

# *Performance Analysis of Energy Based Spectrum Sensing over Nakagami-m Fading Channels with Noise uncertainty*

*Amir ESLAMI<sup>1</sup>*

*Saeid KARAMZADEH<sup>1</sup>*

## **Abstract**

Spectrum is getting rare day after day and in order to mitigate the scarcity of spectrum, cognitive radio (CR) is being introduced which the licensed spectrum is being sensed by a device that is allowed to use in the case of spectrum under-utilization. The device which is sensing the licensed spectrum is called secondary user for that spectrum. In the case of vacancy detection, secondary user could use that spectrum till the licensed spectrum user which is called primary user begin to use its allocated spectrum again. The key factor in CR systems is spectrum sensing. Secondary user should reliably ensure that the spectrum band is not being used by primary user and there should be no interference with it. One of the key factors in spectrum sensing is simplicity of the algorithm to be used in detection devices. Because of low computational complexity, energy detection-based (ED) spectrum sensing is very popular. In this paper, the performance of ED spectrum sensing method and parameters effecting its performance over Nakagami-m fading channels is studied and analyzed with and without noise uncertainty.

**Keywords:** *cognitive radio, spectrum sensing, energy detection based spectrum sensing, nakagami-m fading, noise uncertainty*

## **INTRODUCTION**

In recent years, by the development of new wireless communication services, applications and need for higher speeds in communication systems, demand for spectrum resources has get more obvious than before. In each country, spectrum is allocated by governmental or non-governmental centers. Fixed spectrum allocation has forced wireless service providers to use the spectrum allocated to them only even if it needs more bandwidth whether there is some vacant spectrum at that moment in other spectrum bands which are called white spaces. Federal Communications Commission (FCC) has reported the temporal variations in spectrum utilization is range between 15% and 85% [2]. In order to mitigate the scarcity of

spectrum, cognitive radio (CR) is being introduced which the licensed spectrum is being sensed by a device that is allowed to use in the case of spectrum under-utilization which is called secondary user. In the case of vacancy detection, Secondary user could use that spectrum till the licensed spectrum user which is called primary user begin to use its allocated spectrum again. One of the most important factors of CR systems is detection. Secondary user should ensure that the spectrum band is vacant and there is no interference with the licensed user. So, reliability in spectrum sensing method is very important to prevent such interferences. There are many sensing algorithms such as ED [1], [8] and [9], Double

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<sup>1</sup>Electric and Electronics Engineering Department, Istanbul Aydin University, [eslamiam@itu.edu.tr](mailto:eslamiam@itu.edu.tr); [karamzadeh@itu.edu.tr](mailto:karamzadeh@itu.edu.tr)

threshold energy detection (DTED) [7] and [11], Matched filtering [3], Wavelet Based detection [10], cyclostationary detection [4] and covariance based spectrum sensing methods [5-6]. Each of these sensing methods have different advantages, disadvantages and different parameters to detect the presence of signal. For example, cyclostationary spectrum sensing method needs to know the cyclic frequency of the primary user and matched filter requires to know the wave forms of the primary user. Among these sensing methods covariance based spectrum sensing is considered as a blind spectrum sensing method. In covariance based spectrum sensing, for determining the threshold of the method no information is needed about the environment and the signal sent by the primary user. Energy to minimum eigenvalue (EME) and maximum to minimum eigenvalue (MME) sensing methods are two most known sensing methods in covariance based spectrum sensing methods [5]. ED is considered as a semi-blind spectrum sensing method as well. The reason is, ED needs to know the noise variance in order to calculate the threshold of the method which is considered as a disadvantage for it. Any error in determining the noise variance effects the performance of this method that is studied in this paper. The biggest advantage of ED method is its simplicity among all other sensing methods which makes it unique in the sensing methods to be used in real life.

The rest of paper is organized as follows. In section II, energy detection spectrum sensing method is described. In section III, simulation

results and analysis is provided, followed by concluding remarks in section IV.

## ENERGY DETECTION SPECTRUM SENSING METHOD

Spectrum detection in cognitive radio systems has two hypotheses. Hypothesis  $H_1$  is when the primary user is using the spectrum and uses its spectrum allocated and hypothesis  $H_0$  is when the primary user is absent and not using the spectrum allocated. Both hypothesis can be shown as follow:

$$y(t) = \begin{cases} \eta(t) & : H_0 \\ hx(t) + \eta(t) & : H_1 \end{cases}$$

where  $x(t)$  is the signal transmitted by the primary user,  $h$  is the gain of the channel signal is passing through,  $y(t)$  is the signal received by the cognitive radio detector and  $\eta(t)$  is additive white Gaussian noise (AWGN) that is assumed to be a stationary process satisfying variance of  $\sigma_\eta^2$ ,  $E(\eta(t))=0$  and  $E(\eta(t)\eta(t+\tau))=0$  for any  $\tau \neq 0$ . The term  $h$  is the gain of the channel that effects the sent signal by the primary user is mostly modeled as Nakahami-m fading channels in cell sized environments. Nakagami-m fading channel can mathematically modeled as below?

$$P_p = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega^m} \exp(-\frac{mx^2}{\Omega}) , m \geq \frac{1}{2}, x \geq 0$$

$$\Gamma(m) = \int_0^\infty y^{m-1} e^{-y} dy , m > 0$$

$\Gamma(\cdot)$  is a gamma function where  $\Gamma(1) = 1$ . The most known Rayleigh and Gaussian channels can be shown by Nakagami-m distribution also. If we consider  $m = 1$ , Nakagami fading

channel becomes a Rayleigh channel and if  $m$  goes to infinity, Nakagami fading distribution becomes a Gaussian channel one. An analog energy detector is based on the normalized energy of the received signal by the receiver or detector. The normalized received signal energy is as follow:

$$e(t) = \frac{1}{N} \sum_{n=0}^{N-1} |y(n)|^2$$

Number of collected samples by detector is considered to be equal to  $N$ . Samples can be treated as a random process as the received signals are unknown. the sample transmitted signals follows an independent and identically distributed (i.i.d) random processes with zero mean and variance of  $\sigma_s^2$  [1], [8]. So that the received signal SNR in a channel with gain of  $h$  can be shown as  $\alpha = \frac{|h|^2 \sigma_s^2}{\sigma_\eta^2}$ . In the case that collected signals are large enough, using CLT, under hypothesis  $H_0$ , the probability density function (PDF) of  $e(t)$  becomes a normal distribution with mean =  $N\sigma_\eta^2$  and variance =  $N\sigma_\eta^4$ . The PDF of  $e(t)$ , under hypothesis  $H_1$ , it is a normal distribution with mean =  $N(1+\alpha)\sigma_\eta^2$  and variance =  $(1+2\alpha)N\sigma_\eta^4$ . Considering the distributions above, the probability of false alarm ( $P_{fa}$ ) and probability of detection ( $P_d$ ) can be shown as [1], [7]:

$$P_{fa} = \text{prob}(e(N_s) > \lambda | H_0) = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} = Q\left(\frac{\lambda - \sigma_\eta^2}{\sqrt{2\sigma_\eta^4/N}}\right)$$

$$P_d = \text{prob}(e(N_s) > \lambda | H_1) = Q_u(\sqrt{2\alpha}, \sqrt{\lambda}) = Q\left(\frac{\lambda - (|h|^2 \sigma_s^2 + \sigma_\eta^2)}{\sqrt{2(|h|^2 \sigma_s^2 + \sigma_\eta^2)/N}}\right)$$

In the IEEE802.22,  $P_{fa}$  is equal to 0.1 as minimum but generally for any  $P_{fa}$  we can calculate threshold based on  $P_{fa}$  as follow:

$$\lambda_{fa} = \sigma_\eta^2 \left(1 + \frac{\sqrt{2}Q^{-1}(p_{fa})}{\sqrt{N}}\right)$$

In the case of hypothesis  $H_1$ , we can calculate the threshold based on  $P_d$  for any signal to noise ratio ( $\alpha$ ) as follow:

$$\lambda_d = \sigma_\eta^2 (1+\alpha) \left(1 + \frac{\sqrt{2}Q^{-1}(p_d)}{\sqrt{N}}\right)$$

In ED based spectrum sensing method the threshold calculated based on  $P_{fa}$  is compared with the received signal to detect if the primary user is using the spectrum allocated or not. If the energy is bigger than the found threshold, the detector concludes the presence of the signal and absence in other case. The algorithm can be shown as follow:

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Algorithm 1

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Input :  $\lambda, \sigma_\eta$

Output :  $R_i$

- 1: **for** each sensing period **do**
  - 2:  $e(t) \leftarrow$  Energy of the  $N$  samples
  - 3: **if**  $e(t) > \lambda$  **then**
  - 4:  $R_i \leftarrow H_1$
  - 5: **else**
  - 6:  $R_i \leftarrow H_0$
  - 7: **return**  $R_i$
  - 8: **end for**
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ED sensing method is a semi blind spectrum sensing method. It is called semi blind as for measuring the threshold, instead of the  $P_{fa}$ , it needs the variance of noise also. Measuring the exact variance of noise is not possible mainly and there could be some error in the calculation. Assume that  $\zeta$  dB is the error accrued in noise estimation.  $\theta = 10^{\zeta/10}$  is the power of the error so that  $P_{fa}$  and  $P_d$  can be calculated as:

$$P_{fa} = \text{prob}(T(N_s) > \lambda | H_0) = \Gamma(u, \frac{\theta\lambda}{2}) / \Gamma(u)$$

$$P_d = \text{prob}(T(N_s) > \lambda | H_1) = Q_u(\sqrt{2\alpha}, \theta\sqrt{\lambda})$$

**SIMULATION RESULTS AND ANALYSIS**

In this section, a Monte-Carlo simulation model is developed in MATLAB software with QPSK modulated random primary signals and i.i.d noise samples with Gaussian distribution are used. It is assumed that the channel is stable and doesn't change during the period of sampling. To calculate the sensing threshold, only the noise variance and  $P_{fa}$  is needed for ED spectrum sensing algorithm. The probability of false alarm is  $P_{fa} \leq 0.1$  and probability of detection is  $P_d > 0.9$  as required by IEEE 802.22 standard.  $P_{fa}$  is chosen as 0.1 in the simulations and results are averaged over  $10^4$  tests. The SNR is chosen between -15 and +5 dBs and noise variances are 1 and 2 dBs. Figure 1 shows the performance of ED sensing in different channels. As mentioned earlier as the  $m$  goes to infinity the channels distribution function gets nearer to Gaussian channel and if the  $m=1$ , the Nakagami- $m$  channels gets the same distribution function as Rayleigh fading

channel. The Gaussian channel has the best performance and Rayleigh fading has the worst performance among Nakagami- $m$  fading channels. In Figure 2 shows the performance in Gaussian channel with noise uncertainty of 0, 0.5, 1, 1.5 and 2dBs. The ED sensing algorithm has the best performance among other sensing methods but its performance is very dependent to noise uncertainty. As the uncertainty of noise gets higher, the performance of the ED sensing algorithm gets worth as it effects the threshold. Figure 3 shows the performance of ED sensing is shown in Rayleigh channel with noise uncertainty of 0, 0.5, 1, 1.5 and 2dBs. In figure 4 The performance of ED is compared with DTED sensing method with RAC = 0.5, MME and EME. DTED has a better performance in high SNRs but its performance highly decreases in lower SNRs. MME and EME are not dependent to noise variance but as can be seen have worth performance compared to ED with exact calculation of noise variance.

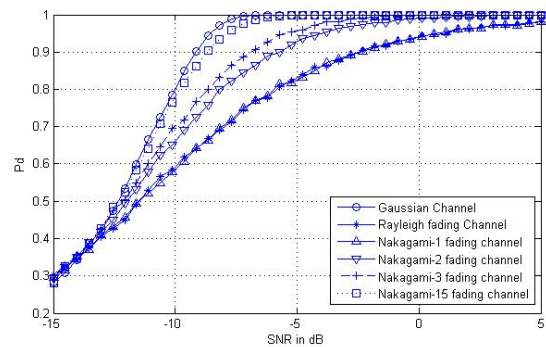


Figure 1.ED sensing SNR performance of QPSK in Gaussian, Rayleigh and Nakagami- $m$  fading channels with  $m=1,2$  and 15

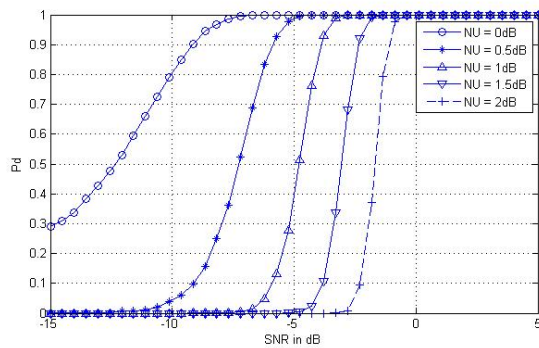


Figure 2. ED sensing method performance with 0, 0.5, 1, 1.5 and 2dB noise uncertainty (NU) of QPSK in Gaussian channel

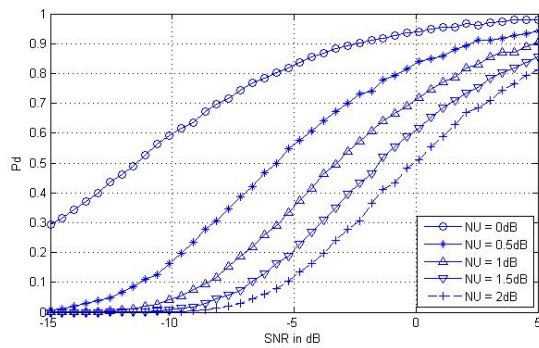


Figure 3. ED sensing method performance with 0, 0.5, 1, 1.5 and 2dB noise uncertainty (NU) of QPSK in Rayleigh channel

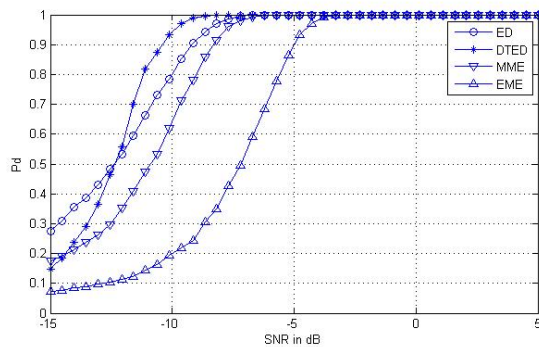


Figure 4. ED, DTED, MME and EME sensing method performance comparison of QPSK modulation in Gaussian channel

## CONCLUSION

Energy detection sensing method has the best performance in different Nakagami-m channels among semi blind and blind spectrum sensing methods. Double threshold energy detection has better performance compared to ED but in low SNRs, because of loss of information, its performance is highly lower than the ED sensing method. MME and EME are the most popular sensing methods in covariance based spectrum sensing method. These are blind spectrum sensing methods that the threshold needs only the value defined for  $P_{fa}$ . The only drawback of the ED is its threshold dependence to noise variance. In the case of noise uncertainty, as shown in the simulation results and simulation section, its performance decrease in a way that it becomes non reliable. So in a case that ED sensing method is planned to use in an environment, noise measurement error in that environment should be considered in the evaluation of the performance of this method. The research findings help to understand the parameters should be considered in using an energy detection based spectrum sensing method in an environment which effects the its performance.

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