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HYBRID APPROXIMATION METHOD FOR TIME RESPONSE IMPROVEMENT OF CFE BASED APPROXIMATE FRACTIONAL ORDER DERIVATIVE MODELS BY USING GRADIENT DESCENT ALGORITHM

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Abstract: Due to its high computational complexity, fractional order (FO) derivative operators have been widely implemented by using rational transfer function approximation methods. Since these methods commonly utilize frequency domain approximation techniques, their time responses may not be prominent for time-domain solutions. Therefore, time response improvements for the approximate FO derivative models can contribute to real-world performance of FO applications. Recent works address the hybrid use of popular frequency-domain approximation methods and time-domain approximation methods to deal with time response performance problems. In this context, this study presents a hybrid approach that implements Continued Fraction Expansion (CFE) method as frequency domain approximation and applies the gradient descent optimization (GDO) for step response improvement of the CFE-based approximate model of FO derivative operators. It was observed that GDO can fine-tune coefficients of CFE-based rational transfer function models, and this hybrid use can significantly improve step and impulse responses of CFE-based approximate models of derivative operators. Besides, we demonstrate analog circuit realization of this optimized transfer function model of the FO derivative element according to the sum of low pass active filters in Multisim and Matlab simulation environments. Performance improvements of hybrid CFE-GDO approximation method were demonstrated in comparison with the stand-alone CFE method.

Keywords: CFE approximation method, FO realization, Optimization, Time response improvement

Gradyan İniş Algoritması Kullanarak CFE Tabanlı Yaklaşık Kesirli Dereceli Türev Modellerinin Zaman Cevabının İyileştirilmesi İçin Hibrit Yaklaşım Yöntemi

Öz: Yüksek hesaplama karmaşıklığı nedeniyle, kesirli dereceli (KD) türev operatörleri, yaygın olarak rasyonel transfer fonksiyonu yaklaşım yöntemleri kullanılarak gerçekleştirilmektedir. Bu yöntemler genelde frekans alanı yaklaşım tekniklerini kullandığından, zaman cevapları zaman bölgesi çözümleri için yeterince iyi olmayabilir. Bu nedenle, yaklaşık KD türev modellerinin zaman cevaplarının iyileştirilmesi, KD uygulamaların gerçek hayattaki kullanım performanslarına katkıda bulunabilir. Son zamanlardaki çalışmalar, zaman cevabı performans problemlerinin üstesinden gelebilmek için popüler frekans alanı yaklaşımı yöntemlerinin ve zaman alanı yaklaşım yöntemlerinin hibrit kullanımını ele almaktadır. Bu bağlamda, bu çalışma, frekans alanı yaklaşımı olarak Sürekli Kesir Açılımı (SKA) yöntemini uygulayan ve KD türev operatörlerinin SKA tabanlı yaklaşık modelinin basamak cevabı iyileştirmesi için gradyan iniş optimizasyonunu (GİO) uygulayan hibrit bir yaklaşım sunmaktadır. GİO'nun SKA tabanlı rasyonel transfer

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fonksiyonu modelinin katsayılarını hassas şekilde değiştirebildiği ve bu hibrit kullanımın, SKA tabanlı yaklaşık türev operatör modellerinin birim basamak ve impuls cevaplarını önemli ölçüde iyileştirebildiği gözlemlenmiştir. Ayrıca, KD türevin optimize edilmiş transfer fonksiyonu, Multisim ve Matlab simülasyon ortamlarında alçak geçiren aktif filtrelerin toplamı şeklinde analog devre olarak gerçekleştirilmesini göstermekteyiz. Hibrit SKA-GİO yaklaşımının performans iyileştirmesi klasik SKA yöntemi ile karşılaştırmalı olarak gösterilmiştir.

Anahtar Kelimeler: SKA yaklaşım yöntemi, KD gerçekleştirme, Optimizasyon, Zaman cevabı iyileştirme

1. INTRODUCTION

Fractional calculus is widely preferred in different fields for modeling real-world systems since it can more accurately express or represent some real-world phenomenon in daily life (Caponetto et al., 2010; Elwakil, 2010; Radwan et al., 2021; Sun et al., 2018; Tepljakov, 2017). It is often used for the system modeling in control engineering, energy systems, electronics, mechanics, etc (Chen et al., 2009; Delghavi et al., 2016; Homaeinezhad & Shahhosseini, 2020; Sidhardh et al., 2020; Silva-Juárez et al., 2020; Tapadar et al., 2022; Tepljakov et al., 2021; Tzounas et al., 2020; Vigya et al., 2021; Yang et al., 2020). However, the practical realization of FO systems is not an easy problem because ideal realization of FO elements is computationally expansive because of the long-memory effect (Tepljakov et al., 2021). Some approximate realization methods have been utilized to partially overcome this problem. The transfer function for a FO system model includes FO elements, and these elements are generally written in the form of approximate integer order transfer functions in a finite operating frequency ranges by using many integer order approximation methods; for instance Carlson, Oustaloup, Continued Fraction Expansion (CFE), Matsuda and modified stability boundary locus (MSBL) (Bingi et al., 2019; Colín-Cervantes et al., 2021; Deniz et al., 2016, 2020; Krishna, 2011; Tepljakov et al., 2021). There are numerous works in literature, and many of them present applications of the aforementioned integer order approximation methods(Monje et al., 2010; Tepljakov et al., 2021; Tufenkci et al., 2020).

The CFE approach, which is a well-known integer order approximation method to implement FO elements, is basically a series expansion technique and widely used in simulation and realization of the approximate FO elements. In this method, continued fractions are used to express the FO operator in the form of rational function (Deniz et al., 2020; Krishna, 2011; Vinagre et al., 2000). Many studies have been made to realize the FO circuits according to the CFE method. However, some drawbacks were encountered in the application of this method. It was expressed in previous studies: The time response approximation performance may not be satisfactory, and the operating frequency range is not configurable by users, and it can work in the low-frequency region (Colín-Cervantes et al., 2021; Deniz et al., 2020).

For discrete time domain approximation of fractional order elements, a unified method based on delta domain has been suggested, and the CFE was used for expansion of fractional power of discrete derivative elements that were expressed by using discrete-time frequency variables (Dolai et al., 2022; Swarnakar et al., 2019). Detailed surveys of fractional order elements and their application potential have been presented in several recent works, and these works reveal growing importance of approximation methods in practical realization of fractional order elements (Colín-Cervantes et al., 2021; Deniz et al., 2020; Shah et al., 2019; Tepljakov et al., 2021). Recently, employment of optimization methods has been shown to improve synthesis and approximate implementation of fractional order elements and functions. For instance, a genetic algorithm was employed in the synthesis of fractional order elements (Kartci et al., 2019). The GDO algorithm is applied to improve time-domain approximation performance of MSBL method (Koseoglu, 2022). Koseoglu's work was a useful contribution that can enhance practical performance of frequency-domain based approximate fractional order element realizations by improving time responses of results of other frequency domain approximation methods.

In the current study, the time response of the CFE approximation method has been improved with cooperation of the GDO algorithm. Here, similar to Koseoglu's work, where the time response of MSBL transfer function was enhanced by GDO (Koseoglu, 2022), the GDO algorithm is used to improve the time-response of the CFE method by optimizing the coefficients of the CFE based approximate transfer functions. This hybrid algorithm has been used to improve the step response of the CFE approximation method because the step response is very substantial in many system design applications such as control system design. We also observed that the algorithm can contribute to the frequency domain approximation performance of the CFE method to some extent. To realize the obtained approximate derivative model as an analog circuit implementation, the transfer functions is decomposed into the sum of the low pass filters form according to partial fraction expansion (PFE) and analog realization circuit is designed by using active first order filters in the Multisim environment (Bertsias et al., 2019; Koseoglu, Deniz, Alagoz, & Alisoy, 2021; Koseoglu, Deniz, Alagoz, Yuce, et al., 2021; Yüce & Tan, 2020).

2. METHODOLOGY

This section briefly introduces the cooperation of CFE method and GDO algorithm. Previously, Koseoglu (2022) demonstrated hybrid utilization of the MSBL method and the GDO algorithm in order to improve step response of the approximate FO derivative models for MSBL method (Koseoglu, 2022). The current study aims to use this hybrid method to integrate the CFE method and GDO algorithm and thus improves step response of the CFE-based approximate FO derivative models. The mathematical foundations of the CFE method can be found in (Colín-Cervantes et al., 2021; Deniz et al., 2020; Krishna, 2011; Vinagre et al., 2000).

To realize the FO element, firstly, the FO derivative function is expressed as an approximate rational transfer function by using the CFE method as follows (Deniz et al., 2020; Koseoglu, Deniz, Alagoz, & Alisoy, 2021; Koseoglu, Deniz, Alagoz, Yuce, et al., 2021):

$$s^{\alpha} \cong \frac{r_n s^n + r_{n-1}s^{n-1} + r_{n-2}s^{n-2} + \cdots + r_2 s^2 + r_1 s + r_0}{p_n s^n + p_{n-1}s^{n-1} + p_{n-2}s^{n-2} + \cdots + p_2 s^2 + p_1 s + p_0} \tag{1}$$

where, the parameter n is the integer order of the approximation method and the parameter $\alpha \in [0,1]$ is the FO of the derivative function s^{α}. The $r_0 \cdots r_n$ and $p_0 \cdots p_n$ are the coefficients of numerator and denominator polynomials, respectively. To realize (1), it is decomposed by employing PFE and expressed in the form of the sum of the first order filters as follows (Koseoglu, Deniz, Alagoz, & Alisoy, 2021; Koseoglu, Deniz, Alagoz, Yuce, et al., 2021):

$$T_{CFE}(s) = \frac{b_1}{s-a_1} + \frac{b_2}{s-a_2} + \frac{b_3}{s-a_3} + \cdots + \frac{b_{n-1}}{s-a_{n-1}} + \frac{b_n}{s-a_n} + k \tag{2}$$

where $b_1 \cdots b_n$, $a_1 \cdots a_n$ and k represent residues, poles and the direct gain, respectively. To obtain the approximate step response in the time domain, firstly, the step response is obtained in s domain as follows (Koseoglu, 2022):

$$Y(s) = \frac{T_{CFE}(s)}{s} = \frac{1}{s} \cdot \left(\frac{b_1}{s-a_1} + \frac{b_2}{s-a_2} + \cdots + \frac{b_{n-1}}{s-a_{n-1}} + \frac{b_n}{s-a_n} + k \right) \tag{3}$$

Then, the time domain approximate step response is written by using the inverse Laplace transform as (Koseoglu, 2022):

$$y(t) = \frac{b_1\, e^{a_1 t} - b_1}{a_1} + \frac{b_2\, e^{a_2 t} - b_2}{a_2} + \cdots + \frac{b_{n-1}\, e^{a_{n-1} t} - b_{n-1}}{a_{n-1}} + \frac{b_n\, e^{a_n t} - b_n}{a_n} + k \tag{4}$$

The exact (analytical expression) step response for FO derivative operator s^{α} can be expressed as follows:

$$y_{FO}(t) = \frac{1}{\Gamma(1-\alpha)\cdot t^{\alpha}}, \tag{5}$$

where $\boldsymbol{\Gamma}(.)$ represents the Gamma function. Using the expressions (4) and (5), the cost function that depends on the difference between the analytical and the approximate step responses is defined as

$$J = \frac{1}{2m}\sum_{i=1}^{m}\left(y_{FO}(t_i) - y(t_i)\right)^2 \tag{6}$$

where the parameter *m* is the total sampling point number for time t_i. For more accurate results, the step response was sampled in the interval of 0.001 s to 100 s with a time increment of 0.001 s. This cost function is minimized by the GDO algorithm that was expressed as follows (Koseoglu, 2022):

$$\frac{\partial J}{\partial a_j} = -\frac{b_j + a_j b_j\, t\, e^{a_j t} - b_j e^{a_j t}}{{a_j}^2}\cdot e_r\ ;\ \frac{\partial J}{\partial b_j} = -\frac{e^{a_j t}-1}{a_j}\cdot e_r\ ;\ j = 1\cdots n\ ;\ \frac{\partial J}{\partial k} = -\,e_r \tag{7}$$

By using these partial derivatives (sensitivity derivatives), the coefficient updates by using GDO can be written as (Koseoglu, 2022):

$${a_j}^{q+1} = {a_j}^{q} - \eta\frac{\partial J({a_j}^{q})}{\partial a_j}\ ;{b_j}^{q+1} = {b_j}^{q} - \eta\frac{\partial J({b_j}^{q})}{\partial b_j}\ ;\ j = 1\cdots n\ ;k^{q+1} = k^{q} - \eta\frac{\partial J(k^{q})}{\partial k} \tag{8}$$

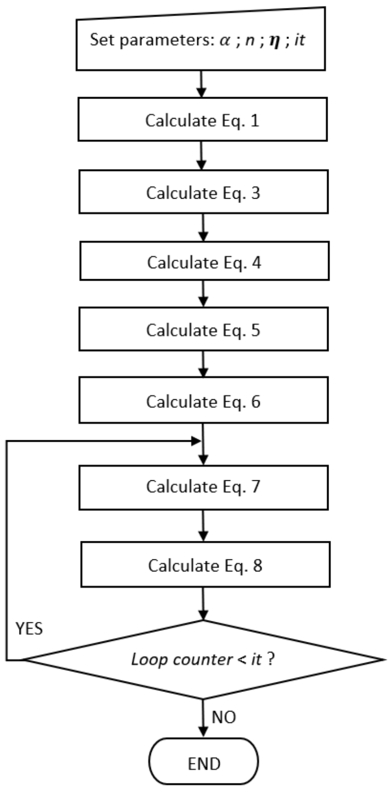

Figure 1:
A flowchart of the CFE-GDO approximation method

A flowchart of hybrid CFE-GDO approximation method is shown in Figure 1. In summary, the GDO method performs fine-tuning of both denominator and numerator coefficients of the CFE based approximate transfer function. The initial values for the transfer function coefficients are calculated by using the classical CFE approximation method, and GDO method is used to optimize these coefficients (Figure 1) to improve the step response obtained by the approximate model. To illustrate application of the proposed algorithm, the derivative operator $s^{0.5}$ is approximated by using CFE-GDO method in the following section.

3. NUMERICAL RESULTS AND DISCUSSIONS

This section is composed of two subsections. The first subsection shows the use of the proposed CFE-GDO approximation method to obtain an approximate rational transfer function model for FO derivative operator $s^{0.5}$. Results of the CFE-GDO method are compared with results of the classical CFE method, and contributions of the proposed CFE-GDO algorithm are demonstrated. In the second subsection, analog circuit realization results of this CFE-GDO based approximate model are demonstrated in the Matlab Simulink environment and Multisim analog circuit simulation environment (Matlab-R2020b, 2020; NI-Multisim-14.1, 2017).

3.1. Approximate Modeling Results for FO Derivative $s^{0.5}$

In this section, we present an illustrative example to approximate the FO derivative $s^{0.5}$. In the first step, the 5th order approximate transfer function was obtained for $s^{0.5}$ by classical CFE method (Krishna, 2011) as follows:

$$T_{CFE}(s) = \frac{11 \cdot s^5 + 165 \cdot s^4 + 462 \cdot s^3 + 330 \cdot s^2 + 55 \cdot s + 1}{s^5 + 55 \cdot s^4 + 330 \cdot s^3 + 462 \cdot s^2 + 165 \cdot s + 11} \qquad (9)$$

Then, the CFE-GDO algorithm was performed to improve the step response of the model given in (9). For the GDO algorithm, $\boldsymbol{\eta}$=0.004 was taken as the learning coefficient, and the number of iterations was taken as $it = 2 \cdot 10^5$. For these initial configurations, the cost function decreased and converged as shown in Figure 2, and the optimized 5th order approximate transfer function of the CFE-GDO method, T_o, was obtained as:

$$T_O(s) = \frac{10.95 \cdot s^5 + 161.5 \cdot s^4 + 436.8 \cdot s^3 + 286.4 \cdot s^2 + 32.59 \cdot s + 0.2453}{s^5 + 54.95 \cdot s^4 + 325.8 \cdot s^3 + 432.4 \cdot s^2 + 126.7 \cdot s + 4.442} \qquad (10)$$

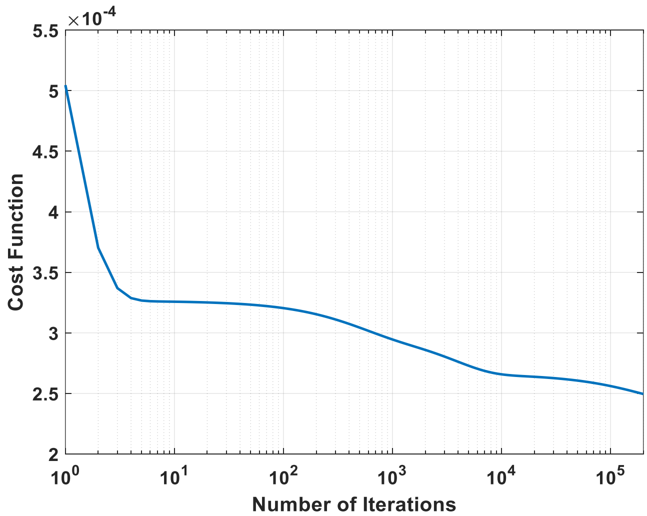

Figure 2:
The change in the cost function as the number of iterations increases

When (9) and (10) are considered, one can observe that the coefficients of numerator and denominator have changed slightly during the GDO optimization. This fine-tuning on the coefficients has enabled the optimized transfer function to yield more accurate impulse and step responses in comparison with those obtained by classical CFE method as seen in Figures 3 and 4, respectively.

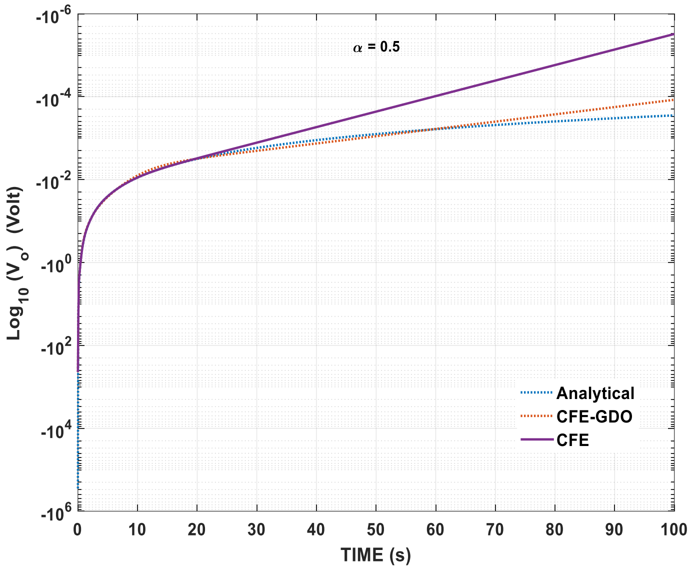

Figure 3:

The comparison of analytical (exact) impulse responses with approximate impulse responses based on CFE and CFE-GDO methods

Root Mean Squared Error ($RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(y_{FO}(t_i) - y(t_i)\right)^2}$) and Mean Absolute Percentage Error ($MAPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{y_{FO}(t_i)-y(t_i)}{y_{FO}(t_i)}\right|$) performances were calculated for the impulse response. The RMSE is a measure of accuracy that is more sensitive to larger errors. MAPE is a measure used to evaluate the performance of regression or forecasting models. RMSE values for both CFE and CFE-GDO methods were about 125.526, and RMSE performances are very close to each other for impulse responses. Slight difference between results of the models can be better evaluated by considering the MAPE performance. MAPE value for the CFE-GDO method was calculated as 19.297 while this value is 57.123 for the classical CFE method. Considering the exact impulse response, Figure 3 clearly shows the improvement of the derivative $s^{0.5}$ due to the CFE-GDO method and supports the improvement in MAPE values.

Figure 4 also clearly shows a considerable improvement in the step response of the classical CFE method by using the CFE-GDO method. In figure, the CFE diverges from the exact (analytical calculation) step response for the FO derivative $s^{0.5}$. The RMSE values were calculated as $5.540 \cdot 10^{-3}$ for the CFE-GDO algorithm and $5.854 \cdot 10^{-3}$ for the CFE method. When the MAPE values were considered, the degree of the improvement was revealed better. The calculated MAPE values were 2.147 for the CFE-GDO and 21.848 for the CFE. Such improvement in time response of the approximate model is very important for more accurate modeling and realization of the FO systems and this can significantly contribute to the FO system practice.

Furthermore, to investigate frequency domain approximation performance, Figure 5 compares the frequency responses of the CFE and the CFE-GDO methods. The magnitude and phase responses in the figure indicate that the CFE-GDO can provide frequency domain approximation in a wider frequency range, particularly by expanding the lower frequency part of the CFE approximation.

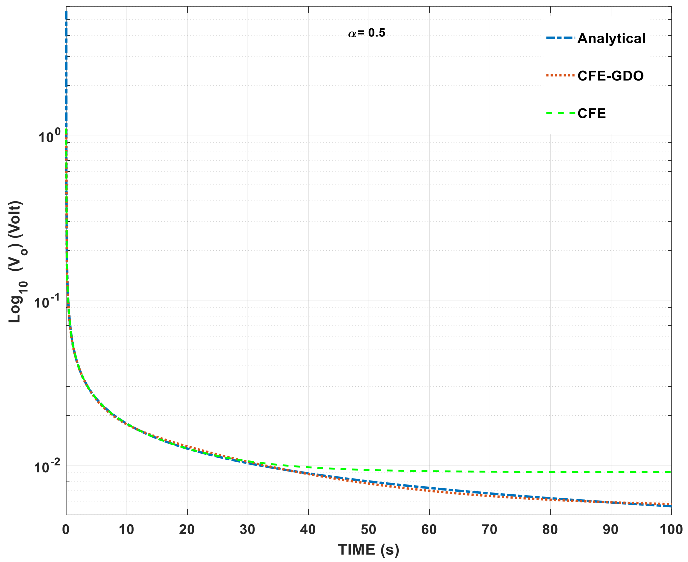

Figure 4:
The comparison of step responses with CFE and CFE-GDO based models of FO derivative $s^{0.5}$

Table 1. Comparative Magnitude and Phase Response error values for CFE and CFE-GDO methods

Frequency Response	Magnitude Response		Phase Response	
	CFE	CFE-GDO	CFE	CFE-GDO
RMSE	$3.128 \cdot 10^{-1}$	$2.950 \cdot 10^{-1}$	7.222	5.193
MAPE	3.910	4.078	8.018	5.542

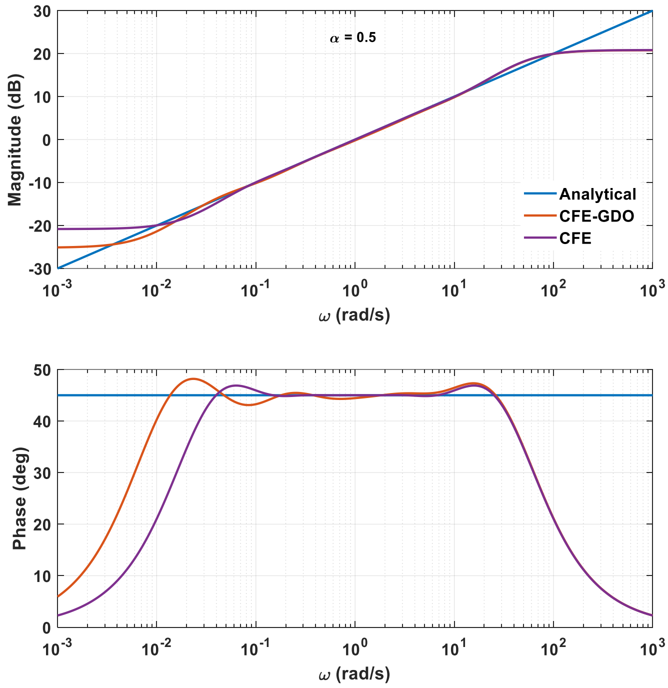

Figure 5:
The comparison of exact magnitude and phase responses with approximate magnitude and phase responses based on CFE and CFE-GDO methods

For better evaluation of frequency domain approximation performances, the RMSE and MAPE values for both CFE and CFE-GDO methods were calculated within the frequency interval $\omega \in [0.01\ 100]\ rad/s$. Table 1 shows RMSE and MAPE values, and these values also indicate considerable improvements in frequency responses arising from the CFE-GDO method.

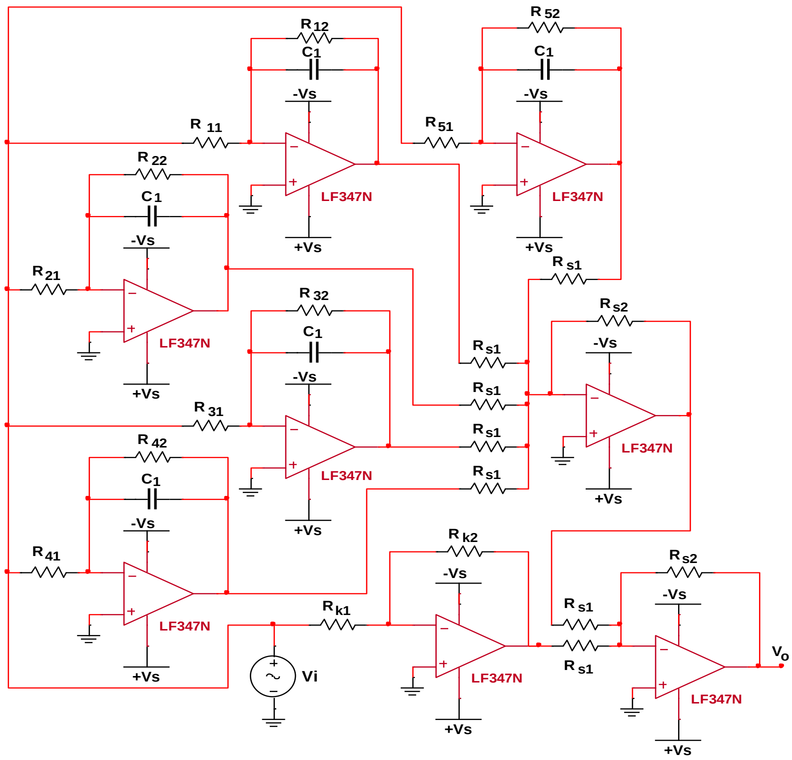

Figure 6:
The Multisim circuit schematic that realizes $T_O(s)$ function in the sum of the filter form (Koseoglu, 2022)

3.2. Analog Circuit Realization Results

To demonstrate an analog circuit implementation of CFE-GDO based approximation of the FO derivative $s^{0.5}$ by using the analog circuit in Figure 6, we used PFE of the $T_O(s)$ and performed analog circuit design according to the sum of the low-pass filter realization techniques (Bertsias et al., 2019; Koseoglu, 2022; Koseoglu, Deniz, Alagoz, & Alisoy, 2021; Koseoglu, Deniz, Alagoz, Yuce, et al., 2021; Yüce & Tan, 2020). The PFE of $T_O(s)$ is obtained as

$$T_{O-PFE}(s) = \frac{-434.259}{s+48.4029} + \frac{-5.0257}{s+4.8166} + \frac{-0.50697}{s+1.3398} + \frac{-0.11549}{s+0.35138} + \frac{-0.006782}{s+0.04047} + 10.9450 \qquad (11)$$

This equation shows partial fraction terms that include the residues, poles and the constant value. Each partial fraction term, which is shown in Table 2, is realized by using low pass filters with operational amplifiers that were shown in Figure 6. The constant value is realized as a constant gain element by using a basic inverting amplifier. A comprehensive explanation on the realization method was also presented in (Koseoglu, 2022; Koseoglu, Deniz, Alagoz, & Alisoy, 2021; Yüce & Tan, 2020).

Table 2. PFE terms and corresponding component values for analog realization

Term	Component Values	PFE Terms
1	$C_1 = 10^{-5}$ F, $R_{11} = 230.277\ \Omega$, $R_{12} = 2.066\ k\Omega$	$\frac{-434.259}{s+48.4029}$
2	$C_1 = 10^{-5}$ F, $R_{21} = 19.898\ k\Omega$, $R_{22} = 20.762\ k\Omega$	$\frac{-5.0257}{s+4.8166}$
3	$C_1 = 10^{-5}$ F, $R_{31} = 197.251\ k\Omega$, $R_{32} = 74.636\ k\Omega$	$\frac{-0.50697}{s+1.3398}$
4	$C_1 = 10^{-5}$ F, $R_{41} = 865.905\ k\Omega$, $R_{42} = 284.592\ k\Omega$	$\frac{-0.11549}{s+0.35138}$
5	$C_1 = 10^{-5}$ F, $R_{51} = 14.7455\ M\Omega$, $R_{52} = 2.47079\ M\Omega$	$\frac{-0.006782}{s+0.04047}$
6	$R_{k1} = 1\ k\Omega$, $R_{k2} = 10.945\ k\Omega$	10.9450

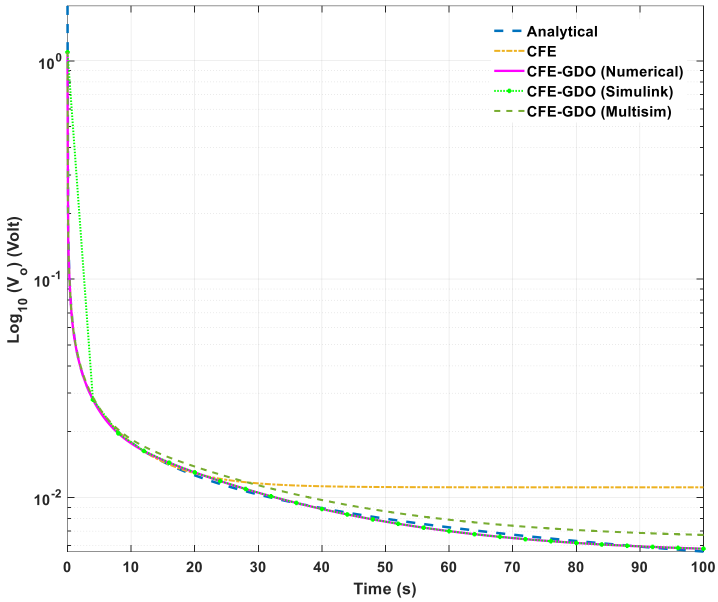

Figure 7:
The comparison of analytical step responses with approximate step responses based on the CFE method, the CFE-GDO method and its realizations

When a 0.1 V step input is applied to the designed circuit for 100 s simulation time, the step responses of the analog circuit from Multisim simulation and Simulink simulation are shown in Figure 7 in comparison with the exact step response, the numerical step responses of the CFE approximate model ($T_{CFE}(s)$) and the CFE-GDO based approximate model ($T_o(s)$). The Simulink simulation environment uses ideal component models. Therefore, Simulink simulation results are very similar to the step response of $T_o(s)$ transfer function model. The Multisim simulations allow the use of the non-ideal component models, and its simulation results are more realistic.

In Figure 8, the response of the designed circuit is shown for a square input wave with a frequency of 1 rad/s. As expected for an FO derivative circuit, sharp rises of falls at the circuit output were observed at edges of the square wave, and this validates the derivative operation of the circuit for the square waveform.

When the overall results are considered, it is observed that the proposed method can contribute to time responses (the step and impulse responses) of the CFE method, and this contribution can be useful for practical system applications such as control engineering applications. It is also seen that the configuration of the learning coefficient in the GDO algorithm is difficult, and it needs trial and error for proper setting of these parameters in order to obtain effective results from the CFE-GDO method.

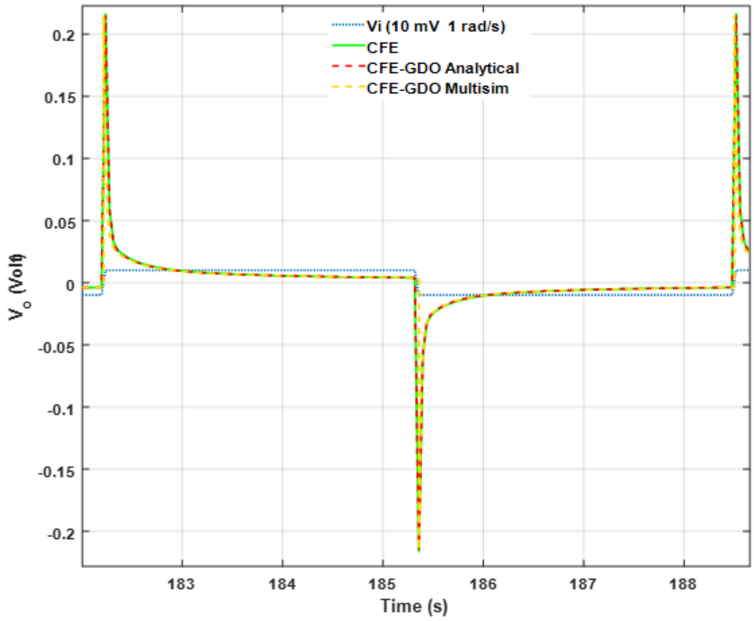

Figure 8:
The comparison of square wave responses of CFE method and the CFE-GDO based methods

4. CONCLUSIONS

In this study, time domain performance of the CFE approximation method is further improved by adopting the GDO algorithm. The CFE-GDO method is used for fine-tuning of transfer function coefficients of the CFE method, and this hybrid method is implemented to compute more accurate step responses from approximate transfer function models of the FO derivative elements (s^{α}). We demonstrated analog circuit implementation for the CFE-GDO based approximate model of FO derivative function $s^{0.5}$. Simulation results clearly indicated improvement in step and impulse responses by using CFE-GDO method for analog system applications.

As a future work, the improvements of the proposed hybrid approximation method on the other approximation methods can be investigated for realization of both integral and derivative operators. On the other hand, some other optimization methods may be tested for more accurate realization. Rounding errors (Round-off errors) of circuit element values can negatively affect approximation performance, and it can reduce performance of analog circuit realization in experimental studies. This problem can also be addressed in application works.

CONFLICT OF INTEREST

There are no conflicts of interest with any person or institution in the prepared article.

AUTHOR CONTRIBUTION

Murat Köseoğlu: Conceptualization, Methodology, Software, Writing.
Furkan Nur Deniz: Conceptualization, Methodology, Visualization, Writing.
Barış Baykant Alagöz: Conceptualization, Methodology, Visualization, Writing.

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