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VARIANTS OF THE CUTTING STOCK PROBLEM AND THE SOLUTION METHODS

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─Abstract ─

Over the years, column generation based algorithms such as branch and price have been the preferred solution techniques for the classical cutting stock problem (CSP). However, most cutting stock problems encountered in the real world are variants of the classical CSP with many more complexities. The exact algorithms have been found wanting for these variant problems such as cutting stock with knives setup considerations, pattern minimization, ordered cutting stock, order spread minimization, minimization of open stacks, CSP with contiguity, CSP with due dates and service level considerations, CSP with multiple objectives and integration of the cutting stock problem with other production processes. This paper studies the cutting stock variants with an emphasis on the optimality of solutions obtained by the approximate methods.

Key Words: Cutting Stock Problems, Computational Techniques, Optimization

JEL Classification: C610, C630

1. INTRODUCTION

The classical cutting stock problem (CSP) with only a trim minimization goal is best solved with the column generation method. The fractional values in the solution can be removed by resorting to rounding heuristics or by the branch and price algorithm. However, most industrial cutting stock problems are extensions of the classical CSP with added constraints and objectives. The effectiveness of an exact solution approach like column generation and 'branch and price' is limited for such complex problems leaving heuristics as the preferred option. The exact or global solution is not pursued because it is perceived to be intractable or too difficult to obtain and instead, good solutions but not necessarily the best ones are deemed to suffice. For example, Haessler (1975) identified knife setup cost as an important factor during the conversion stage of a paper mill and solved it with a sequential heuristic. Chen, Hart & Tham (1996) solved a combined objective function comprising trim loss and knife setup cost with simulated annealing. Pattern minimization problem (PMP) is a related problem where minimization of knife setup cost is carried out as a secondary objective to the trim loss criterion. McDiarmid (1999) classified the pattern minimization problem as 'strongly NP-hard' even if the trim loss solution was trivial implying PMP's enhanced complexity. Forester & Wáscher (2000) used lexicographic search to minimize the number of setups or different patterns. Umetani, Yagiura & Ibaraki (2006) proposed heuristics for similar CSP variants. Golfeto & Neto (2009) used multi-objective evolutionary optimization algorithm to solve the cutting stock problem with knife setup considerations..

The Cutting Stock Problem (CSP) with contiguity, open stacks or order spreads are related computationally challenging cases because of the integration of two NP-hard problems of pattern generation and sequencing (Becceneri, Yanasse, & Soma, 2004). Hinterding & Khan (1995), Ragsdale & Zobel (2004) and Respicio & Captivo (2005) used genetic algorithms whereas Forester & Wascher (1998) used simulated annealing to solve this CSP variant. Belov & Scheithauer (2007) developed a stepwise sequential heuristic for a combined problem of minimizing trim loss, open stacks and setup cost. Araujo, Constantino & Poldi (2011) used a multi-objective evolutionary algorithm for a similar problem.

The assessment of above literature shows that the trim minimization problem has an efficient exact solution approach in column generation or branch and price (B&P) algorithm but the effectiveness of B&P is limited when variants of cutting stock problem are encountered in various industries, leaving heuristics as the preferred option to obtain good solutions. Heuristic methods and meta-heuristics are well known for their ability to solve very difficult or nearly intractable problems but the optimality of resulting solutions is not guaranteed. Since, in most these cases, the global optimal is also unknown, the effectiveness of meta-heuristics in solving difficult problems cannot be quantitatively measured. Most of the computationally challenging cutting stock variants mentioned above are essentially two NP-hard problems solved simultaneously. It can be argued that if a solution method can efficiently solve one of the two NP-hard problems or part of a difficult joint optimization problem, it may be reasonable to assume that solution method will also be effective in obtaining a good solution to the overall problem. It may also help to quantify the robustness of the heuristic and meta-heuristic methods to solve a particular type of optimization problem by comparing the results with exact solutions which are relatively easy to obtain for the classical cutting stock problem. This paper studies the computational complexity of the cutting stock variants and the appropriateness of the applied solution methods with an emphasis on the quality of obtained solutions by genetic algorithms (GA) so as to extrapolate its performance for bigger and more complex CSP variants.

2. META-HEURISTCS AND THE CUTTING STOCK PROBLEM

The one dimensional (1-D) cutting stock problem involves optimal allocation or grouping of a finite set of items into a number of categories subject to constraints and, has been referred to as a ‘grouping optimization problem’. Meta-heuristics such as evolutionary algorithms, simulated annealing and tabu search have been used as solution methods for the classical cutting stock problem and similar ‘grouping optimization problems’ such as the bin packing problems (BPP), timetabling problems, knapsack problems and vehicle loading problems. However, there are divergent views on the effectiveness of a standard genetic algorithm in solving grouping optimization problems. Falkenauer (1996) proposed a new GA mapping scheme called Grouping Genetic Algorithm (GGA) for the BPP wherein each gene hosted a group of candidate solutions. It was deemed that with this arrangement, traditional genetic operators like crossover would be less disruptive. The improved experimental results were also backed by Gen & Cheng (2000) in their analyses of genetic algorithms for bin-packing problems. On the contrary, Reeves (1996) did not approve of grouping genetic algorithm and instead used the standard genetic algorithm hybridized with a local heuristic to reduce the size of the bin packing problem and found it to be effective.

Peng & Chu (2010) utilized a two chromosome genetic representation for the classical cutting stock problem and the CSP with contiguity. The first chromosome consisted of different cutting patterns with each cutting pattern being represented by a single gene while the second chromosome represented the frequency of cutting patterns. Experimental results showed that their mapping scheme showed better or equally good results as those obtained by Liang et al (2002) which used mutation as the only genetic operator for their cutting stock problem. Other application of evolutionary algorithms that utilized mutation as the only genetic operator include Chiong & Beng (2007) for the cutting stock problem and Stawowy (2008) for the bin packing problem. Nevertheless, the utility of both crossover and mutation in obtaining improved solutions has been confirmed by Peng & Chu (2010) when their genetic algorithm performed equally well as or better than Liang's evolutionary algorithm. Therefore, it is reasonable to assume that instead of jettisoning crossover altogether, a better genetic representation coupled with careful parametric settings is more likely to succeed.

3. COMPARISON OF GENETIC AGORTHIMS (GA) AND EXACT SOLUTIONS TO THE CLASSICAL CSP

In the following sub-sections, two separate genetic representations are applied to the classical cutting stock problem to ascertain their usefulness in solving the variants of the cutting stock problem. The rationale behind testing multiple representations for suitability stems from the findings of the previous section that different GA representations have corresponded to different results

3.1 GA Representations for the Classical Cutting Stock Problem

3.1.1 Single Chromosome Representation

The single chromosome representation for five ordered lengths *(m = 5)* is shown in Figure 1-(a) where each gene represents an ordered length. If a cutting pattern is used more than once, it is treated as a new pattern i.e if a pattern is used *'n'* times it appears *'n'* times in the chromosome with the same combination of ordered lengths. Each pattern represents a possible combination of ordered lengths to be cut from the jumbo reel, therefore, the number of patterns in the Figure 1-(a) represent the required number of jumbo reels to meet the customer demand. The figure represents a relatively small problem where seven jumbo reels are sufficient to fulfill the demand. Although easy to model, this representation's major flaw is that the length of the chromosome increases considerably with increasing problem size, thus, accentuating the effects of crossover.

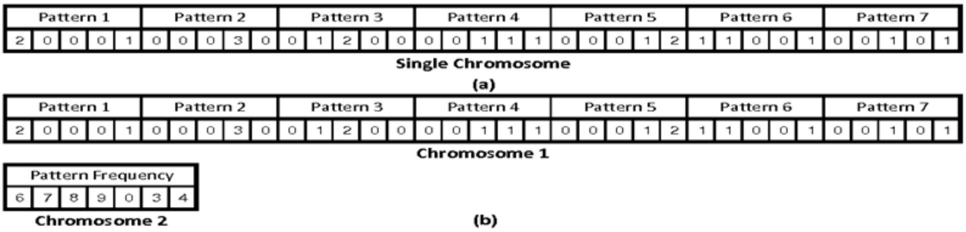

Figure 1: Genetic Representations of 1Dimensional Cutting Stock Problem

3.1.2 Two Chromosome Representation

The two chromosome representation models the usage of each cutting pattern and its combination in two separate chromosomes as shown in Figure 1-(b). Possible combinations of ordered lengths are represented in the chromosome 1 and the chromosome 2 represents the frequencies of patterns which is different from the single chromosome representation where a repeat pattern was treated as a new pattern. Pattern 1 is used six times, pattern 2 seven times, pattern 3 eight times, pattern 4 nine times, pattern 5 not used at all, pattern 6 is used three times and pattern 7 is used four times. The total number of jumbo reels used to meet customer's demand is the sum of all the frequencies and for the example shown in Figure 1-(b), the required number of jumbo reels is forty three. Figure 1 shows that the two chromosome representation can handle much bigger problems with an additional chromosome having only a few genes. The two chromosome representation is similar to Peng & Chu (2010) except that they assigned a group of ordered lengths to a single gene and here, each gene corresponds to a single ordered length.

3.1.3 GA Implementation

Evolver, a GA application in the Palisade Decision Tools, is used to solve the problem. It has the capability to deal with two chromosome representations by treating its chromosome as comprising two distinct parts i.e the Evolver chromosome will have *'n+m'* genes with *'n'* representing the cutting patterns and *'m'* the frequencies. All the genetic operators are applied separately on these two parts. Evolver also comes with different 'solving methods', each of which is a different type of genetic algorithm with customized attributes. The Recipe solving method is a genetic algorithm that treats each decision variable as an ingredient in a recipe, trying to find the best mix by changing each decision variable independently. Grouping solving method is a special type of recipe genetic algorithm with a reduced search space; it involves multiple variables to be grouped together in sets. The Grouping method is applied on the cutting patterns chromosome whereas the Recipe method is suitable for the frequency chromosome. A uniform crossover value of 0.5 is used across all experiments and auto mutation is used (Palisade, 2009). The initial population of 2500 gave good results without significant increase in the computational load and was therefore selected for all the experiments. The global known optimum obtained by an exact solution method was used as the stopping criterion and in case optimal solutions were not obtained by GA, experiments showed that GAs converged before 200 equivalent GA generations or 500,000 iterations; therefore, it was used as the stopping criterion.

3.2 The Exact Solution Approach

The exact solutions were obtained by solving the continuous relaxation of the original integer problem with the column generation method. With Excel built-in functions and a Visual Basic for Application (VBA) program, Solver was automated for repeated exchange of information between the restricted master and dual problem of column generation approach. Once an optimal solution to the linear relaxation was obtained, an appropriate rounding heuristic as proposed by Wascher & Gau (1996) was applied to obtain integer solutions.

3.3 Random Test Instances Generator

Gau & Wascher (1995) introduced a test set generator (CUTGEN) for the cutting stock problem wherein three important input parameters number of cuts or number of ordered lengths *(m)*, demand factor $(\tilde{d})$ and length factor *(b)* are varied one at a time to randomly generate several classes of test instances. Although no genetic algorithm representation has been tested against CUTGEN, it has been used as a test data generator for various other solution approaches to the CSP (Poldi & Arenales, 2009; Umetani et al., 2006; Wascher & Gau, 1996). The number of ordered lengths *(m)* to be cut from the jumbo reel is an important input parameter for the cutting stock problem. Different values of '*m*' used in the comparison are 3, 5, 7, 10 and 15. The second input parameter for the cutting stock problem is the individual demand of ordered lengths which has been treated as a random variable. Average demand per order $\tilde{d}$ is the determinant of the total demand for ordered lengths with two values of $\tilde{d} = (10, 50)$ to differentiate between the low and high demand cutting stock problems. The total demand of all ordered lengths is given by (1).

$$D = m * \tilde{d} \qquad (1)$$

The individual demand (d_i) of each ordered length (l_i) is represented by the following equation:

$$d_i = (D \times R_i)/(R_1 + R_2 + \cdots + R_m) \;\; where\; i = 1, \ldots, m-1 \qquad (2)$$

R_i is a random variable drawn from a uniform distribution $[0,1]$. The demand for mth ordered length is determined by subtracting the individual random demands obtained from the above equation from total demand D.

$$d_m = D - \sum_{i=1}^{m-1} d_i \qquad (3)$$

The ratio of ordered lengths (l_i) to the jumbo length (L) is defined as the length factor (b). The length factor is third important input parameter which may affect the solution quality. Consideration of length factor for checking the effectiveness of different GA representations appears to have been ignored in all previous studies. The jumbo length for all problem instances is fixed at 10,000 length units, while different cases have been distinguished with respect to the order lengths. The order lengths are modelled as uniformly distributed integer random variables which were allowed to vary between one and a certain percentage b of L, i.e. $l_i = [1000, b \times L]$. The values of the length factor b that have been investigated were $b = 0.25, 0.5, 0.75$ and 1.0. It is felt that comparing any solution technique on these 40 test problems will give a good measure of its effectiveness

4. RESULTS

Of the two proposed single item per gene representations, the two chromosome representation performance is far superior and its deviation from the optima is plotted in Figure 2 against variations in the three input parameters. Figure 2 shows that the two chromosomes GA performed very well when the number of ordered lengths was less than 10, irrespective of demand and length factor variations. With $m = 10$, GA was still able to reach the global optimal for five of the eight test instances belonging to the same class but when faced with high demand and high ratio of length factor, the optimal values obtained by GA were two to three units (jumbo reels) away from the exact solution. When the number of ordered lengths increased to 15, GA was still able to match the exact solution approach when the demand and length factor were low but it fell short when the demand and length factor increased.

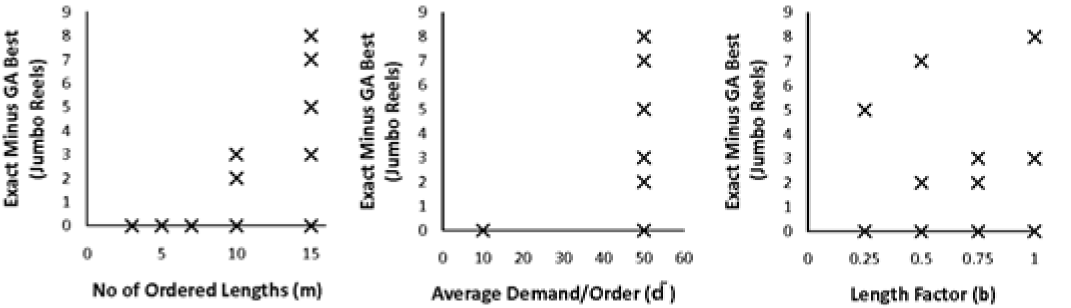

Figure 2: GA Performance against Three Input Variables

5. DISCUSSION

Two different genetic representations were tested against several classes of cutting stock problem and it is noted that because of fewer number of genes, the two chromosome representation performed better than the single chromosome representation. With smaller problems ($m \leq 5$ and $\tilde{d} = 10$), their performance was similar but as the length of chromosome increased, the single chromosome GA started yielding poor results because of the involvement of the genetic operators with a greater number of genes. This suggests that the grouping GA introduced by Hinterding & Khan (1995) and further transformed by Peng & Chu (2010) into a two chromosome representation is likely to perform better because of its much reduced size

The comparison between the exact and approximate solution obtained by genetic algorithms also gives us confidence that GA is a good choice to solve CSP variants against certain classes of input data. For example, in this study, if the number of ordered lengths is equal to or less than ten, the proposed representation is likely to perform as well as the exact solution technique with an ability to tackle the complexities of CSP variants.

6. CONCLUSION

The classical cutting stock problem is best solved with the help of column generation or branch and price algorithms but the real world scenarios often involve non-linearities and added complexities which can only be captured by variants of cutting stock problem. Application of exact solution approaches is limited in such scenarios where approximate methods such as heuristics and meta-heuristics can give good solutions. However, the optimality of solutions is not guaranteed because the global optimum is unknown. For such instances, applying the approximate method to the classical cutting stock problem with a readily available exact solution will help us determine the performance of the approximate method for a bigger and more complex CSP variant. This is particularly valid when genetic algorithm is chosen as the approximate solution technique because it has been shown in this paper that different genetic representations result in dissimilar performances.

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