMS and $\lim_{t\to 0^+} A(u, v, t) > 0$. If the sequel items hold:

i. $\Im$ is continuous,

ii. $\Im$ is a F_{H^0}-fuzzy contraction,

then $\Im$ has a unique fixed point in Y.

Proof : The proof will be examine in two parts.

I. $t > 0$;

This case was proved in Theorem1's proof [2]. Because,

$A^0{}_{u,v}(t) = A_{u,v}\ (t)\ \ \forall\ u, v\ \in Y$ and $t > 0$, it is similar in FMS.

II. $t = 0$;

$$\{u_n\} \text{ is a Cauchy } \Leftrightarrow \lim_{m,n} A^0(u_n, u_m, 0) = 1$$

Let $u_0 \in Y$ and the sequence $\{u_n\}$ in Y with $u_{n+1} = \Im u_n,\ \forall\ n \in \mathbb{N}$.

Provided that $u_{n+1} = u_n = \Im u_n$, for some $n \in \mathbb{N}$, then $u^* = u_n$ is fixed point of $\Im$.

We pressume that, $u_{n+1} \neq u_n;\ \forall\ n \in \mathbb{N}$.

From (ii), using (1) and implementing (2) with $u = u_{n-1}, v = u_n, t = 0$, we obtain;

$$\begin{aligned} F_H\big(A^0(\Im u_{n-1}, \Im u_n, 0)\big) &= F_H\big(N_A(\Im u_{n-1}, \Im u_n)\big) \\ &> \tau. F_H\big(N_A(\Im u_{n-1}, \Im u_n)\big) \\ &\geq\ F_H(N_A(u_{n-1}, u_n)) \end{aligned}$$

So, we have;

$$F_H\big(N_A(\Im u_{n-1}, \Im u_n)\big) > F_H(N_A(u_{n-1}, u_n))$$

Since F_H is strickly increasing, we get;

$$N_A(u_n, u_{n+1}) > N_A(u_{n-1}, u_n)$$

$\{N_A(u_n, u_{n+1})\}$ is a strickly increasing sequence. Also, since it is bounded from above, the sequence is convergent.

And so, as $n \to \infty$,

$$N_A(u_n, u_{n+1}) = \mu, \qquad \mu \in [0,1] \text{ and } n \in \mathbb{N}.$$

It is obviously that,

$$N_A(u_n, u_{n+1}) < \mu, \qquad \text{for } n \in \mathbb{N}.$$

As $n \to \infty$,

$$F_H\big(N_A(u_n, u_{n+1})\big) = F_H(\mu)$$

We assume that $\mu < 1$,

From (2) with $u = u_n, v = u_{n+1}, t = 0$,

$$F_H\big(N_A(\Im u_n, \Im u_{n+1})\big) > \tau. F_H\big(N_A(\Im u_n, \Im u_{n+1})\big) \geq F_H(N_A(u_n, u_{n+1}))$$

as $n \to \infty$,

$$F_H(\mu) > \tau. F_H(\mu) \geq F_H(\mu)$$

Then $F_H(\mu) = 0$. It is a contradiction.

So, as $n \to \infty$,

$$N_A(u_n, u_{n+1}) = 1, n \in \mathbb{N}.$$

Whether the sequence $\{u_n\}$ is Cauchy or not is very important the proof. Assume that $\{u_n\}$ is not Cauchy sequence.

$\exists\ \varepsilon \in (0,1)$ and $\{u_{m_k}\}$ and $\{u_{n_k}\}$ such that $\forall\ k \in \mathbb{N}$ and $m_k > n_k \geq k$, we obtain

$$N_A\big(u_{m_k}, u_{n_k}\big) \leq (1-\varepsilon)$$

$$N_A\big(u_{m_{k-1}}, u_{n_{k-1}}\big) > 1-\varepsilon \text{ and } N_A\big(u_{m_{k-1}}, u_{n_k}\big) > (1-\varepsilon)$$

And so using ($SFMS_4$), we have

$$(1-\varepsilon) \geq N_A\big(u_{m_k}, u_{n_k}\big) \geq N_A\big(u_{m_{k-1}}, u_{n_{k-1}}\big) * N_A\big(u_{n_{k-1}}, u_{n_k}\big)$$

As $k \to \infty$,

$$\lim_{k\to\infty}(1-\varepsilon) \geq \lim_{k\to\infty} N_A\big(u_{m_k}, u_{n_k}\big) \geq \lim_{k\to\infty} N_A\big(u_{m_{k-1}}, u_{n_{k-1}}\big) * \lim_{k\to\infty} N_A\big(u_{n_{k-1}}, u_{n_k}\big)$$

And so,

$$(1-\varepsilon) \geq \lim_{k\to\infty} N_A\big(u_{m_k}, u_{n_k}\big) > (1-\varepsilon)$$

We get,

$$\lim_{k\to\infty} N_A\big(u_{m_k}, u_{n_k}\big) = (1-\varepsilon).$$

In addition to, by (2) with implementing $u = u_{m_{k-1}}, v = u_{n_{k-1}}, t = 0$, we obtain

$$F_H\Big(N_A\big(\Im u_{m_{k-1}}, \Im u_{n_{k-1}}\big)\Big) > \tau. F_H\Big(N_A\big(\Im u_{m_{k-1}}, \Im u_{n_{k-1}}\big)\Big) \geq F_H(N_A(u_{m_{k-1}}, u_{n_{k-1}}))$$

Since F_H is strictly increasing on [0,1],

$$(1-\varepsilon) \geq N_A(u_{m_k}, u_{n_k}) > N_A(u_{m_{k-1}}, u_{n_{k-1}}) > (1-\varepsilon)$$

It is a contradiction. So, we get that $\{u_n\}$ is a Cauchy. Because, Y is complete, $\exists\, u^* \in Y$: as $n \to \infty$ and $u_n \to u^*$.

Now we will prove that $\Im u^* = u^*$.
From the contiunity of $\Im$,

$$u^* = \lim_{n\to\infty} u_{n+1} = \Im(\lim_{n\to\infty} u_n) = \Im u^*.$$

Now we want to show that whether u^* is unique or not. Presume that $u^*$and v^* are two different fixed points of $\Im$; we get*,

$$F_H(N_A(\Im u^*, \Im v^*)) > \tau. F_H(N_A(\Im u^*, \Im v^*)) \geq F_H(N_A(u^*, v^*))$$

And so, we obtain

$$N_A(\Im u^*, \Im v^*) > N_A(u^*, v^*) = N_A(\Im u^*, \Im v^*).$$

It is a contradiction. That is u^* is unique.
So, we complete the proof. ■

Now we want to introduce and prove a new theorem. In fact, this theorem can be consider as a modified version proved in [2] (Theorem 3).

Theorem 3.2: Let $(Y, A^0, *)$ be a complete EFMS and $\lim_{t\to 0^+} A(u,v,t) > 0$, $\Im: Y \to Y$ be a mapping, $F_H \in \mathcal{F_H}$ and $\forall\, u, v \in Y\ (u \neq v), t \geq 0$ there exists $\tau \in (0,1)$ such that

$$\tau.F_H(A^0(\Im u, \Im v, t)) \geq F_H(\min\{A^0(u,v,t), A^0(v, \Im v, t), A^0(u, \Im u, t)\}) \qquad (3)$$

If F_H or $\Im$ is continuous, $\Im$ has a unique fixed point in Y.

Proof : The proof will be examine in two parts.
I. $t > 0$;
This case was proved in Theorem3's proof [2].
Because, $A^0{}_{u,v}(t) = A_{u,v}(t)\ \forall\, u, v \in Y$, it is similar in FMS.
II. $t = 0$;
Let $u_0 \in Y$ and the sequence $\{u_n\}$ in Y with $u_{n+1} = \Im u_n$, $\forall\, n \in \mathbb{N}$.
Provided that $u_{n+1} = u_n = \Im u_n$, for some $n \in \mathbb{N}$, then $u^* = u_n$ is fixed point of $\Im$.
If $u_{n+1} \neq u_n, \forall\, n \in \mathbb{N}$;
Using (1) and (3) with $u = u_{n-1}, v = u_n, t = 0$, we obtain

$$\begin{aligned} F_H(A^0(\Im u_{n-1}, \Im u_n, 0)) &= F_H(N_A(\Im u_{n-1}, \Im u_n)) \\ &> \tau. F_H(N_A(\Im u_{n-1}, \Im u_n)) \\ &\geq F_H(\min\{N_A(u_{n-1}, u_n), N_A(u_n, \Im u_n), N_A(u_{n-1}, \Im u_{n-1})\}) \\ &\geq F_H(\min\{N_A(u_{n-1}, u_n), N_A(u_n, u_{n+1}), N_A(u_{n-1}, u_n)\}) \\ &\geq F_H(\min\{N_A(u_{n-1}, u_n), N_A(u_n, u_{n+1})\}) \end{aligned}$$

And so, we get,

$$F_H(N_A(\Im u_{n-1}, \Im u_n)) = F_H(N_A(u_n, u_{n+1})) > F_H(\min\{N_A(u_{n-1}, u_n), N_A(u_n, u_{n+1})\})$$

If $\min\{N_A(u_{n-1}, u_n), N_A(u_n, u_{n+1})\} = N_A(u_n, u_{n+1})$,

$$N_A(u_n, u_{n+1}) > N_A(u_n, u_{n+1}).$$

It is a contradiction.

If $\min\{N_A(u_{n-1}, u_n), N_A(u_n, u_{n+1})\} = N_A(u_{n-1}, u_n)$,

$N_A(u_n, u_{n+1}) > N_A(u_{n-1}, u_n)$.)

We know that $\lim_{n\to\infty} u_n = u^*$, $u^* \in Y$ by the proof of Theorem 3.1.

Assume that F_H is continuous;

Using (3) with $u = u_{n+1}, v = u_n, t = 0$, we obtain

$$F_H\big(A^0(\Im u_{n+1}, \Im u_n, 0)\big) = F_H\big(N_A(\Im u_{n+1}, \Im u_n)\big)$$

$$> \tau. F_H\big(N_A(\Im u_{n+1}, \Im u_n)\big)$$

$$\geq F_H(\min\{N_A(u_{n+1}, u_n), N_A(u_n, \Im u_n), N_A(u_{n+1}, \Im u_{n+1})\})$$

For all $n \in \mathbb{N}$ and $t = 0$.

If $\Im u^* \neq u^*$ and as $n \to \infty$,

$$F_H\big(N_A(u_{n+1}, \Im u^*)\big) > \tau. F_H\big(N_A(u_{n+1}, \Im u^*)\big)$$

$$\geq F_H(\min\{N_A(u_n, u^*), N_A(u_n, u_{n+1}), N_A(u^*, \Im u^*)\})$$

And we obtain,

$$F_H\big(N_A(u^*, \Im u^*)\big) > \tau. F_H\big(N_A(u^*, \Im u^*)\big)$$

$$\geq F_H(\min\{N_A(u^*, u^*), N_A(u^*, u^*), N_A(u^*, \Im u^*)\})$$

$$= F_H(\min\{1,1, N_A(u^*, \Im u^*)\})$$

$$= F_H\big(N_A(u^*, \Im u^*)\big)$$

So we obtain,

$$F_H\big(N_A(u^*, \Im u^*)\big) = 0.$$

It is a contradiction. Therefore $\Im u^* = u^*$, that is u^* is a fixed point of $\Im$.

Pressume that $\Im$ is continuous;

Since $\{u_n\}$ is a sequence in Y with $u_{n+1} = \Im u_n$ and $\lim_{n\to\infty} u_n = u^*$, we obtain $\Im u^* = u^*$.

That is u^* is a fixed point of $\Im$.

Now we prove the uniquesness of u^*.

Presume that $\Im$ have two different fixed points; u^* and v^*.

Using (1) and (3) implementing with $u = u^*$ and $v = v^*$, $t=0$ we obtain,

$$F_H\big(A^0(\Im u^*, \Im v^*, 0)\big) = F_H\big(N_A(\Im u^*, \Im v^*)\big)$$

$$> \tau. F_H\big(N_A(\Im u^*, \Im v^*)\big)$$

$$\geq F_H(\min\{N_A(u^*, v^*), N_A(v^*, \Im v^*), N_A(u^*, \Im u^*)\})$$

$$\geq F_H(\min\{N_A(u^*, v^*), N_A(v^*, v^*), N_A(u^*, u^*)\})$$

$$= F_H(\min\{N_A(u^*, v^*), 1,1\})$$

$$= F_H(\min\{N_A(u^*, v^*)\})$$

And so, we get,

$$F_H\big(N_A(u^*, v^*)\big) > F_H\big(N_A(u^*, v^*)\big).$$

It is a contradiction. Therefore $\Im$ has a unique fixed point.

The proof is completed. ■

4. Conclusion

In this article, we proved some fixed point theorems in the literature, in extended fuzzy metric spaces, using new concepts. In the proofs, we specifically examined for "$t=0$", in which case we worked with stationary fuzzy metrics. The difference between fuzzy metrics and extended fuzzy metrics comes from "$t=0$"point. The case of F "$t>0$" is already the same as fuzzy metrics. So, we provide some methods to the researchers who want to work on fixed point theorems via various contractive mappings in the extended fuzzy metrics. If we can inspire researchers, it will be a source of happiness for us.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

Authors' Contributions

All authors contributed equally to the writing of this paper and they read and approved the final of it.

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References

[1] Wardowski, D., 2012. Fixed points of a new type of contractive mappings in complete metric spaces, Fixed Point Theory Appl., 2012, 94.

[2] Huang, H., Caric, B., Dosenovic, T., Rakic, D. and Brdar, M., 2021.Fixed point theorems in fuzzy metric spaces via fuzzy F-contraction, Mathematics Journal, 9, 641.10

[3] Gregori, V., Minana, J. J. and Miravet, D., 2019. Extended fuzzy metrics and fixed point theorems, Mathematics Journal, 7, 303.

[4] Zadeh, L.A., 1965. Fuzzy sets, Inform. Control, 8, 338-353.

[5] Kramosil, I., Michalek, J., 1975. Fuzzy metrics and statistical metric spaces, Kybernetika, 11, 336-344.

[6] George, A., Veeramani, P., 1994. On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64, 395-399.
[7] Banach, S., 1922. Sur les oprations dans les ensembles abstrails et leur application aux quations intgrales, Fund Math., vol. 3, 133-181.
[8] Grabiec, M., 1988. Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems, 27, 385-389.
[9] Gregori, V., Romaguera, S., 2014. Characterizing completable fuzzy metric spaces, Fuzzy Sets and Systems, 144, 411-420.
[10] Gregori,V., Sapena, A., 2002. On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems, 125, 245-252.
[11] Mihet, D., 2008. Fuzzy ψ- contractive mappings in non-Archimedean fuzzy metric space, Fuzzy Sets and Systems, 159, 739-744. 10.
[12] Mihet, D., 2014. A note on fuzzy contractive mappings in fuzzy metric spaces, Fuzzy Set. Syst., 251, 83-91,16.
[13] Di Bari, C., Vetro, C., 2005. Fixed points, attractors and weak fuzzy contractive mappings in a fuzzy metric space, J. Fuzzy Math., 13(4), 973-982.
[14] Gopal, D. and Vetro, C., 2014. Some new fixed point theorems in fuzzy metric spaces, Iranian Journal of Fuzzy Systems,Vol.11, No.3, 95-107.
[15] Samet, B. Vetro, C. Vetro, P., 2012. Fixed point theorems for α-ψ contractive type mappings, Nonlinear Analysis Theory, Methods and Applications, 75, 2154-2165.
[16] Schwizer, B., Sklar, A., 1960. Statistical metric spaces, Pacific Journal of Mathematics 10, 315-367.