MULTIFRACTAL BEHAVIOUR IN NATURAL GAS PRICES BY USING MF-DFA AND WTMM METHODS

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-Abstract-

We make a comparative study of Multifractal Detrended Fluctuation Analysis (MF-DFA) and the Wavelet Transform Modulus Maxima (WTMM) method to detect multifractal character of natural gas daily returns. We give a brief introduction on above methods and compare their effectiveness. The results from this methodoligies show that behaviour of natural gas daily returns were multifractal. The major sources of multifractality are long-range correlations of small and large fluctuations and Fat-tail distributions of the series.

Key Words: Natural Gas, Multifractal, MF-DFA, WTMM

JEL Classification: G17 Financial Forecasting and Simulation

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1. INTRODUCTION

Energy plays an essential role in the world economy. The dynamics of energy prices are of great interest among researchers and market participants. Modeling and forecasting has increasingly become very important in analysis of trends in commodity markets, particularly in high frequency trades. It is difficult to predict the price return, i.e. profit or loss, due to many unknown variables including social and political unrest, catastrophic events, etc.

Natural gas prices are a function of market supply and demand. Because of limited alternatives for natural gas consumption or production in the short run, even small changes in supply or demand over a short period can result in large price movements to bring supply and demand back into balance. Natural gas has several interesting characteristics. First, gas is costly to transport internationally, so prices and forward curves vary regionally. Second; once a given well has begun production, gas is costly to store. Third, demand for gas in the United States is highly seasonal, with peak demand arising from heating in winter months. Thus, there is a relatively steady stream of production with variable demand, which leads to large and predictable price swings (http://www.eia.gov/).

A traditional assumption used in the early studies of financial time series, considered that returns are independent, Gaussian random variables. However, uncountable number of empirical studies, initiated by B. Mandelbrot, have shown that empirical returns reveal instead very rich and non trivial statistical features, such as fat tails, volatility clustering and multiscaling. This paper aims to empirically test whether returns on the Natural Gas spot prices exhibit long-range correlations and multifractal patterns. The term fractal was coined by Mandelbrot (Mandelbrot,1982:15) to characterize a rough or fragmented geometric shape that displays a large degree of self similarities within its own fractional dimensions.

In principle there are two competitive methods for detecting the multifractality which are commonly used to eliminate trends and concentrate on the analysis of fluctuations. Multifractal Detrended Fluctuation Analysis (MF-DFA)

(Kantelhardt,2002:87) and Wavelet Transform Modulus Maxima (WTMM) (Muzzy,1994:245) .

2. METHODOLOGY

2.1.Multifractal Detrended Fluctuation Analysis

By using the MF-DFA analysis, we estimate the generalized Hurst and the Renyi exponents for price fluctuations. By deriving the singularity spectrum from the above exponents, we quantify the multifractality of a financial time series and compare the multifractal properties . In this method, we use the logarithmic return of the time series for each step u. Methodlogy of MF-DFA is as follows. The deviation in return from the mean of the return is

$$y(u) = \sum_{u}^{n} [\left(r(u) - \bar{r}]\right) \qquad (u=1,2,....N) \tag{1}$$

The length N of the time series is partitioned into n segments, each of length s, N = ns. A least squares method can be used to identify trends in running deviation over each segment k by a polynomial $g(u)$. The average fluctuation $F_{k(s)}$ in each subregion k is

$$\left(F_{k(s)}\right)^2 = \frac{1}{s} \sum_{u=(k-1)s+1}^{ks} \left((y_k\,](u) - g(u)\right)^2 \;) \tag{2}$$

The average moment of the fluctuation of order q over n segments of the time series is

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$$F_{q(s)} = \left\{ \frac{1}{n} \sum_{k=1}^{n} [\ (\ldots] F_{k(s)} \Big)^{n} \right\}^{\frac{1}{q}}$$)

(3)

As q - > 0 (4)

The power-law dependence of the q-th order moment of the fluctuation Fq(s) in interval s of the time series provides an estimate of the Hurst exponent h(q), i.e.,

In general, the exponent h_q may depend on q . When h_q is constant for all q the time series are mono-fractal. For stationary time series, h_2 is identical to the well-known Hurst exponent H. Thus, we will call the function h_q generalized Hurst exponent. Positive values of q are used for magnifying the effects of large price variations in the scaling analysis, and negative values of q are used for magnifying the effects of small price variations.

When $h_2 > 0.5$,the kinds of fluctuations related to are persistent. An increase (decrease) is always followed by another increase (decrease). When $h_2 < 0.5$, the kinds of fluctuations related to are anti-persistent. An increase (decrease) is always followed by another decrease (increase). However, $h_2 = 0.5$ the, the kinds of fluctuations related to display random walk behavior. The richness in multifractality is associated with high variability of h_q and the degree can be quantified as $\Delta h = h_{qmin} - h_{qmax}$. As large fluctuations are characterized by smaller scaling exponent h_q than small fluctuations, h_q for $q < 0$ are larger than those for $q > 0$, and Δh is positively defined . Multifractality degree can be used to measure the efficient extent of a finance market. When multifractality degree is weaker, for all q value, generalized Hurst exponents are closer to 0.5. This shows that no matter the fluctuation is big or small, its change of state is closer to random walk, so the market is more efficient.

The analytical relationship between generalized Hurst exponents based on MF-DFA and Renyi exponent τ_q is, $\tau_q = qh_q - 1$. The exponent τ_q represents the temporal structure of the time series as a function of the various moments q , or τ reflects the scale dependence of smaller fluctuations for negative values of q , and larger fluctuations for positive values of q . If τ_q increases nonlinear with q , then the series is multifractal.

Via a Legendre transform, another important variable set $\alpha - f(\alpha)$ is defined by $\alpha = h_q + qh'_q$ $f(\alpha) = q(\alpha - h_q) + 1$ Here, α is the Holder exponent or singularity strength which characterizes the singularities in a time series. Singularity basically points at the rapid changes in the time series values for small changes in time. In the multifractal case, the different parts of the dataset are characterized by different values of α , or the singularity spectrum .

2.2.Wavelet transform modulus maxima method (WTMM)

As proven by Mallat and Hwang (1990:549), multifractal formalism based on wavelet transform modulus maxima (WTMM) allows us to determine the whole singularity spectrum directly from any experimental signal. Muzy et al. (1991:3515) define the scaling behavior of partition functions (a) from the WTMM. The slope of the partition function determines the scaling τ() of moments of the distribution. Linearity of the scaling function suggests monofractal behavior of the time series, (all moments exhibit the same H scaling with time). The procedures of calculating the multifractal singularity spectrum based on WTMM is described in Yalamova (2003). Wavelet transform has proved to be a particularly efficient tool for measuring the local regularity of a function. The wavelet transform of $f(t) = P(t)$ is defined as:

$$W(\tau, a) = \int_{\infty}^{+\infty} f(t)\psi_{\tau,a} t\,(dt) \qquad (5)$$

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where the analyzing wavelet ψ is a function with local support, centered around zero and the family of wavelet vectors is obtained by translation τ and dilatation a. The modulus maxima (largest wavelet transform coefficients) are found at each scale a as the suprema of the computed wavelet transforms such that:

$$\frac{\partial W(\tau, a)}{\partial \tau} = 0 \qquad (6)$$

The originality of the WTMM method is in the calculation of the partition function $Z(q, a)$ from these maxima lines. The space-scale partitioning given by the wavelet tiling or skeleton defines the particular Gibb's partition function:

$$Z(q, a) = \sum_{\tau, a} \sup_{a} |W(\tau, a)|^{q} \qquad (7)$$

The WTMM method uses continuous wavelet transform rather than Fourier transforms to detect singularities – that is discontinuities, areas in the signal that are not continuous at a particular derivative. Another interesting property of the wavelet transform is that the coefficients at these maxima-which are a small fraction of the total number of coefficients-are enough to encode the information contained in the signal (Muzzy,1994:245), Moreover, as one follows a maxima line from the lowest scale to higher and higher scales, one is following the same singularity. This fact allows for the calculation of h_q by a power law fit to the coefficients of the wavelet transform along the maxima line (Struzik,2000:163).

Wavelet Skeleton is an aggregate of all Local Maxima Lines (LML) on each scale of Wavelet coefficient matrix. The idea of Skeleton matrix construction is to remove all wavelet coefficients in absolute wavelet coefficients matrix that are not maximal. Skeleton matrix is a scope of all local maxima points that exist on each scale a. If scaling exponential function is everywhere convex that indicates multifractal behaviour of the signal. It assumes that the signal does not have some decent fractal measure, but is characterized by the scope of fractal measures. In case of monofractal behaviour, the scaling exponential function is line. (Puckovs,2012:83)

3. DATA ANALYSIS

The Natural Gas Spot prices data were taken from Energy Information Administration in the US Department of Energy (http://www.eia.gov/) The data constitutes of daily closing prices over the period from Jan 01, 1997 to April 01, 2013 for 4065 observations in Figure1. In our analyses we used log return of the spot prices illustrated as Figure 2.

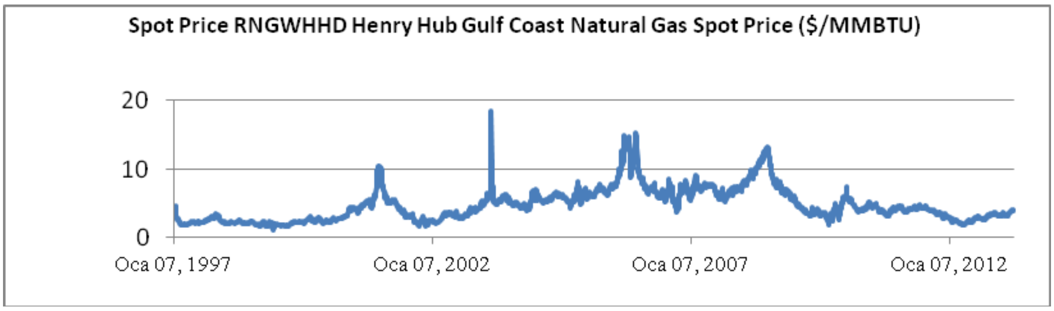

Figure1- Natural Gas Spot Price

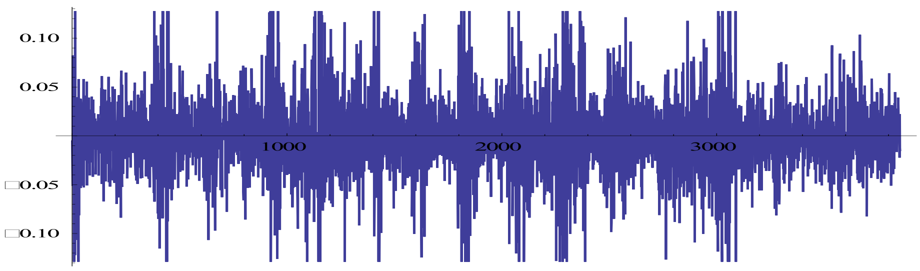

Figure 2:Log Return Of Spot Prices

4. EMPRICAL RESULTS

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4.1. MDF-FA Method

We have used matlab codes to implement MF-DFA on log-return data of NaturalGas Spot prices . (Ihlen,2012:3) We estimate the generalized Hurst and the Renyi exponents for price fluctuations. Deriving the singularity spectrum from the exponents, we quantify the multifractality of a financial time series. In Figure 3 (Upper Right Corner), The generalized Hurst exponents for time scales is given. When q varies from -5 to 5, $\mathbf{h_q}$ decreases from 0.4465 to -0.16511. $\mathbf{h_q}$ is not a constant, indicating multifractality in time series. Due to H < 0.5, the system displays fractional Brownian motion and anti-correlation and The antipersistent behavior and deviations of one sign generally followed by deviations with the opposite sign is an indication of the high degree of natural gas prices nervousness and uncertainty. It means that the analysis of these events doesn't give support to the Efficiency Market Hypothesis.

There are two factors contributing to multifractal properties, namely long-range temporal correlations for small and large fluctuations and the fat-tailed probability distributions of variations. (K.Matia, 2003:422) As shown in Figure 3 (Lower Left Corner), multifractal scaling function almost linear with q for negative moments, but show significant non-linearities for positive moments. This means that the temporal structure of the larger fluctuations play an important role in the multifractality.

The spectrum, in Figure 3 (Lower Right Corner) as an upside-down parabola, peaks at and stretches from min to max. $\Delta\alpha = \alpha_{max} - \alpha_{min}$ conventionally quantifies the degree of multifractality which is the width of singularity spectrum, while $f(\alpha)$ tells how frequently events with α scaling exponent occur. The width of the fractal spectrum, which shows the distinction between the maximum probability and the minimum probability,The larger the value of $\Delta\alpha$, the more uneven is the distribution of time series, and thus the stronger is the multifractality. The long-range and short correlations lead to a relatively narrow width of the spectrum (lower risk) or vice versa.In our case,the widht of spectrum 0.57409-(-0.37938)=0.95347 reveals that there is strong multifractility in Natural Gas daily returns.

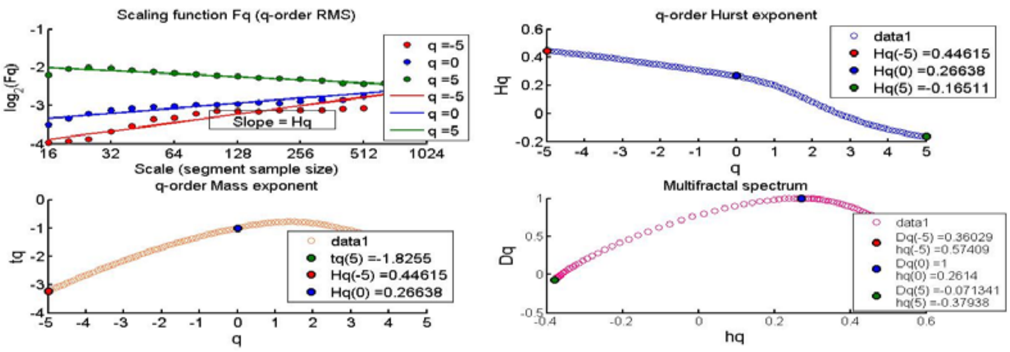

Figure:3 Generalized Hurst Exponent, Reyni Exponents, Multifractal Spectrum

4.2. Wavelet Transform Modulus Maxima (WTMM)

We draw the Wavelet Skeleton by using Mathematica. Time shifting coefficients (b) are drawn on x axis, Scales (a) are drawn on Y axis. Local maxima lines are constructed using Wavelet coefficient matrix, selecting local maxima points on each scale parameter (Figure 4). The scope of all local maxima lines builds the so called Skeleton function. This function illuminates periodicity of the signal on decent scales. Dark colours correspond to lower absolute wavelet coefficient values. Light colours indicate higher absolute wavelet coefficient values. Continuous maxima line from small scales to large scales determine the time of the singularity at different scales. According to WTMM methods, we see that scaling exponential function is convex which shows that Natural Gas daily returns has multifractal properties.

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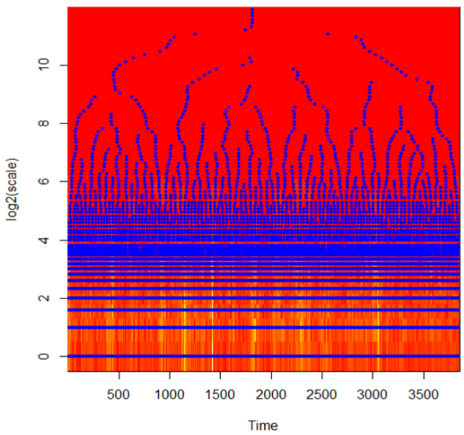

Figure 4:Wavelet Sceleton

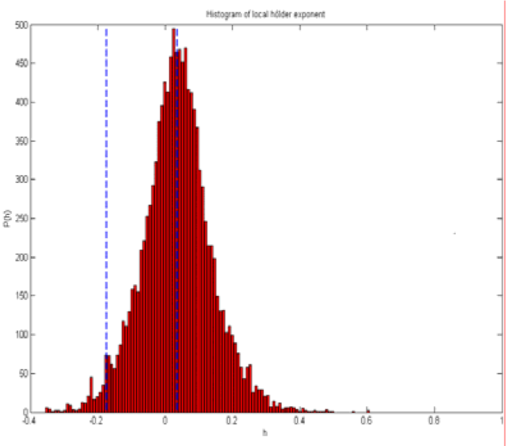

Figure 5:Histogram Of Local Holder Exponent

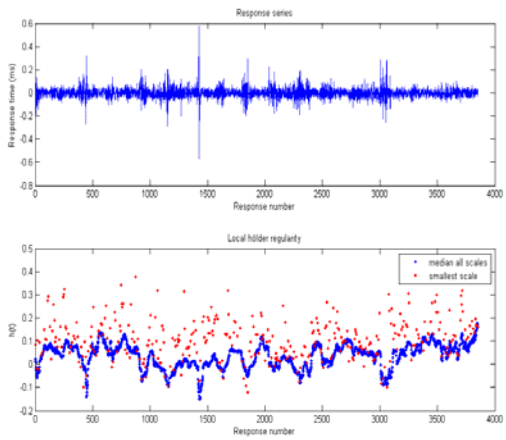

Figure 6: Local Holder Regularity

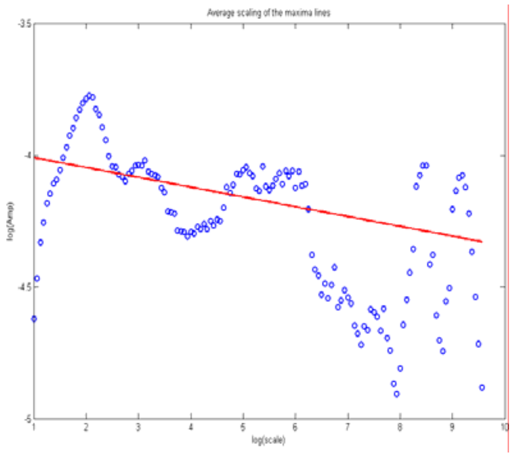

Figure 7. Average Scaling of Maxima Lines

Unlike Multifractal, If we look at the scaling function, the Monofractal price series' behaviour is observed to be less deviated from the trend line (Figure 7). And also fat tails in both ends of the Local Holder Exponents indicates multifractality of the series (Figure 5).

5. CONCLUSION

Using the multifractal detrended fluctuation analysis (MF-DFA) together with WTMM, we showed the multifractal properties of the USA Natural Gas daily returns. Anti-persistent behaviour of prices which means that an increase (decrease) is always followed by another decrease (increase) is an indication of the high degree of market nervousness and uncertainty as stated in the literature. The fat-tailed distributions (probability distribution of returns), also contribute to the multifractal behaviour of the natural gas daily returns.

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