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Design of a Kinematic Model Based Backstepping PID and SMC for Mobile Robots

Sinan YİĞİT [*1], Aziz SEZGİN[1]

Abstract

Mobile robot concept is one of the most commonly used nonholonomic system for industrial and academic autonomous applications. There are many types of mobile robot design concepts and control strategies which have been continuously developed by researchers. In this study, two wheeled differential drive mobile robot (DDMR) is used for trajectory tracking study under different conditions. Reference trajectory, dynamic and kinematic motion models of DDMR are defined as mathematical expressions in computer software. For tracking the reference trajectory, error between current pose and reference pose was decreased by sliding mode controller (SMC) and proportional–integral–derivative controller (PID) with kinematic based backstepping controller (KBBC) respectively. A reference path which consists of sinusoidal and linear parts tracked by both controller combinations in first simulation to examine controller tracking performances. In order to examine and compare; responsiveness, stability and robustness of the controllers, an additional mass which affects motion dynamics of DDMR vertically added to the mobile robot body during trajectory tracking application. All results and discussions are comparatively stated at the end of the study with related error figures and evaluations.

Keywords: Differential drive mobile robot, PID and sliding mode control, kinematic based backstepping control, trajectory tracking, robustness test

1. INTRODUCTION

Autonomous ground vehicles have several types such as legged, tracked and wheeled robots which are widespreadly used for agriculture, industry and military applications. Mahmud et al. worked on multi objective path planner for mobile robot usage in greenhouse environment [1]. Li, H., and Savkin's study is proposed that a sensor assisted navigation and collusion free tracking application for dynamic industrial areas [2]. Dang et al. studied about real time integration of autonomous mobile robots to the industrial production systems via heuristic approaches [3]. In mobile robot applications, environment recognition, motion planning and real time navigation systems were being studied as a main research subject from past to present

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[4]. Wheeled mobile robots are commonly preferred by researchers because of the convenient mechanical design properties and low energy usage. Mobile robots can be designed and operated in various types in terms of number of wheels, motor types, controller strategies and navigation techniques.

As a frequently used wheeled mobile robot model, differential drive mobile robots consist of two coaxial, separately rotatable wheels, one free turning wheel, robot chassis, necessary motor components, control units and also necessary additional parts. Two driven wheeled design of differential drive mobile robot provides versatility of usage and also provides pose detection and correction convenience with only control of motor angular velocities. Besides DDMR advantages, its control applications contain rough problems arise from nonholomic constraints which gives nonlinearity to the system. On account of this, DDMR system is hard to control smoothly and time invariantly, which can be understood from Brockett's necessary condition. To overcome these issues several assumptions are applied to the system such as non-slipping wheel conditions, exactly detectable dc motor speeds and robot pose.

The trajectory tracking which can be defined as cruise from instant position to next desired position in a defined trajectory which is main concern of recent autonomous mobile robot studies. This tracking problems consist of defining a trajectory which gives reference control variables to drive mobile robot on a desired way with using mathematical models and changing motor speeds for making robot pose close to the reference poses momentarily.

Until recent studies, various kind of control methods are used for mobile robot applications to get optimum path tracking performance. On the other hand, in common problems is that conventional control strategies are dependent to the system parameters and external influences which affects robustness and certainty of the controller. As an effective solution kinematic based backstepping control, PID control and SMC are used in different combinations and reached to the convincing results at the end of the study. Kinematic based backstepping controller is useful for nonlinear systems to get asymptotic stability at first. Panahandeh's study stated that kinematic based Lyapunov approach provides system asymptotic stability and ability to obtain efficient control law for obstacle avoidance [5]. Sliding mode control law is also extensively used for tracking of reference trajectories with velocity control because of the robustness, quick response time and better transient behavior in comparison to the conventional control strategies.

Although it has been studied about SMC and PID control methods on mobile robots, kinematic based backstepping controller assistanted trajectory tracking performance analysis and robustness test have the potential to be an approach that can contribute to the literature. The use of multiple controllers together, the use of variable shaped trajectory and the sudden change in system parameters make this study valuable in terms of giving a holistic perspective.

In this study, SMC method is used and durable control performance has been achieved against unexpected environmental changes and system parameters uncertainty of our DDMR trajectory tracking problem. And it would be possible to obtain sliding mode control law without measuring and defining all system parameters which gives reliability and simplicity to the control law. The common downside of sliding mode controller is chattering problem which has been solved with using additional filter in computer software.

The path tracking problem is examined in different conditions for kinematic based back stepping control assisted SMC and PID control with using computer software for two wheeled differential drive mobile robot.Firstly, DDMR is introduced and mathematically modeled as and dynamic models. Secondly, DC motor control model and controller combinations are obtained respectively by mathematical expressions. Controlled parameters of the system are obtained again via backstepping control, which give control simplicity and pose correction at first step, which means that the control problem can be converted from position (X, Y) and heading angle θ error to the linear and angular velocity control. As a next step SMC and PID control methods are tested for tracking reference trajectories with non- additional mass and mass added DDMR simulations in Simulink Software. In conclusion, trajectory tracking performance and robustness of the control methods are evaluated with respect to the X,$Y$and θ error values.

2. DIFFERENTIAL DRIVE MOBILE ROBOT SYSTEM OVERVIEW

A commonly used design of DDMR is two coaxial wheels settled on the rear axle and one castor wheel settled on the front axle which is illustrated Fig. 1 below.

The motion of DDMR can be modeled based on the mathematical relation between angular velocities of wheels and current position of robot. For mathematical modelling approach, robot coordinate system X^r, Y^r which is to fixed robot body and inertial coordinate system X^i, Y^i in universal plane are need to be defined by considering physical constraints of DDMR as Fig. 1.

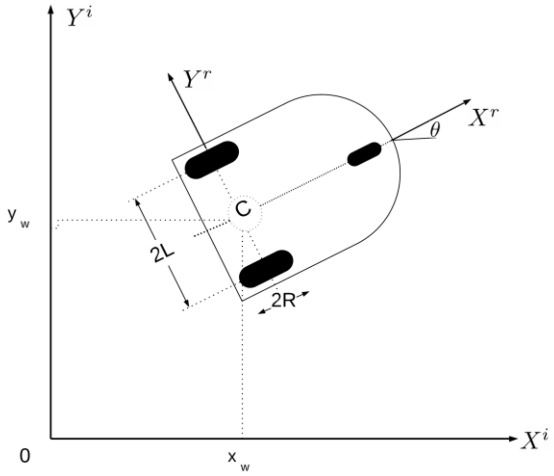

Figure 1 Coordinate system

3. KINEMATIC MODELING OF DDMR

The position and orientation (pose) of the differential drive mobile robot in general form in Eqn 1.

$$q = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (1)$$

The inertial coordinate system pose q_i and robot coordinate system pose q_r can be represented respectively by vectors below

$$q_i = \begin{bmatrix} x^i \\ y^i \\ \theta^i \end{bmatrix}, \quad q_r = \begin{bmatrix} x^r \\ y^r \\ \theta^r \end{bmatrix} \quad (2)$$

Inertial coordinate system and robot coordinate system have a relation which can be defined by orthogonal rotation matrix R_θ.

$$R(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Kinematic model of the DDMR is based on two main assumptions.

No slip motion along the lateral axis Y_r of DDMR. Midpoint of the rear axle (C) is accepted as a reference point in order to identify robot pose and velocities. And slip motion and velocity of point C,$({y_c}^r)$

are accepted zero along axis Y_r.

$$\dot{y}_c^r = 0 \tag{4}$$

Pure rolling constraints accepted which means any slipping along the X_r axis. And also, it is assumed that each driving wheel has one contact point with floor during the robot frame motion.

Pure rolling velocities V_p are stated based on wheel diameter (R), right wheel angular velocity ω_r and left wheel angular velocity ω_l as can be seen below;

$$V_p r = R\omega_r, \qquad V_p l = R\omega_l \tag{5}$$

Constraint Eqn 1 and 2 and R_θ can be defined as matrix form below;

$$\delta(q) = \begin{bmatrix} -sin(\theta) & cos(\theta(& 0 & 0 & 0 \\ cos(\theta) & sin(\theta) & L & -R & 0 \\ cos(\theta) & sin(\theta) & -L & 0 & -R \end{bmatrix} \tag{6}$$

$$\delta(q)\dot{q} = 0 \tag{7}$$

$$\dot{q} = [\dot{x} \quad \dot{y} \quad \dot{\theta} \quad w_r \quad w_l]^T \tag{8}$$

Velocities of the DDMR will be defined as a function of driving wheels velocity and the dimensions of robot frame and additional parts. V is the linear velocity and W is the angular velocity of DDMR. [5, 6]

$$V = \frac{v_r + v_l}{2} = R\left(\frac{\omega_r + \omega_l}{2}\right) \tag{9}$$

$$\omega = \frac{v_r - v_l}{2L} = R\left(\frac{\omega_r - \omega_l}{2L}\right) \tag{10}$$

The DDMR velocities can be formulated in robot coordinate system (r) and inertial coordinate system (i) respectively as follows;

$$\dot{q}^r = \begin{bmatrix} \dot{x}_c^r \\ \dot{y}_c^r \\ \dot{\theta}_c^r \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ 0 & 0 \\ R/2 & R/2 \end{bmatrix} \begin{bmatrix} w_r \\ w_l \end{bmatrix} \tag{11}$$

$$\dot{q}^i = \begin{bmatrix} \dot{x}_c^i \\ \dot{y}_c^i \\ \dot{\theta}_c^i \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ 0 & 0 \\ R/2 & R/2 \end{bmatrix} \begin{bmatrix} w_r \\ w_l \end{bmatrix} \tag{12}$$

4. DYNAMIC MODELING OF DDMR

Trajectory tracking by controlling wheel speeds has some issues during fast maneuvers and loaded operations due to the difference between the velocities given by the software and the actual velocities. In order to compensate these situations and get correct trajectory tracking, dynamic model of DDMR is need to be designed.

DDMR as a nonholonomic system with n generalized coordinates $(q_1, q_2 \dots q_n)$ and dependent to m constraints can be described by equation below; [7, 10]

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q) \tag{13}$$

M(q) is a nxn symmetric positive definite inertia matrix, $F(\dot{q})$ is the surface friction matrix, G(q) is the gravitational vector,τ_d is the disturbance vector, B(q) is the input matrix,τ is the input vector, $A^T(q)$ is kinematic constraint matrix, and λ is the Lagrange multipliers vector.

Lagrange Method is very effective way to formulate dynamic model of a mechanical system with using Kinetic energy and Potential energy.

$$L = T - U \tag{14}$$

The Lagrange equation is stated as an equation as below;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \left(\frac{\partial L}{\partial q_i}\right) = F - A^T(q)\lambda \tag{15}$$

The potential energy of DDMR is accepted as zero because DDMR moves just in $X_i Y_i$ plane.

The equation of motion can be defined by using Eqn. 15 and Lagrange Function(L = T) as stated in appendix section (A.1-A.2). The equations of motion A.3-A.7 can be rearranged in general form seen in A.13 as below;

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} = B(q)\tau - A^T(q)\lambda \tag{16}$$

Next, the Eqn. 16 can be organized in alternative form to simulate and control the system with ease. In this alternative form $A^T(q)\lambda$ term will be eliminated. Using kinematic Eqn. 12 transformation matrix can be redefined according to the point D.

Dynamic equations can be written with new matrix forms as follow;

$$M'(q)\dot{\eta} + V'(q,\dot{q})\eta = B'(q)\lambda \tag{17}$$

5. TRAJECTORY TRACKING OF DDMR

5.1. Actuator Modelling

The wheels of DDMR are commonly driven by dc motors with armature control method via armature voltage as control input for the system.
The armature circuit of a permanent magnet dc motor can be represented by equations (A1-A4) [9, 10]. Where; i_a:armature current, R_a:resistance of armature, L_a:inductance of armature, e_a:back emf, ω_m:rotor angular speed, τ_m:motor torque, K_t:torque constant and K_b:back emf constant, N:gear ratio, and τ is the output torque of dc motor.

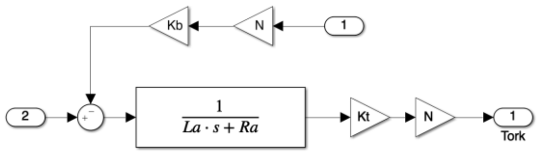

Figure 2 Actuator modelling

Dc motors are mechanically linked to the wheels with gears, therefore mechanical equations of dc motors and dynamics of the system are connected to each other. Input one is angular velocity feedback and input two is control input as shown in Fig. 2.

$$\omega_m r = N\omega_r, \qquad \omega_m l = N\omega_l \tag{18}$$

6. CONTROLLER DESIGN METHODOLOGY

Kinematic based backstepping control (KBBC) and sliding mode control are used for trajectory tracking of DDMR in two stages which are kinematic and dynamic modeling stages. For the kinematic control stage, for any initial conditions, status of the robot can be characterized as reference q_r and real pose q.

$$q_r = [x_r \quad y_r \quad \theta_r]\,, \; q = [x \quad y \quad \theta] \tag{19}$$

And the difference between reference pose and real pose states pose error in the inertial frame which is shown in matrix; [11]

$$e_p = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{20}$$

Kinematic based backstepping control is preferred to abolish error at the reference linear velocity and heading angle which give twist parameters ($v_r = [v_r \quad \omega_r]$).
This linear velocity and heading angle of DDMR come from backstepping controller are used as input parameters for dynamic control stage.

7. KINEMATIC BASED BACKSTEPPING CONTROL

The first derivative of the pose error is written in Eqn. 21 [12],

$$\dot{e_p} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta \end{bmatrix} = \begin{bmatrix} \omega_d e_y - v_d + v_r cos(e_\theta) \\ -\omega_d e_x + v_r sin(e_\theta) \\ \omega_r - \omega_d \end{bmatrix} \quad (21)$$

In this step, Lyapunov stability method is preferred to examine stability, [11, 13]

$$L_1 = \frac{\left(e_x^2 + e_y^2\right)}{2} + \frac{1 - cos(e_\theta)}{k_2} \quad (22)$$

In this equation, k_2>0 and $L_1 \geq 0$.

$$\dot{L_1} = \dot{e_x}.e_x + \dot{e_y}.e_y + \frac{sin(e_\theta).\dot{e_\theta}}{k_2} \quad (23)$$

$$\dot{L_1} = -e_x\left(v_d - v_r cos(e_\theta)\right) - sin(e_\theta)\left(\frac{\omega_d - \omega_r}{k_2} - v_r e_y\right) \quad (24)$$

The first derivative of L_1 has to be lower than zero in order to obtain asymptotically stable system. Therefore, the KBBC law can be determined as Eqn. 25;

$$v_d = \begin{bmatrix} k_1 e_x + v_r cos(e_\theta) \\ \omega_r + k_2 v_r e_y + k_3 v_r sin(e_\theta) \end{bmatrix} \quad (25)$$

8. SLIDING MODE CONTROL

Sliding mode control is one of the most robust and strong controller which works based on the sliding surfaces and equilibrium point approach method. SMC method does not affected from the system parameter changes and disturbances, so increasingly preferred in control applications.

Linear velocity, angular velocity and reference velocity values are used as controlled parameters in controller design stage.

If Eqn. 17 is rearranged via Eqn 9, 10 to obtain equations below [10];

$$\left(m + \frac{2I_w}{R^2}\right)\dot{V} - m_c d\omega^2 = \frac{1}{R}(\tau_r + \tau_l) \quad (26)$$

$$\left(I + \frac{2L^2}{R^2} I_w\right)\dot{\omega} - m_c d\omega V = \frac{1}{R}(\tau_r - \tau_l) \quad (27)$$

$$\dot{V} = \frac{(m_c d\omega V)}{\left(I + \frac{2L^2}{R^2} I_w\right)} + \frac{(\tau_r + \tau_l)}{\left(I + \frac{2L^2}{R^2} I_w\right)}\left[\frac{1}{R}\right] \quad (28)$$

$$\dot{\omega} = \frac{(m_c d\omega V)}{\left(I + \frac{2L^2}{R^2} I_w\right)} + \frac{(\tau_r - \tau_l)}{\left(I + \frac{2L^2}{R^2} I_w\right)}\left[\frac{1}{R}\right] \quad (29)$$

In order to obtain sliding surfaces, DDMR model is defined mathematically as Eqn. 30.

$$\underline{\dot{x}} = \underline{f}(x) + [B]\underline{u} \quad (30)$$

σ is defined as sliding surface.

$$\sigma = G\Delta X \quad (31)$$

$$G = \begin{bmatrix} \alpha_1 & 0\,1 & 0 \\ 0 & \alpha_2 0 & 1 \end{bmatrix} \quad (32)$$

Γ is a diagonal matrix which is defined in order to improve performance of the SMC.

As SMC control input, equivalent control force u_{eq} is defined as equation below [14].

$$u = u_{eq} + (GB)^{-1}\Gamma \quad (33)$$

SMC control has chattering problem and a low pass filter is applied to the control signal in order to eliminate the difference between theoretical and practical equivalent control force.

$$\tau\dot{\hat{u}}_{eq} + \hat{u}_{eq} = u \quad (34)$$

The equation of SMC is defined at Eqn. 35.

$$u(t) = \hat{u}_{eq}(t) + (GB)^{-1}\Gamma \quad (35)$$

9. PID CONTROL

PID is a control algorithm which works with a continuous error detection and correction approach. Calculated errors are decreased by mathematical operations using proportional, derivative and integral terms. Proportional term is a multiplication for decreasing errors, integral term and derivative terms are respectively used for decreasing past and future errors.

PID control parameters are stated in Eqn. 35 as k: Proportional coefficient, k_i: Integral coefficient, k_d: Derivative coefficient.

$$u(t) = ke(t) + k_i \int_o^t e(\tau) d_\tau + k_d \frac{d_e}{d_t} \qquad (36)$$

10. SIMULATION AND RESULTS

DDMR model is created as a software model as stated Fig. 3 to compare tracking performances of SMC and PID control methods.

Identical reference trajectory conditions are used for both controllers at all simulations. Reference trajectory values taken from related block as inputs 1-3 to KBBC. KBBC supplies angular and linear velocity values as control inputs for SMC and PID controllers. Controllers identify optimum speed and voltage conditions of left and right dc motors which generate torque inputs for dynamic model. Dynamic model converts torque values to right and left wheel angular velocity for kinematic model. Also, linear and angular velocities values come from dynamic model are used to feedback the control system.

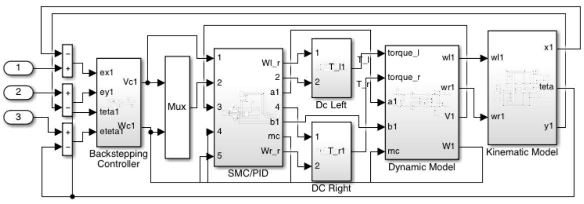

Figure 3 DDMR Simulink software model

As a first simulation, DDMR velocities controlled in order to track reference trajectory under non-additional mass condition. Afterward, additional mass applied to the simulation of DDMR dynamic model in order to examine error decrease performances and robustness of controllers.

Linear velocity reference value selected as $V_r = 1\ m/s$.

For the trajectory tracking performance simulation and robustness test of controllers, control parameters are selected as Table A.T1 and all physical parameters of DDMR are given in Table A.T2 which is stated in appendix section.

10.1. Reference Trajectory

X, Y coordinate and heading angle values are defined based on time as a reference trajectory at the beginning of the simulation. Selected reference trajectory defined with mathematical expressions as $x = t, y = \sin(t)$ and $\theta = atan(y/x)$. These values are selected in such a way that to get sinusoidal shaped trajectory and given to the system to track by controllers.

Reference trajectory was turned into the continuous linear path at 2π moment of simulation in order to test controllers in reference trajectory type transition zone and to observe possible continuous state errors of controllers. Reference trajectory is given by Fig. 4 as a graph of change of X, Y coordinate values.

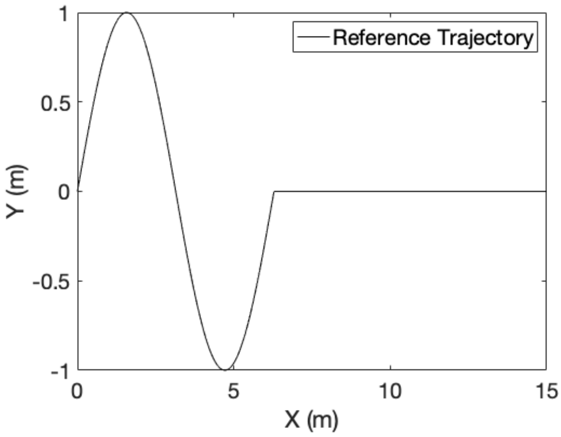

Figure 4 Reference trajectory

10.2. Trajectory Tracking Performance of Controllers

In this simulation step, kinematic based backstepping assisted PID and SMC are used for controlling pose of DDMR in defined trajectory without adding mass. DC motor speeds are changed by controllers to decrease X, Y and θ errors.

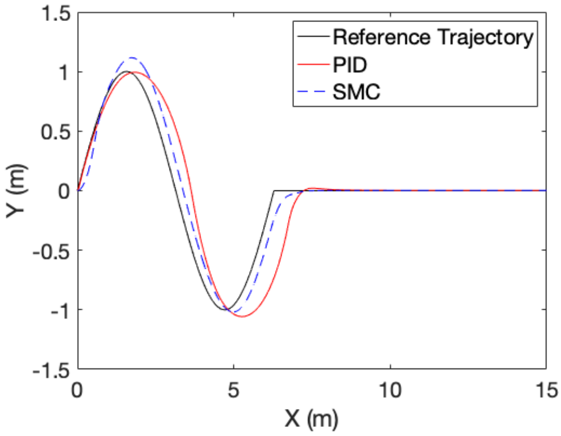

Figure 5 DDMR trajectory tracking

SMC method and PID control methods are similarly good at trajectory tracking which is shown at Fig. 5. Nevertheless, Fig. 6, 7 and 8 states that SMC has better error decrease performance at the beginning of the simulation and linear path transition zone. SMC control is able to react reference trajectory change quicker than PID control method because of the robust and effective control structure.

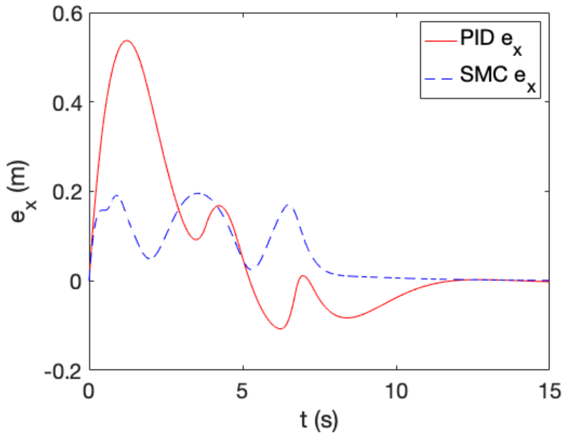

Figure 6 X coordinate error

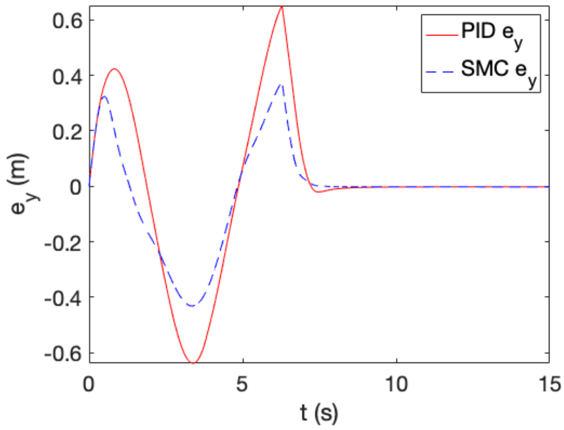

Figure 7 Y coordinate error

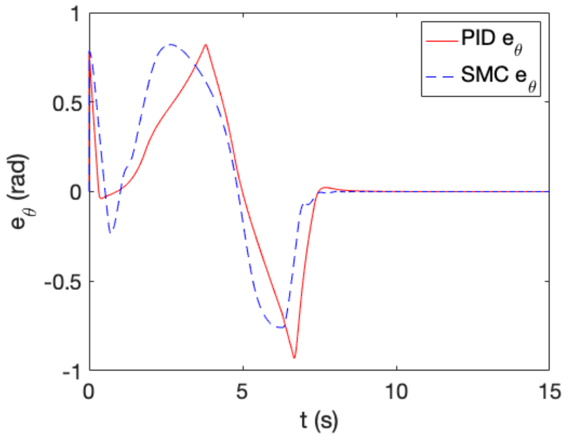

Figure 8 Heading angle error

As shown in Fig. 9 and Fig. 10 both controllers have same angular and linear velocity profile during the simulation but SMC decides velocities slightly higher than PID in order to decrease errors rapidly.

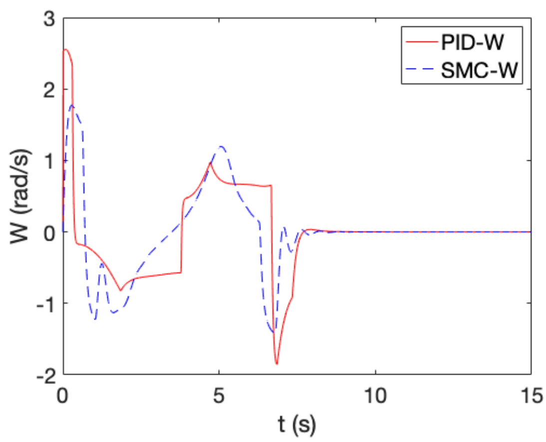

Figure 9 Angular velocity

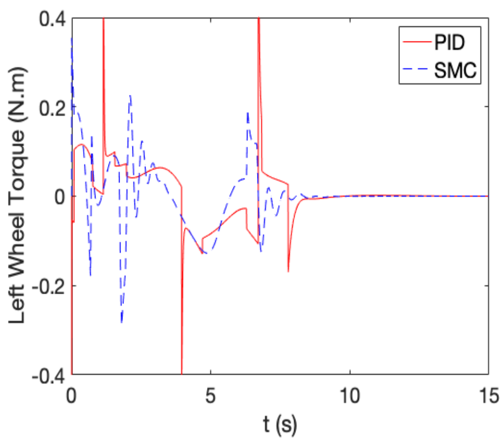

Figure 11 Left Wheel Torque

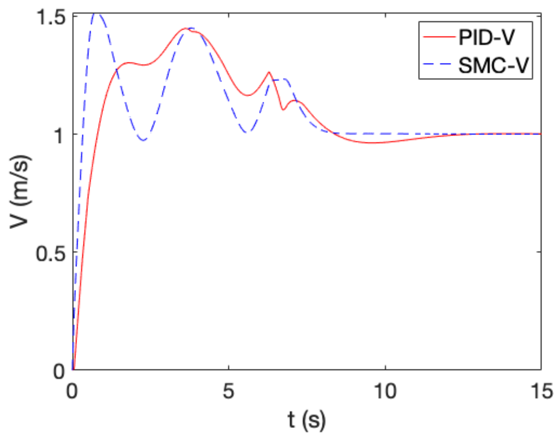

Figure 10 Linear velocity

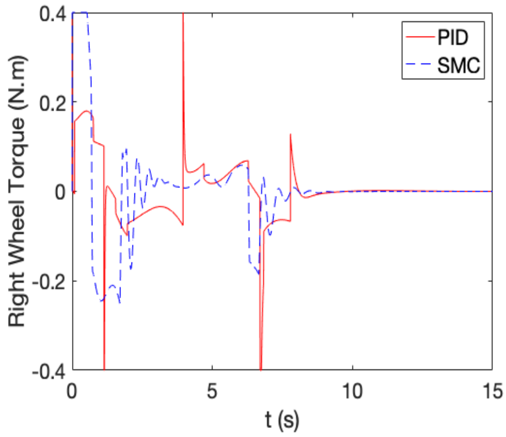

Figure 12 Right Wheel Torque

The torque values of dc motors are given by Fig. 11 and Fig. 12 for the zero additional mass condition. Both controllers appear to set torque values within reasonable and variable ranges. The torque values of the wheels are obtained higher values at the turning points of the trajectory and values are closer to the zero at the linear part of the trajectory. In addition to this, it is seen that SMC maximum torque values are lower than PID controller values and SMC changes values more often and in a narrower range than PID controller.

In addition to this, it is seen that SMC maximum torque values are lower than PID controller values and SMC changes values more often and in a narrower range than PID controller.

10.3. Robustness Test

The detection of the more robust control algorithm is the main purpose of this step so additional mass included to the DDMR body in order to test controller independency to the changes of the system parameters which is called robustness.

Total mass of DDMR is 0.630 kg and mass added value is 1.630 kg is shown in Fig. 13. Mass addition changes the dynamic model inputs as moment of inertia and control forces which increases robot position and heading angle errors. To what extent the controllers will be affected by this situation will be the main topic of this section.

As it is seen in Fig. 14, after 5 seconds of simulation PID control method is not capable of working successfully feasible about the reference trajectory and actual DDMR trajectory convergence. In contrast to this, SMC control trajectory tracking performance almost the same with zero additional condition. Fig. 14 shows that SMC control is able to capture linear part of the trajectory in 7.5th seconds which is 7 seconds earlier than PID control.

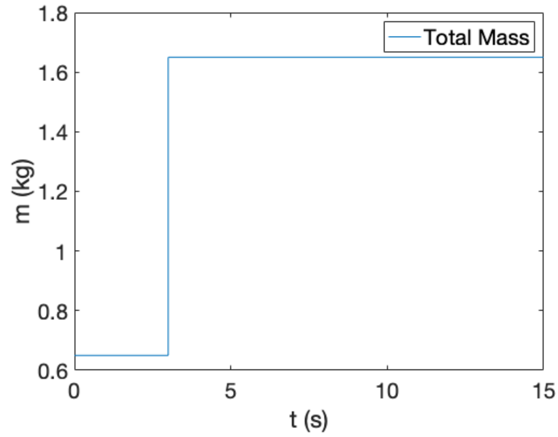

Figure 13 Total mass of DDMR

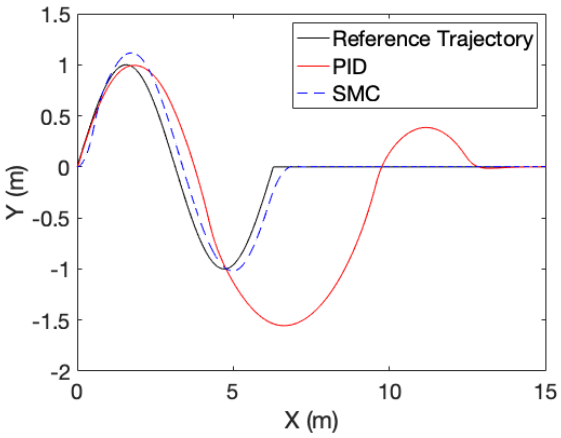

Figure 14 Trajectory tracking performance

SMC method more robust than PID control method against unexpected changes in simulation parameters.

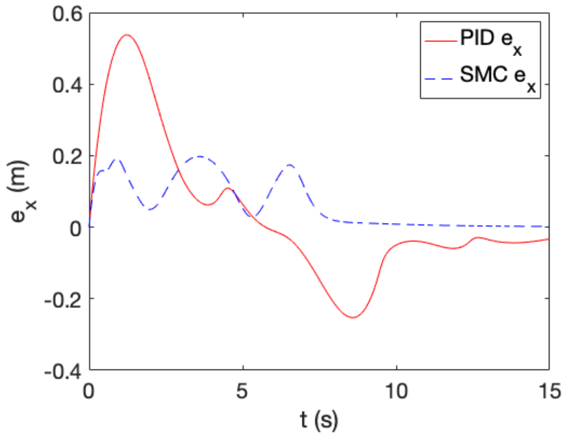

Figure 15 X coordinate error

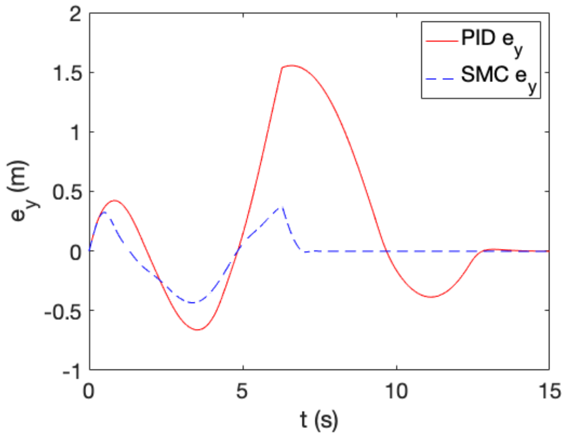

Figure 16 Y coordinate error

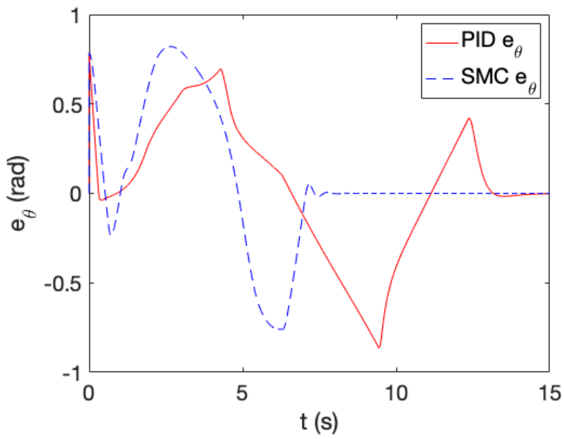

Figure 17 Heading angle error

This can be deduced from 5 to 10 seconds range of error graphics in Fig. 15, 16 and 17. SMC method is succeeded about error elimination at sinusoidal part and transition part of the reference trajectory under instantaneous mass addition.

Fig. 18 and 19 shows that angular and linear velocity values of SMC methods have sharp turns because of the the ability of reacting quickly and positively of this controller to mass change of DDMR body. PID controller cannot response to this change as effective as SMC method, but it seems to have succeeded albeit belatedly.

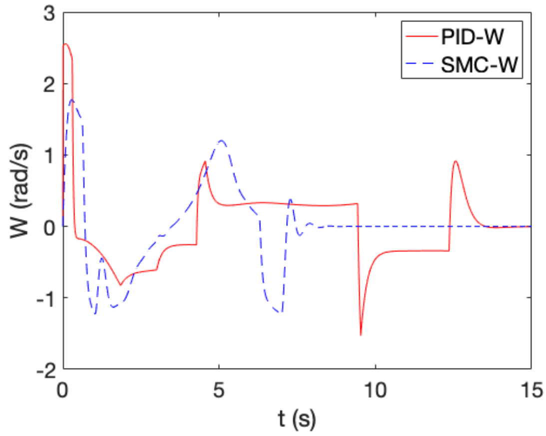

Figure 18 Angular velocity

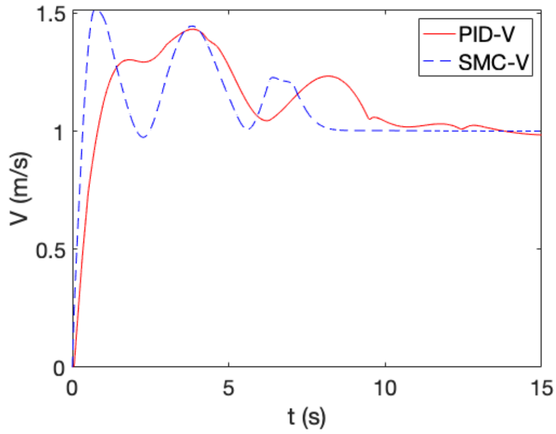

Figure 19 Linear velocity

The effects of the mass addition at 5th seconds to torque values of dc motors are shown in Fig. 20 and Fig. 21. The torque values are also represented to effects of controller forces to the DDMR system. As a similar inference with velocity analysis of the dc motors, SMC seems to be more robust and more successful than PID controller against the unexpected mass addition. Because SMC set the torque values significiantly lower and more stable than PID controller. The torque values are determined in a variable way by PID controller in order to eliminate the imbalance caused by added mass.

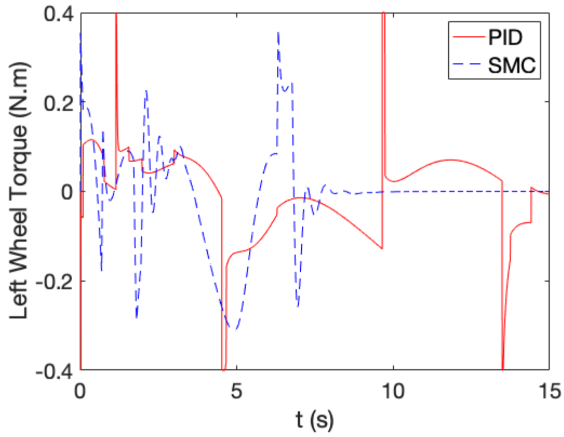

Figure 20 Left Wheel Torque

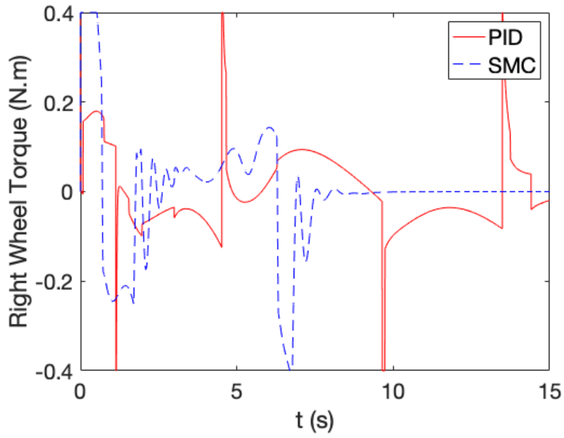

Figure 21 Right Wheel Torque

11. CONCLUSIONS AND DISCUSSION

In this study, kinematic based backstepping controller assisted SMC method and PID control method are used for different trajectory tracking simulations of differential drive mobile robot. As a first simulation, a reference trajectory which consists of sinusoidal and linear parts is tracked by SMC and PID control. These control methods performed quite similar trajectory tracking performances with stable system and environmental parameters. Both controllers seem to be successful about X , Y and θ errors elimination according to first simulation. Nonetheless, SMC has better performance at the turning points of the sinusoidal part of the trajectory and transition to linear part than PID control method.
As a more challenging test for controllers, additional mass applied to the DDMR body

at a certain time of simulation. Mass addition physically affects the system and robot motion which makes harder to keep DDMR pose values close to the reference values for controllers.

The result, obtained from this parameter change, robustness and functionality of SMC are clearly important reason for preference based on the error reduction of DDMR trajectory tracking parameters. PID control method is also track the reference trajectory, but it was strictly affected mass addition and respond prominently later than SMC method.

The figures of X, Y coordinate and heading angle errors show that SMC controller keeps the errors narrower range and closes to errors in a shorter time periods. Although the velocity setting profile is very similar for both controllers, SMC has been tracking the reference path with smaller amount of velocity changing.

Also, for both simulations SMC requires less wheel torque level and changes the torques in a practical way which means that controller forces are lower than PID controller. At the additional mass simulation, figures of the right and left wheel torques demonstrate that PID controller is struggling with the dynamic effects off the additional mass to the DDMR model.

Based on the result of this study, these conclusions were drawn which are SMC control is also preferred for different control systems and under various disturbances on the system because of the stable and robust structure of it.

APPENDIX

The total kinetic energy of the system can be obtained by adding kinetic energy of wheels and actuators to the kinetic energy of robot body. Where; m: total mass of robot, m_c: mass of robot body, m_w: mass of the wheels, I: total moment of inertia, I_c: robot body moment of inertia, I_w: wheel moment of inertia.

$$m = m_c + 2m_w, \quad I = I_c + m_c d^2 + 2m_w L^2 + 2I_w \tag{A.1}$$

$$T = \frac{1}{m}\left(\dot{x_c}^2 + \dot{y_c}^2\right) - m_c d\dot{\theta}\left(\dot{y}_c cos(\theta) - \dot{x}_c sin(\theta)\right) + \frac{1}{2}I_w\left(\dot{\omega_r}^2 + \dot{\omega_l}^2\right) + \frac{1}{2}I\dot{\theta}^2 \tag{A.2}$$

$$m\ddot{x}_c - md\ddot{\theta}sin(\theta) - md\dot{\theta}^2 cos(\theta) = A_1 \tag{A.3}$$

$$m\ddot{y}_c - md\ddot{\theta}cos(\theta) - md\dot{\theta}^2 sin(\theta) = A_2 \tag{A.4}$$

$$I\ddot{\theta} - md\ddot{x}_c sin(\theta) + md\ddot{y}_c cos(\theta) = A_3 \tag{A.5}$$

$$I_w\ddot{\theta}_r = \tau_r + A_4 \tag{A.6}$$

$$I_w\ddot{\theta}_l = \tau_l + A_5 \tag{A.7}$$

$$A^T(q) = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} \tag{A.8}$$

$$M(q) = \begin{bmatrix} m & 0 & -md\sin\theta & 0 & 0 \\ 0 & m & md\cos\theta & 0 & 0 \\ -md\sin\theta & md\cos\theta & I & 0 & 0 \\ 0 & 0 & 0 & I_\omega & 0 \\ 0 & 0 & 0 & 0 & I_\omega \end{bmatrix} \tag{A.9}$$

$$V(q,\dot{q}) = \begin{bmatrix} 0 & m & md\dot{\theta}\cos\theta & 0 \\ 0 & m & md\dot{\theta}\cos\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.10}$$

$$B(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{A.11}$$

$$A^T(q)\lambda = \begin{bmatrix} -\sin\theta & \cos\theta & \cos\theta \\ \cos\theta & \sin\theta & \sin\theta \\ 0 & L & -L \\ 0 & -R & 0 \\ 0 & 0 & R \end{bmatrix} \times \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix} \quad \text{(A.12)}$$

$$\begin{bmatrix} \dot{x}_a^i \\ \dot{y}_a^i \\ \dot{\theta}^i \\ \omega_r \\ \omega_l \end{bmatrix} = \begin{bmatrix} \frac{R}{2}\cos\theta & \frac{R}{2}\cos\theta \\ \frac{R}{2}\sin\theta & \frac{R}{2}\sin\theta \\ \frac{R}{2L} & -\frac{R}{2L} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \quad \text{(A.13)}$$

$$S(q) = \frac{1}{2}\begin{bmatrix} R\cos\theta & R\cos\theta \\ R\sin\theta & R\sin\theta \\ \frac{R}{L} & -\frac{R}{L} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \eta = \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \quad \text{(A.14)}$$

$$\dot{q} = S(q)\eta \Rightarrow \dot{q} = \dot{S}(q)\eta + S(q)\dot{\eta} \quad \text{(A.15)}$$

Substituting $\dot{q}$ and $\ddot{q}$;

$$M(q)\left[\dot{S}(q)\eta + S(q)\dot{\eta}\right] + V(q,\dot{q})\left[S(q)\eta\right] = B(q)\tau - A^T(q)\lambda \quad \text{(A.16)}$$

By multiplying both elements with $S^T(q)$ and rearranging for simpler form,

$$S^T(q)M(q)\dot{S}(q)\eta + S^T(q)M(q)S(q)\dot{\eta} + S^T(q)V(q,\dot{q})S(q)\eta = S^T(q)B(q)\tau - S^T(q)A^T(q)\lambda \quad \text{(A.17)}$$

$$S^T(q)M(q)S(q)\dot{\eta} + S^T(q)\left[M(q)\dot{S}(q) + V(q,\dot{q})S(q)\right]\eta = S^T(q)B(q)\tau - S^T(q)A^T(q)\lambda \quad \text{(A.18)}$$

$$V_a = R_a + L_a\frac{di_a}{dt} + e_a \quad \text{(A.19)}$$

$$e_a = K_b\omega_m \quad \text{(A.20)}$$

$$\tau_m = K_t i_a \quad \text{(A.21)}$$

$$\tau = N\tau_m \quad \text{(A.22)}$$

State space representation of control variables;

$$x_1 = V,\ x_2 = \omega,\ x_3 = \dot{V},\ x_4 = \dot{\omega}.$$

Controller input signals are shown in Eqn. A.23;

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \tau_r + \tau_l \\ \tau_r - \tau_l \end{bmatrix} \quad \text{(A.23)}$$

Table A1 T1 Model parameters

Term	***Value***	***Unit***
m_c	***0.65***	***kg***
m_ω	***0.05***	***kg***
I_c	***0.001***	***kg.m²***
I_ω	***0.0005***	***kg.m²***
I_m	***0.0012***	***kg.m²***
R	***0.10***	***m***
L	***0.275***	***m***
d	***0.12***	***m***

Table A2 T2 Control parameters

Symbol	***Value***
K_p	***95***
K_i	***65***
K_d	***2.55***
α_{12}	***300***
Γ_{12}	***55***
k_1	***1.5***
k_2	***100***
k_3	***25***

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Authors' Contribution
The authors contributed equally to the study.

The Declaration of Conflict of Interest/ Common Interest

No conflict of interest or common interest has been declared by the authors.

The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University ƒJournal of Science.

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