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EVALUATION OF COMPUTER ALGEBRA SYSTEMS USING FUZZY AHP

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Abstract. *The paper proposes an evaluation model based on fuzzy AHP to help users select CAS that best matches their requirements. The subjectiveness and imprecision of the evaluation process are modeled using linguistic terms. The evaluation criteria framework based on the usability and problem solving capability of CAS is developed. Fuzzy (AHP) is employed to determine the relative importance weights of criteria and the preference order of alternatives. The applicability and effectiveness of the proposed methodology is illustrated.*

Keywords: CAS, fuzzy AHP, usability, problem solving capability, linguistic evaluation.

1 INTRODUCTION

It has been appreciated that the goal of promoting mathematical exploration (through symbolic, numerical and graphical experimentation) is well served by computer algebra system (CAS) [1]. CAS(s) are computer based software packages for performing mathematical symbolic computations [2].

However, there are dozens of CAS available for users: Derive, Maple, Mathematica, Maxima, and etc. Hence, users are faced with the challenge to select the most appropriate CAS that meets s/he requirements. From the human computer interaction perspective the usability dimension and from the functional perspective the problem solving capability dimension are the most wanted requirements for a software package [3,4]. Thus, evaluation (or selection) of CAS can be viewed as a complex multicriteria decision making (MCDM) problem [5]. Recent research studies have demonstrated the applicability and flexibility of MCDM approach to evaluation of educational software [6,7]. They employed the analytical hierarchy process (AHP) method of MCDM, ich was developed by T.L.Saaty [8]. However, HP does not give reliable results under fuzzy environment.

This paper proposes an evaluation model based on fuzzy AHP to help users select CAS that best matches their requirements. The subjectiveness and imprecision of the evaluation process are modeled using linguistic terms. The evaluation criteria framework based on the usability and problem solving capability of CAS is developed. Fuzzy (AHP) is employed to determine the relative importance weights of criteria and the preference order of alternatives. The applicability and effectiveness of the proposed methodology is illustrated.

The paper is organized as follows. The description of fuzzy AHP and evaluation model is given in Section 2. A case study of evaluation of CAS is presented in Section 3. Finally, in Section 4 we present results and conclusion.

DESCRIPTION OF FUZZY AHP AND EVALUATION ALGORITHM

In this section, we present an overview of the literature on FAHP and describe the proposed evaluation algorithm. AHP is a powerful decision making tool of multi-criteria decision making methods. Its aim is to select the best alternative among different criteria. The main idea of AHP is to decompose a complex problem into several small problems by means of a systematic hierarchy structure [9]. A decision maker makes a reciprocal comparison for each element and layer of the structure using ratio scales. A reciprocal matrix is constructed. Then, using matrix algebra, the relevance weights of elements are calculated. However, AHP is not able to make decision under the environment of uncertain, vague, incomplete, fuzzy information. Hence, there is a need to modify AHP for fuzzy environment. It is presented in [10, 11], where fuzzy comparison ratios were introduced. The work [12] proposed an extent analysis method to handle fuzzy reciprocal matrix. Using this method we propose the following evaluation algorithm.

Step1. Identify the goal.

Step 2.Identify a set of alternatives: $A_j, (j = 1,2,..., n)$

Step 3.Identify a set of evaluation criteria (or sub criteria): $C_i, (i = 1,2,..., m)$ and construct a tree type hierarchy structure of criteria and sub-criteria.

Step 4.Get decision makers' evaluation judgments in the form of comparison scores a_{ij} in pairs of criteria $C_{ij}, (i, j = 1,2,..., m)$. Each comparison score should show how much important one criterion is than the other. The comparison scores form the matrix of pairwise comparisons $A = \lfloor a_{ij} \rfloor$ that should satisfy the conditions: $a_{ij} = 1/ a_{ji}$ and $a_{ii} = (1,1,1)$ for $i = 1,2,\ldots,m.$ The comparison scores a_{ij} represent linguistic terms [13] expressed by triangular fuzzy numbers $k = (k_1, k_2, k_3)$, where $-\infty < k_1 \le k_2 \le k_3 < \infty$, and described in Table 1.

Table 1. Linguistic scores for comparison and ratings

Linguistic scores	Fuzzy number
Just equal	(1,1,1)
Equally important (EqI)	(1,1,3)
Moderate important (MI)	(1,3,5)
Strong important (SI)	(3,5,7)
Very strong important (VSI)	(5,7,9)
Extremely important (ExI)	(7,9,9)

Step 5.Calculate the relative importance weight w_i for each criterion $C_i, (i = 1,2,..., m)$ using the equation:

$$w_i = \sum_{j=1}^{m} a_{ij} \otimes \left[\sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij} \right]^{-1} , \tag{1}$$

where addition, multiplication and division operations for two fuzzy triangular numbers $a = (a_1, a_2, a_3)$, and $b = (b_1, b_2, b_3)$ are defined as [14]:

$$a \oplus b = (a_1 + b_1, a_2 + b_2, a_3 + b_3),$$

$a \otimes b = (a_1 b_1, a_2 b_2, a_3 b_3)$, and

$a / b \cong (a_1 / b_3, a_2 / b_2, a_3 / b_1)$, respectively [13].

Step 6.Similar to Step4 we obtain decision maker'spreferences, $d_{ij}, (i = 1,2,..., m, j = 1,2,..., n)$, about the performance of each alternative A_j within each criterion C_i using Table 1. These values form the decision matrix $D = \lfloor d_{ij} \rfloor$. Then, it is normalized as follows:

$$\left[\hat{d}_{ij}\right] = \frac{d_{ij}}{\sum_{j=1}^{n} d_{ij}} , \quad i = 1,2,..., m, j = 1,2,..., n . \tag{2}$$

Step 7.Calculate the fuzzy score of each alternative:

$$X_i = \hat{d}_{ij} \otimes w_j \tag{3}$$

Step 8.Calculate the ranking score of each alternative using the graded mean integration representation of the fuzzy number a [15]

$$R(a) = \frac{a_1 + 4a_2 + a_3}{6} , \tag{4}$$

where $a > (=,<) b \Leftrightarrow R(a) > (=,<) R(b)$.

Step 9.Choose the alternative whose ranking score is maximum as the best alternative.

2 EVALUATION OF CAS.

In this section, we present an empirical study concerning the application of the proposed algorithm. It is carried out through a survey among students. Three alternatives of CAS are identified: A_1- Maple, A_2- Mathematica, A_3- Maxima. They are open source and easily available. Students are given a questionnaire about the user interface and problem solving capability of CAS. The user interface is closely related to the concept of usability that is central dimension in human computer interaction [3]. The usability is considered to be inherent in human computer interface, because it implies the interaction of users with the software product [16,17]. The guidelines for the mathematical problem solving software design proposed in [4] are adopted in our case study. The results of the survey, is analyzed and the following criteria set hierarchy is derived.

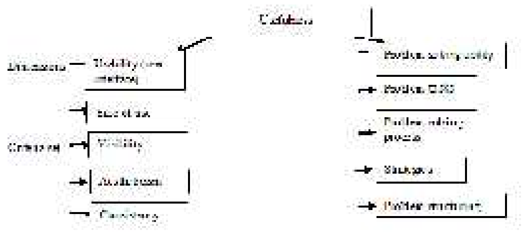

Fig. 1. Hierarchical structure of criteria set.

We give a short description of each criterion [3,4]. Ease of use - C_{11} - use of an interface with a minimum effort; Visibility- C_{12} - how interface looks indicates how it can be used; Aesthetics - C_{13} - the 'look and feel' of the user interface intended to make the interface attractive and appealing; Consistency - C_{14} - makes the interface familiar and predictable by providing a sense of stability; Problem tasks – C_{21} - refers to a situation in which a person wants something and does not know immediately what sorts of action he/she can perform to get it; Problem solving process – C_{22} – means understanding , planning, solving, reviewing the problem and the solution; Strategies – C_{23} - refers to the ways to proceed that are planned and carried out; Problem structuring – C_{24} - enables students to recognize problems by their structure rather than their contextual setting. After obtaining criteria structure, we follow the algorithm described in section two.

Step 4.Matrices of pairwise comparisons obtained from the survey analysis are shown in Tables 2 and 3.

Table 2. The pairwise comparison matrix of the dimensions

Dimensions	C_1	C_2
C_1	(1,1,1)	(1,3,5)
C_2	1/(1,3,5)	(1,1,1)

Table 3. The pairwise comparison matrix of the usability criterions

	C_{11}	C_{12}	C_{13}	C_{14}
C_{11}	(1,1,1)	(1,3,5)	(1,3,5)	1/(1,3,5)
C_{12}	1/(1,3,5)	(1,1,1)	(1,1,3)	(1,1,3)
C_{13}	1/(1,3,5)	1/(1,1,3)	(1,1,1)	(1,1,3)
C_{14}	(1,3,5)	1/(1,1,3)	1/(1,1,3)	(1,1,1)

Step 5. Based on Eq. (1) the relative importance weights of dimensions and criteria are computed and presented in Table 4.

Table 4. Priority weights of dimensions and criteria in the AHP decision tree

Criteria	Weight between dimensions	Weight within the criteria(s)	Weight among the sub-criteria
C_1	(0.25,0.76,1.86)		
C_{11}		(0.11,0.42,1.12)	(0.027,0.32,2.1)
C_{12}		(0.11,0.19,0.74)	(0.027,0.14,1.37)
C_{13}		(0.082,0.19,0.56)	(0.02,0.144,1.41)
C_{14}		(0.061,0.19,0.37)	(0.015,0.14,0.69)
C_2	(0.15,0.25,0.62)		
C_{21}		(0.13,0.35,1.04)	(0.02,0.086,0.64)
C_{22}		(0.11,0.19,0.7)	(0.016,0.047,0.43)
C_{23}		(0.077,0.23,0.52)	(0.01,0.57,0.32)
C_{24}		(0.07,0.23,0.35)	(0.01,0.57,028)

Step 6. Based on Eq. (2) the normalized decision matrix is computed and given in

Table 5. Normalized decision matrix

	A1	A2	A3
C_{11}	(0.27,0.33,0.43)	(0.21,0.37,0.71)	(0.15,0.37,0.43)
C_{12}	(0.24,0.57,1.023)	(0.08,0.18,0.27)	(0.16,0.25,0.8)
C_{13}	(0.14,0.43,1)	(0.12,0.37,1)	(0.12,0.2,0.71)
C_{14}	(0.27,0.33,0.43)	(0.21,0.37,0.71)	(0.15,0.37,0.43)
C_{21}	(0.19,0.33,1.14)	(0.15,0.33,0.71)	(0.11,0.33,0.43)
C_{22}	(0.25,0.23,0.56)	(0.18,0.48,1.1)	(0.1,0.3,0.34)
C_{23}	(0.06,0.11,0.3)	(0.25,0.6,1.3)	(0.12,0.38,0.12)
C_{24}	(0.19,0.33,1.14)	(0.15,0.33,0.71)	(0.11,0.33,0.43)

Step7. The scores and ranking of alternatives are computed using Eq. (3) and Eq. (4) and presented in Table 6.

Table 6. Scores and ranks of alternatives

	Scores X_j	graded mean integration R_j	Rank

A1	(0.028,0.59, 5.4)	
1.3		
2		
A2	(0.021,0.83, 5.3)	1.44
1		
A3	(0.017,0.77, 3.25)	1
3		

The chart representation of Table 6 is shown in Fig. 2.

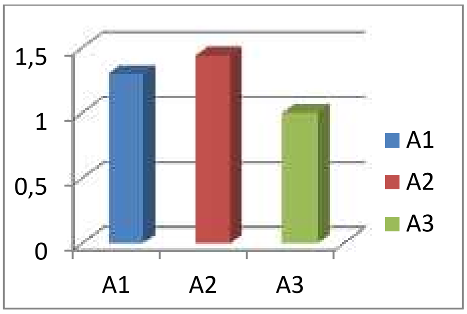

Fig. 2. Ranking of alternatives

4 RESULTS AND CONCLUSIONS

This study proposed a fuzzy AHP framework based on the combination of the extent analysis and the graded mean integration methods to effectively solve the problem of evaluation of CAS under fuzzy environment. Vague, incomplete, fuzzy preferences of decision makers are represented in linguistic terms by triangular fuzzy numbers. This enabled to have a more accurate, reliable and convincing evaluation process. As a result of evaluation of CAS alternatives, Mathematica is ranked first, Maple is ranked second and Maxima is ranked third. Of course, any evaluation process is context dependent. The proposed framework can be used for evaluation of any kind software product. The future work will be devoted to the research on integration of other MCDM into the proposed decision making methodology.

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