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MULTIFRACTAL ANALYSIS OF THE DYNAMICS OF TURKISH EXCHANGE RATE

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─Abstract ─

We perform a comparative study of applicability of the Multifractal Detrended Fluctuation Analysis (MFDFA) and the Wavelet Transform Modulus Maxima (WTMM) method in properly detecting of mono- and multifractal character of data. After summarizing the theory behind both methods, we apply both methods on USD/TRY currency. The results show that our data has multifractal nature but not at high level and multifractality is poorer if WTMM method is used. We also investigated whether other Eastern European country currencies, such as Russian Rubble and Hungarian Forint have multifractal characters by using MFDFA method. Therefore, forecasters have often encountered in trying to predict these exchange rates with models that do not incorporate any notion of inhomogeneity will have little predictive power.

Key Words: *MF-DFA,WTMM, USD/TRY, Eastern European currencies,*

JEL Classification: B23, C6

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1. INTRODUCTION

Multifractal analysis is becoming popular since the real data in different financial instruments in various markets exhibits self similar properties. The reason that multifractals are taking attention is that they overcome the limitation of classical theories in which the occurrence of abrupt events is impossible. Extreme fluctuations at irregular intervals and scale symmetries, which represent exact relationships between fluctuations over different distances, could be analyzed when the dimension of a time series is fractal or multifractal. Analysis of fractal data is based on Hurst exponent, correlation functions and frequency spectrum, or in the multifractal form wavelet transforms and Holder exponent spectrum.

In this paper, we looked at the long-term pattern in USD/TRY following 2001 crisis. We used daily exchange rate of USD/TRY starting from the year 2002 and analyze the multifractal property of our data set using two different methods, Multifractal detrended fluctuation analysis (MDFDA) and Wavelet transform modulus maxima (WTMA). We compare our results in these two methods. The paper is constructed as follows. In section 2, we present the theoretical framework of MDFDA and WTMA methods. Data and our empirical results using these two methodologies are discussed in Section 3. Lastly, we conclude in Section 4 with a summary of our results.

2. THEORETICAL BACKGROUND

Holder exponents or the local Hurst exponents are used to describe the self-similarity of fractal structures. If the fractal is monofractal then it can be associated with only one Holder exponent, on the other hand if the data is multifractal, different parts of the structure are characterized by different values and resulting of the existence of the whole spectrum. There are two methods of computing a singularity spectrum for a time series data:

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- Multifractal Detrended fluctuation analysis (MDFDA)
- Wavelet transform modulus maxima (WTMM)

The Detrended Fluctuation Analysis (DFA) is a tool in analyzing the scaling properties of monofractal signals and in identifying correlations present in noisy non-stationary time series. The multifractal generalization of this procedure is called Multifractal Detrended fluctuation analysis (MFDFA). In contrast to DFA, that is only using the second moment, MF-DFA widens the scope to q-order moments depending on the time series length and derives a continuous set of scaling exponents referred to as the singular spectrum.

Multifractal analysis distinguishes trends from fluctuations in order to detect long-range correlations. Also, scaling behaviour tools enables long range correlations based on high-frequency data as the calculations are extrapolated across different time scales. The MF-DFA procedure has five steps and among these first three steps is identical to the general DFA procedure.

As an initial step, we determine y_k of the time series $\bar{x}$ is created with k=1…N

and representing the average value of x_t for t=1,2,….N and y_k can be shown as:

$$y_k = \sum_{t=1}^{k}(x_t - \bar{x})^{*} \quad k = 1, 2, \ldots, N$$

In the second step, the profile of y_k is divided into ns non-overlapping boxes of equal lengths s with ns≡floor (N/s) and this process is repeated starting from the opposite end of the dataset to include all data points. Hence, $2N_s$ segments are obtained. Thirdly, time series is fitted by using a local polynomial and the corresponding variance of original versus fitted data is given by

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for λ= and $P_\lambda(i)$, the fitting polynomial with order m in segment λ.

Then by using the above function we take the average over all segments and obtain q order fluctuation for any real value q ≠0 given by:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} \left[F^2(s,\lambda)\right]^{q/2} \right\}^{1/q}$$

Lastly, repeating the third step for various time scales s, the relation between the fluctuation functions $F_q(s)$ and time scale is studied by using log-log plot. We determine the scaling behavior of the fluctuation functions by analyzing log-log $F_q(s)$

plots and s for each value of q. If the series are long-range power-law

correlated, increases, for large values of s, as a power-law, $F_q(s) \sim s^{h(q)}$, $h_($ $h_($

where in general the exponent depends on . Generalized exponents can be

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$F_q(s)$

obtained by observing the slope of log_log plot of versus (s) through least squares method.

h_q

If is constant for all , the time series are mono-fractal. Fluctuations related to q are persistent if h(q)>0.5. On the other hand, an increase (decrease) is always followed by another increase (decrease) if h(q)<0.5 and the fluctuations are considered anti-persistent. Lastly, if h(q)=0.5, the kinds of fluctuations related to q considered as random walk.

The richness of multifractality is a related to the variability of h(q) and it is defined as Δh=h(qmin)-h(qmax). Multifractality is used to test how efficient are the markets. When multifractality low, for all q value, generalized Hurst exponents are closer to 0.5 and this shows the process looks like random walk. Hence, it is a support for efficient market hypothesis.

Alternative method to detect multifractality is Wavelet Transform Modulus Maxima (WTMM) method, which consists in detection of scaling of the maxima lines of the continuous wavelet transform on different scales in the time-scale plane. The scaling and translation are performed by two parameters. One is the scale parameter a that stretches the mother wavelet to the required resolution and the other is the translation parameter b shifts the analysing wavelet to the desired location. After mother wavelet is chosen, the continuous wavelet transform is calculated with the following equation:

$X_W(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt$, where a>0 scaling constant a and b∈ R.

Singularities are identified by calculating $X_W(a,b)$ for subsequent wavelets that are derivatives of the mother wavelet. WTMM, which uses continuous wavelet transform is capable of producing skeleton that partitions the scale and time space by fractal dimension. This wavelet transform could keep local characteristics of the data, namely singularities while filtering out the trends.

We will present our data set and empirical results we got by using both of methods on the USD/TRY exchange rate in the next section.

3. DATA

The data set of daily USD/TRY bid rates are obtained from Central Bank of Turkey's website, while daily prices of Russian Rubble, Czech Koruna and Hungarian Forint obtained from Bloomberg. Each data covers the time period starting from 2 January 2002 to 22 March 2013 and there are 2925 data points in our sample for each currency. Note that the data includes only trading days. Price changes X(t) = Z(t) – Z(t – 1) are initially taken, where Z(t) is a logarithm of the currency, i.e. USD/TRY rate at time t. Below is the visualization of logarithmic return of USD/TRY:

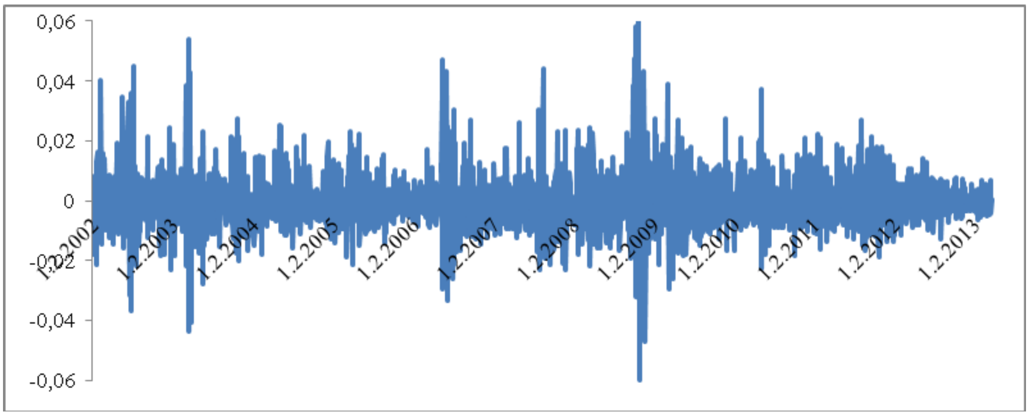

Figure 1: Log return of USD/TRY currency

4. EMPRICAL RESULTS

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First we derived artificial multifractal data using lastwave program and applied WTMM and MFDFA to detect multifractal property of the artificial data. Our results show that in majority of situations multifractal property of a process is better detected by MFDFA. We applied both methods to our log-return USD/TRY data and compared the results of two methods. We also applied MFDFA analysis to find out multifractal behavior of comparable Eastern European currencies to Turkish Lira.

We used matlab program in our MFDFA analysis on our log-return data set of USD/TRY prices and estimated the Renyi exponents and generalized Hurst exponents to derive singularity spectrum. Figure shows our findings related to the generalized Hurst exponents, spectrum and scaling functions.

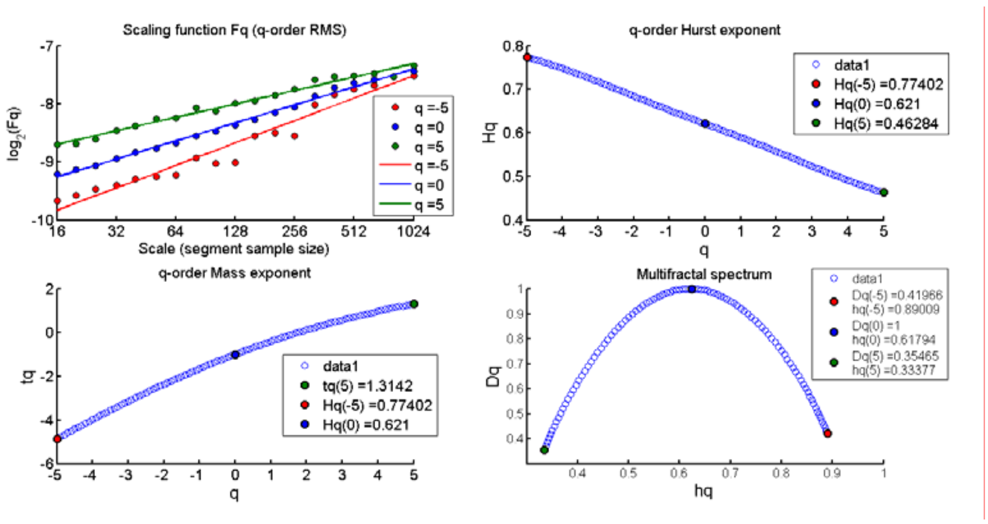

Figure 2: Results of MFDA analysis for USD/TRY

In our results, generalized hurst exponents for time scales are given and when q varies from -5 to 5, h(q) decreases from 0.77402 to 0.46284 and as h(q) is not a constant, we can infer multifractality in our time series data. If H = 0.5, the system

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displays ‘‘Markovian’’ behavior and there is no long-term correlation or memory. If H < 0.5, the system displays fractional Brownian motion and anti-correlation and if H > 0.5, there is a positive long-term correlation or memory exist in the series. Our data exhibits long-term memory. An increase (decrease) is always followed by another increase (decrease). In order to measure strength of multifractality, we can convert q and τ_(q) to α and f (α) where f (α)=aq-τ_(q) andf (α) is the fractal dimension of the time series and f (α) shows the frequency of events with α scaling exponent.The width of the fractal spectrum shows the distinction between the maximum probability and the minimum probability and the larger the value of this difference, the stronger the multifractality. The width of spectrum is 0.6, which indicated not very strong multifractility.

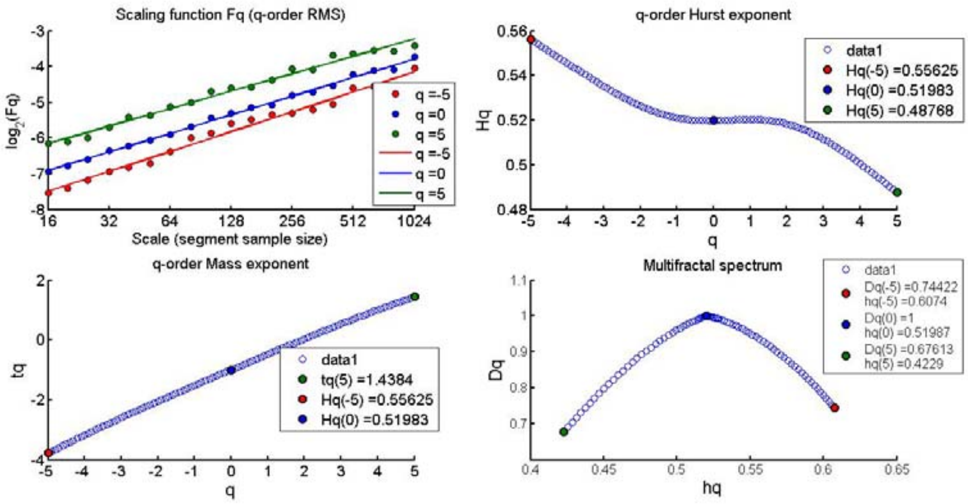

Figure 3: Results of MFDA analysis for USD/HUF

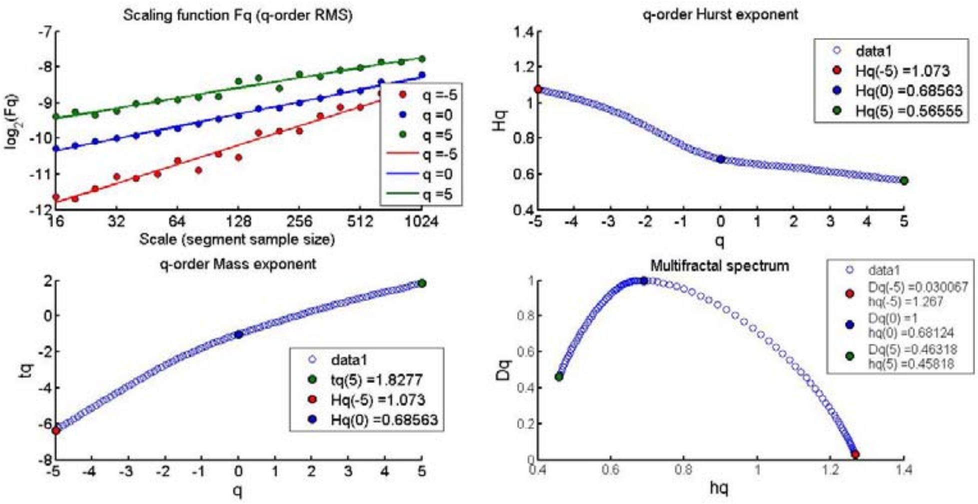

Figure 4: Results of MFDA analysis for USD/RUB

In the figures above we present the generalized exponents and multifractal spectrum of Eastern European country currencies, such as Russian Rubble and Hungarian Forint. While both currencies exhibit multifractal behavior, degree of multifractality is low in Hungrain Forint while opposite is true for Russian Ruble.

In addition to MDFA analysis, we analyzed our USD/TRY data using WTMM method. We drew the Wavelet Skelaton using Mathematica and Last wave programs. Skelaton function is built using all local maxima points and we observed that scalling function is convex, which shows the multifractal behavior of our data.

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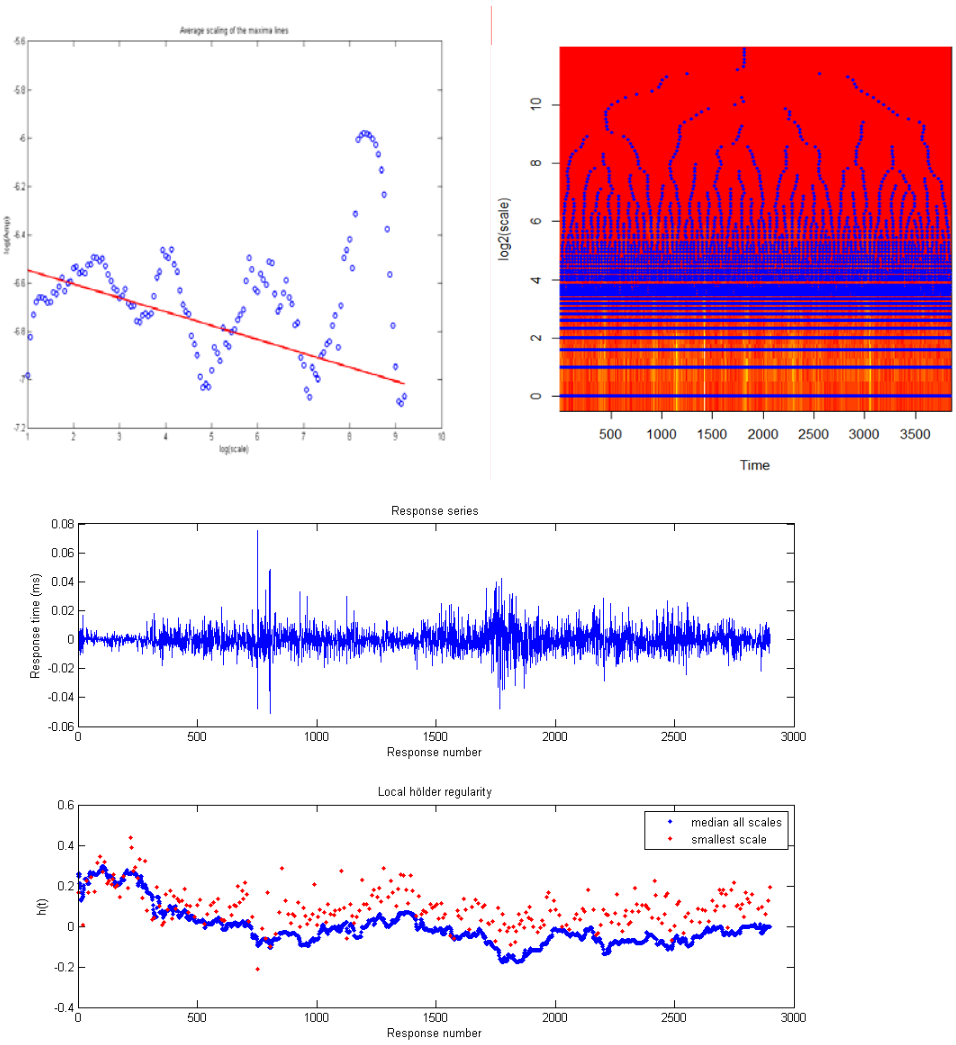

5. CONCLUSION

Using two methods, MF-DFA and WTMM, we demonstrated that USD/TRY exchange rate has fractal properties, and that fractality is pervasive, which is characteristic of foreign exchange markets across a broad range of countries. In addition, our data exhibits long-term memory such at that an increase (decrease) is always followed by another increase (decrease), which shows the persistence of

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shocks to currency. In addition to Turkish Lira, we also showed that Hungarian Forint and Russian Rubble have also multifractal characteristics. After showing the fractal properties of these three different currencies, we can state that forecasters have often encountered in trying to predict these exchange rates with models that do not incorporate any notion of inhomogeneity will have little predictive power.

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