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Testing and Tuning of the Sport Archery Bow and Arrow System

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Abstract

Background. Testing and tuning of the archery bow and arrow system is an important component for successful shooting. The aim of the research is to develop an analytical method of the virtual testing and tuning of the archery bow and arrow system and optimizing of the height of the plunger and arrow rest. *Materials and Methods. Modern recurve bows in the frames of International Archery Federation standards are studied.* Modeling is used to derive a method of preparation of the bow for sport archery competition. A model of the archery bow as a kinematical chain with solid members united with rotated kinematical pairs is investigated. *Results.* The bow and arrow system was studied in the braced and drawn situations, and methods of virtual testing and optimizing of the height of the plunger and arrow rest above the hand is developed. The bow and arrow with a zero angle of attack can not be symmetrical in the main plane because an arrow and a hand that holds a bow could not situated together at the same place. *Conclusion.* The results of modeling are presented in a simple form (as tables and figures) which are suitable for coachers and shooters who are unready to use the mathematical methods.

Keywords: shooting, archer, recurve bow, modeling.

Introduction

The archery shooting is ruled by World Archery Federation documents and standards (World Archery rulebook, 2017). Accuracy of the archery shots is depended upon quality of the bow and arrows. A tuning of the archery bow and arrow system is an important element for successful shooting. There are two adjustments to be made in tuning of any bow. The first is done in the vertical plane and almost always involves only adjustment of the nock point. The second adjustment is done in the horizontal plane and involves an adjustment of the pressure point, the amount of centre shot. Two empirical types of testing are used in the archery practice: the bare shaft tests developed by Max Hamilton and Steve Ellison test (Ellison, 2018). These tests aim the tuning of the bow and arrow system in the vertical plane.A basic tuning test is a simply shooting an arrow horizontally at a target from a distance of about 3–5 m. This quick test gives a fair 'coarse alignment' check. An archer observes whether the tail of the arrow 'lays over' to left or right, or is high or low. This test is more sensitive using a bare shaft instead of a fletched shaft. The nock point adjustment test is affected by tillering; tiller adjustment may be out if the nocking point is hard or impossible to adjust.It is best to check by shooting several arrows, into different places on the target. If the test looks right, the test should be repeated at one or two other distances: 3 and 6 m. If these are right too, things are about right. If not, adjust until the shafts are straight at several distances, or a different test should be used (Ellison, 2016). A walk-back test gives a useful combined test of both center shot and button tension. The test is done in calm conditions, and while an archer are shooting normally. In particular, an archer should be warmed up and has shot enough practice arrows to be close to the typical competition shooting. The 'paper plate' test is suggested by Tim Roberts for compound tiller tuning. However, identical principles apply for any other fine tuning adjustment, so the method could be used for a variety of adjustments. The paper tear test gives a good indication of both vertical and lateral adjustment. It relies on a simple indication of early arrow flight. It uses fletched shafts, but needs other equipment to hold paper in front of a target (Arrow Tuning and Maintenance Guide, 2019).So, the tests which are used to tune the bow arrow system entail considerable effort and much time. They are based on laborious procedures through a lengthy and complicated trials and error phase. Furthermore, these tests allow only the variation of one parameter: the nock height of the arrow. Another variable which is believed to be important is not taken into consideration in the frames of the methods. Among others, it is the initial setup angles of the limbs relatively to the riser. Therefore, the **aim** of the research was to develop analytical methods of the virtual testing and tuning of the archery bow and arrow system and optimizing of the height of the plunger and arrow rest.

Bow and arrow modeling

The first results dealing with the static strains and stresses in a drawn bow have been published by Hickman (1937). His articles have shown the effect of the shape and form of bending of the bow upon these strains and stresses. Some interesting and valuable information was obtained from that work which has materially changed the design of modern bows. He designed Lagrange function of one degree of freedom and corresponding equation of motion. An arrow was modeled as a particle placed at the axis of symmetry of the system. A third part of the string mass was added to the mass of the arrow and the rest – to the limbs.

A successful attempt to creation of the mathematical model of the archery bow and arrow system was done by Klopsteg (1943). He developed a mechanical engineering model of the long bow as a plane symmetrical kinematical chain of solid rods united with rotated kinematical pairs and un-stretched string. Two straight rigid beams hinged with a handle were introduced instead of the real flexible limbs. A stiffness of the bended limbs was modeled with a spiral spring mounted into the rotation kinematic pairs. Approximately linear correlation between the bow force and the drawn distance was derived as a solution of the static problem and as a result of the experimental data.

Ballistics of the modern-working recurve bow and arrow was studied by Schuster (1969). A model of the working recurve bow amenable to analytic solution was developed. The equations of motion of the bow and arrow system were obtained and numerically integrated by computer. It was shown that subject to the approximations, the system is 100% efficient. Variations of the transfer of energy between bow and arrow were discussed in terms of bow parameters and the effects on the archer. A discussion of first-order exterior ballistics and arrow penetration indicated the equipment characteristics most desirable for both the target archer and hunter, subject to the archer's own capabilities.

Marlow (1981) presented analysis of a bow with an elastic string. It is found that arrow exit then takes place when the string and bow limbs still have substantial kinetic energy, and therefore this energy is unavailable for kinetic energy of the arrow. Moreover, the potential energy remaining in the string and bow limb system can also reduce the amount of energy available for the arrow. For the Hickman model of a long bow used in this study, the elastic string prediction of efficiency is 78%, whereas the inelastic prediction is 92%. The analysis utilizes a Lagrangian distributed mass formulation to develop the governing equations of motion and to generate an equivalent point mass model. Estimates of the effect of air resistance were made and found to be less than 2% of the total system energy. The vibratory dynamics of the string and bow limbs subsequent to arrow exit was analyzed.

A wide and accurate experimental and theoretical research in archery was done by Pekalski (1990). The aim of his study was to introduce certain methods and research techniques and to present the results of experiments on parameters of archery equipment to optimize the interaction of the archer – bow – arrow system's elements. The mathematical modeling and computer simulation were used to describe the arrow's movement for various initial conditions and various parameters of the equipment, based on which a nomogram was constructed of the optimum arrow parameters for bows of various draw forces. The device for the mechanical loosing of arrows from a bow was used to study the influence of selected parameters of the archer – bow – arrow system on the accuracy of shots. The film analysis was used to verify the mathematical and mechanical models constructed.

The design parameters associated with the developed model are charted accurately. These and other important problems were studied by Kooi (1991). Bows used in the past and nowadays on shooting meetings such as the Olympic Games are compared. It turns out that the application of better materials which can store more deformation energy per unit of mass and that this material is used to a larger extent, contribute most to the improvement of the bow. The parameters which fix the mechanical performance of the bow appear to be less important as is often claimed.

The arrow needed to get round the bow while being accelerated; this phenomenon is called the Archer's Paradox. In the forties it was observed experimentally with high-speed cameras that the arrow vibrates in a horizontal plane perpendicular to the vertical median plane of the bow. These movements are started and controlled by the movements of the two points of contact with the bow. The middle of the string is in contact with the rear end of the arrow and the grip where the arrow slides along the bow. The latter contact imposes a moving-boundary condition. The numerically obtained results are satisfactorily in agreement with experimental data. The model can be used to estimate the drawing force of ancient bows of which only the contemporary arrows are available and also for the design of new archery equipment (Kooi and Sparenberg, 1997). Vibration processes in the compound and open kinematical chain with an external link, as a model of an archery bow and arrow system, were evaluated. A mechanical and mathematical model of bend oscillations of the system during accelerate motion of the external link was proposed. Correlation between longitudinal acceleration and natural frequencies was obtained. There are recommendations regarding determination of virtual forms to study arrow vibrations and buckling. The models and methods have been adapted for realization into the engineering method using well-known mathematical software packages (Zanevskyy, 2009).

Theoretical and experimental results of research on the problem of archer, bow and arrow behavior in the vertical plane are presented. The aim of the research was to develop a method of computer simulation of static and dynamic interactions between the archer, bow and arrow system in order to provide archery with practical recommendations. A model of an archer's body was presented as a mechanical system composed of a few solid bodies, which are connected to each other and to the ground with viscoelastic elements. Mechanical and mathematical model of bow and arrow geometry in vertical plane in braced and drawn situations was investigated. An asymmetrical scheme, rigid beams, concentrated elastic elements and elastic string are the main features of the model. Numerical results of computer simulation of archer, bow and arrow interactions were presented in graphical form, which makes the methods easy to use by sportsmen and coaches (Zanevskyy, 2006a).

Competition results in sport archery depend to a great extent upon the optimal combination of bow-arrow-archer system parameters. A significant part of bow tuning is vertical adjustment, the aim of which is to give an arrow zero angle of attack. It is conducted in a long and complicated manner and error correction takes a lot of time and effort. The goal of the research was to create an analytical method to determine an optimal combination of bow parameters, which ensures zero angle of attack for an arrow launched from a string. Mechanical and mathematical models of bow and arrow geometry in the vertical plane in braced and drawn situations were investigated. An asymmetrical scheme, rigid beams, concentrated elastic elements and elastic string were the main features of the model. Numerical results of a computer simulation are presented in tabular and graphical form, which makes it easy for sportsmen and coaches to use (Zanevskyy, 2006b).

Park (2009) studied a compound archery bow dynamic model, suggesting modifications to improve accuracy. This paper provides a model for the nock point locus in the vertical plane. While examples are provided for several configurations of compound bow, it is generally applicable to longbows and recurve bows as well. It was noted that asymmetric degrees of freedom in the cam configuration of a compound bow are required if the nocking-point locus is to be both straight and perpendicular to the rest position of the string, and that this cannot be achieved for some compound-bow configurations or for a longbow or recurve bow unless the arrow pass is in the geometric center of the string.

Materials and Methods

Modern recurve bows in the frames of International Archery Federation standards were studied. Mechanical and mathematical modeling was used to derive a method of preparation of the bow for sport archery competition. A drawn bow with an arrow is not symmetrical in its main (vertical) plane because a hand which holds the bow and the arrow cannot be situated simultaneously on the line of symmetry (Figure 1). Therefore, an asymmetrical mechanical chain was used as a modified model of the bow and arrow system in the virtual plane. Because the arrow is situated above the hand which holds the bow, the upper branch of a string should be shorter than the lower branch. Geometry of the scheme model was circumscribed with the equations as follows (Figure 2):

$$l_a = l_U \sin\theta_U + S_U \sin\gamma_U, \ (1) \quad y_A = h_U + l\cos\theta_U - S_U \cos\gamma_U, \ (2),$$

$$l_a = l_L \sin\theta_L + S_L \sin\gamma_L, \ (3), \quad y_A = S_L \cos\gamma_L - l\cos\theta_L - h_L, \ (4),$$

where l_a = length of an arrow, l = length of a limb; S_U, S_L = length of the upper and lower string branches, $\theta_U, \gamma_U, \theta_L, \gamma_L$ = angles between limbs and string branches positions and a handle, $h = h_U + h_L$ = length of the handle and its upper and lower branches. As rule, modern archery bows are equipped with equal limbs: $l_U = l_L = l$. A pivot point situates in the middle of the handle: $h_U = h_L = \frac{h}{2}$.

Because the asymmetry of the bow in the vertical plane is not considerable relatively its size (~3%), the nonlinear system of equations (1) – (4) was transformed to a linear regarding the corresponding symmetrical system (Zanevskyy, 2009):

$$l_a = l\sin\theta + \frac{S}{2}\sin\gamma; \ 2l\cos\theta + h = S\cos\gamma, \ (5)$$

where θ, γ, S are geometrical parameters of this virtual symmetrical system. Parameters of the real asymmetrical model correlate with parameters of the symmetrical model according to the equations below:

$$\theta_U = \theta + \Delta\theta_U; \ \theta_L = \theta + \Delta\theta_L; \ \gamma_U = \gamma + \Delta\gamma_U; \ \gamma_L = \gamma + \Delta\gamma_L; \ \varphi_U = \varphi - \Delta\varphi; \ \varphi_L = \varphi + \Delta\varphi, \ (6)$$

where φ = angle of the initial position of the limbs. There are two sources of the bow asymmetry. One of them is a difference in the initial position of the upper and lower limbs $(\varphi_U < \varphi_L)$, and another is a difference in length of the upper and lower branches of the string $(S_U < S_L)$.

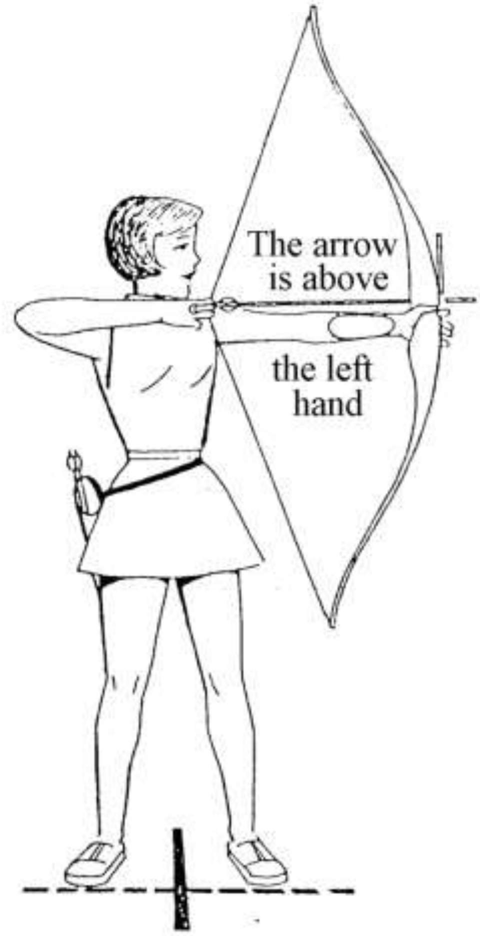

Figure 1. An archer with the drawn bow (Ellison, 2016)

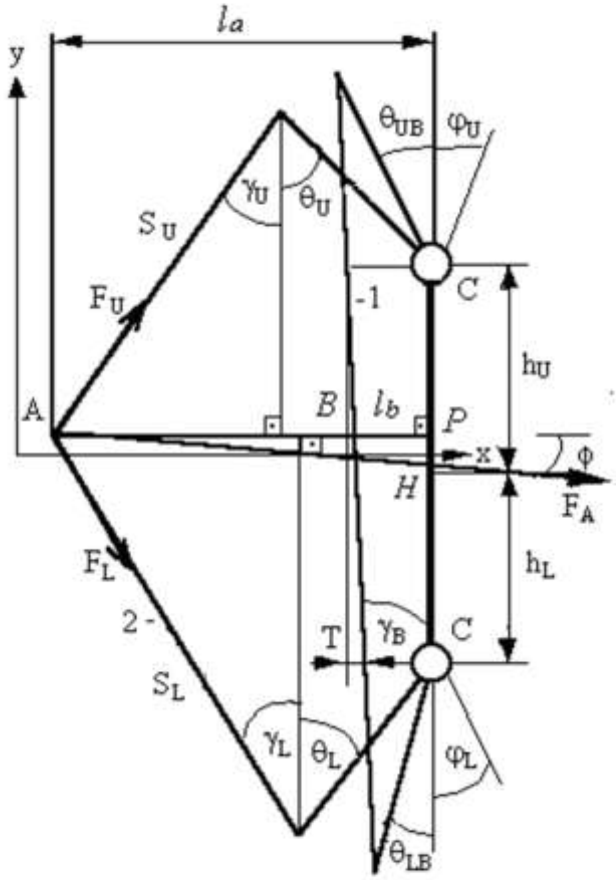

Figure 2. A scheme model of the bow and arrow in the vertical plane: 1 – braced situation and 2 – drawn situation.

The archery bow and arrow system is asymmetrical in its main plane, but the asymmetry of the system is not great: $\frac{S_L - S_U}{S_L + S_U} = \frac{\Delta S}{S} \sim \frac{\varphi_L - \varphi_U}{\varphi_L + \varphi_U} = \frac{\Delta \varphi}{\varphi} \sim 10^{-2}$; therefore, trigonometric functions of small angles were assumed as follow: $\sin \Delta\zeta \approx \Delta\zeta$, $\cos \Delta\zeta \approx 1$, where $S = S_U + S_L$ = length of a string, $\Delta\zeta$ = small angles $(\Delta\theta_U, \Delta\theta_{L,} \Delta\gamma_U, \Delta\gamma_L)$. Products of two small quantities were assumed zero as a number of the second power of a small quantity.

The problem on asymmetry was studied taking into account the length of the string branches: $S_U = \frac{S}{2} - \Delta S$, $S_L = \frac{S}{2} + \Delta S$.

Corresponding equations which made possible to derive expressions of small parameters $\left(\Delta\theta_U,\ \Delta\theta_L,\ \Delta\gamma_U,\ \Delta\gamma_L\right)$ were determined after insertion of the expression (6) into the equations (1)–(4):

$$l\sin\left(\theta + \Delta\theta_U\right) + \left(\frac{S}{2} - \Delta S\right)\sin\left(\gamma + \Delta\gamma_U\right) = l_a,\ y_A = \frac{h}{2} + l\cos\left(\theta + \Delta\theta_U\right) - \left(\frac{S}{2} - \Delta S\right)\cos\left(\gamma + \Delta\gamma_U\right),$$

$$l\sin\left(\theta + \Delta\theta_L\right) + \left(\frac{S}{2} + \Delta S\right)\sin\left(\gamma + \Delta\gamma_L\right) = l_a,\ y_A = \left(\frac{S}{2} + \Delta S\right)\cos\left(\gamma + \Delta\gamma_L\right) - \frac{h}{2} - l\cos\left(\theta + \Delta\theta_L\right).$$

After some algebraic transformations, a system of two linear equations relatively to small parameters $\Delta\theta_U, \Delta\gamma_U$ was derived:

$$l\Delta\theta_U\cos\theta + \frac{S}{2}\Delta\gamma_U\cos\gamma = \Delta S\sin\gamma,\ -l\Delta\theta_U\sin\theta + \frac{S}{2}\Delta\gamma_U\sin\gamma = y_A - \Delta S\cos\gamma. \quad (7)$$

Corresponding solutions of the equations (7) were derived as follow:

$$\Delta\theta_U = \frac{\Delta S - y_A\cos\gamma}{l\sin\left(\theta + \gamma\right)},\ \Delta\gamma_U = \frac{y_A\cos\theta - \Delta S\cos\left(\theta + \gamma\right)}{\frac{S}{2}\sin\left(\theta + \gamma\right)}. \quad (8)$$

After some algebraic transformations, a system of two linear equations relatively small to small parameters $\Delta\theta_L,\ \Delta\gamma_L$ was derived:

$$l\Delta\theta_L\cos\theta + \frac{S}{2}\Delta\gamma_L\cos\gamma = -\Delta S\sin\gamma,\ l\Delta\theta_L\sin\theta - \frac{S}{2}\Delta\gamma_L\sin\gamma = y_A - \Delta S\cos\gamma. \quad (9)$$

Solutions of the equations (9) were derived as follow:

$$\Delta\theta_L = \frac{y_A\cos\gamma - \Delta S}{l\sin\left(\theta + \gamma\right)},\ \Delta\gamma_L = \frac{\Delta S\cos\left(\theta + \gamma\right) - y_A\cos\theta}{\frac{S}{2}\sin\left(\theta + \gamma\right)}. \quad (10)$$

Comparing two pairs of solutions (8) and (10), one can write down: $\Delta\theta_U = -\Delta\theta_L$ and $\Delta\gamma_U = -\Delta\gamma_L$.

Equations of equilibrium of each of two limbs and of the bow itself in the drawn position situation were derived as follow:

$$c(\theta_U + \varphi_U) = F_U l\sin(\theta_U + \gamma_U),\ c(\theta_L + \varphi_L) = F_L l\sin(\theta_L + \gamma_L), \quad (11)$$

$$F_x = F_U \sin\gamma_U + F_L \sin\gamma_L\,,\; F_y = F_U \cos\gamma_U - F_L \cos\gamma_L\,,\; F_A = \sqrt{F_x^2 + F_y^2}\,,\;(12)$$

$$tg\phi = -\frac{F_y}{F_x}\,,\; tg\phi = \frac{y_A + (h_U - h_L)/2}{l_a}\,,\;(13)$$

where ϕ = angle between the line of action of the force vector of the archer and the line of symmetry of the bow, c = virtual bending stiffness of bow limbs concentrated at the ends of the handle, F_A, F_x, F_y = bow force and its projections on the corresponding axes, F_U, F_L = forces of the upper and lower string branches, φ_U, φ_L = angles of limbs setup relatively to the handle. There are positive values of these angles – clockwise for the upper limb and counter-clockwise for the lower limb. The angles in the initial position of the recurve bow limbs are both positive and negative, and zero too.

Like to the transformation of the bow geometry to the linear system (6), the next equations were derived: $\frac{F_U - F_L}{F_U + F_L} = \frac{\Delta F}{F} << 1$, where $F_U = F + \Delta F_U$; $F_L = F + \Delta F_L$. The two last expressions were substituted into the equations (11)–(13), and equations as follow were derived:

$$F(\Delta\theta + \Delta\gamma)\cos(\theta + \gamma) + \Delta F_U \sin(\theta + \gamma) - \frac{c}{l}(\Delta\theta - \Delta\varphi) = 0\,,\;(14)$$

$$-F(\Delta\theta + \Delta\gamma)\cos(\theta + \gamma) + \Delta F_L \sin(\theta + \gamma) + \frac{c}{l}(\Delta\theta - \Delta\varphi) = 0\,.\;(15)$$

A sum of these two equations gave the equation bellow: $(\Delta F_U + \Delta F_L)\sin(\theta + \gamma) = 0$. (16)

Because geometrical parameters of long bare bows $(0 < \theta + \gamma < \pi)$, $\sin(\theta + \gamma) \neq 0$. Taking into account equation (16), one can conclude that $\Delta F_U + \Delta F_L = 0$ or $\Delta F_U = -\Delta F_L = \Delta F$:

$$\Delta F = \frac{\frac{c}{l}(\Delta\theta - \Delta\varphi) - F(\Delta\theta + \Delta\gamma)\cos(\theta + \gamma)}{\sin(\theta + \gamma)}\,.\;(17)$$

Equations of the force parameters (regarding the virtual symmetric system) were derived as follow: $c(\theta + \varphi) = F\sin(\theta + \gamma)$; $F_A = 2F\sin\gamma$. (18)

After a substitution of the last expressions into the system (1)–(4), (11)–(13), the followed system of linear (regarding parameters $y_A, \Delta\theta, \Delta\gamma$) equations was derived:

$$y_A = \Delta S\cos\gamma + \frac{S}{2}\Delta\gamma\sin\gamma - l\Delta\theta\sin\theta\,,\;(19),\quad \Delta S\sin\gamma - \frac{S}{2}\Delta\gamma\cos\gamma - l\Delta\theta\cos\theta = 0\,,\;(20)$$

$$\frac{y_A}{l_a} = \Delta\gamma - \left[\frac{\Delta\theta - \Delta\varphi}{\theta + \varphi} - \frac{\Delta\theta + \Delta\gamma}{tg(\theta+\gamma)}\right] ctg\gamma . \quad (21)$$

Equations of the braced bow were derived as follow:

$$l(\cos\theta_{UB} + \cos\theta_{LB}) + h = S\cos\gamma_B , \; l(\sin\theta_{UB} - \sin\theta_{LB}) = S\sin\gamma_B ,$$

$$F_B l \sin(\theta_{UB} - \gamma_B) = c(\theta_{UB} + \varphi_U) ; \; F_B l \sin(\theta_{LB} + \gamma_B) = c(\theta_{LB} + \varphi_L) , \quad (22)$$

where symbols signed with index "*B*" are used as parameters of the bow in the braced situation (see Figure 2). In the same way as it was done for the bow in the drawn situation, and taking into account geometry of the system $(\theta_{UB} + \varphi_U \approx \theta_{LB} + \varphi_L ; \; \gamma_B << 1)$, one can write down the followed equations: $\theta_{UB} = \theta_B + \Delta\theta_{UB} ; \; \theta_{LB} = \theta_B + \Delta\theta_{LB}$, $\theta_B = \arccos\frac{S-h}{2l}$, $l_b = l\sin\theta_B$, $F_B = \frac{c(\theta_B + \varphi)}{l_b}$,

where θ_B = angle of a limb in the corresponding symmetrical bow system. Then in a similar manner to the equations (8) and (10), the equation was derived: $(\Delta\theta_{UB} + \Delta\theta_{LB})\sin\theta_B = 0$. Because in any position of the bow $0 < \theta_B < \pi$, one can write done: $\Delta\theta_{UB} + \Delta\theta_{LB} = 0$ or $\Delta\theta_{UB} = -\Delta\theta_{LB} = \Delta\theta_B$. As a result, parameters of the braced bow have been derived as follow:

$$\Delta\theta_B = \frac{\Delta\varphi \; tg\theta_B}{tg\theta_B - (\theta_B + \varphi)\frac{h}{S}} ; \; \gamma_B = \Delta\theta_B\left(1 - \frac{h}{S}\right) . \quad (23)$$

Parameters of the system in the drawn situation of the bow were derived using the equations (19)–(21) and the scheme model:

$$\alpha = arctg\frac{h}{2l_a} , \; \theta + \gamma = \arccos\frac{l^2 + S^2/4 - l_a^2 - h^2}{lS} , \; \sin\beta = \frac{l\sin(\theta+\gamma)}{\sqrt{l_a^2 + h^2/4}} , \; \gamma = \frac{\pi}{2} - \alpha - \beta .$$

The nock point distance on the string in its braced situation was determined using the equation as follow (see Figure 2): $y_B = \Delta S - l_b \Delta\theta_B$. (24)

A tiller difference was determined using the equation as follow: $T = h\gamma_B$. (25)

Position of the plunger that obtains a zero angle of attack of the arrow was determined using its coordinate on the handle with the equation bellow: $y_P = y_B - \frac{(y_A - y_B) l_b}{l_a - l_b}$, (26)
where y_A was determined as a solution of the system of equations (19)–(21). The system of equations (17)–(26) is a mathematical model of the optimal tuning of the bow and arrow parameters taking into account a zero angle of attack of the arrow. A scheme model of the testing and tuning procedure is presented on Figure 3.

Figure 3. Bow and arrow scheme system in the vertical plane: a – symmetric, b – asymmetric, c – asymmetric and tuned (A – nock point at the drawn and B – at the braced bow, F – arrow in the free flight just after the launch; ϕ – angle of attack).

Using the proposed mathematical model as equations (17)–(26), calculation experiments regarding the optimal parameters of the moden archery bows during their testing and tuning were done. The method of simple itterations was used for solving a system of transindent algebra equations (19)–(21) and the method of division of a section in half was used when divergence ittarations occurred. Calculations were done using a function FindRoot from a package of computer programs Mathematica (Wolfram Research).

Results

A modern sport archery bow Hoyt GM T/D4 was tested and tuned with aluminum and carbon composite arrows Easton-Beman (Arrow guide, 2015) with a purpose to approbate the mathematical model and the methods developed and described in the research. A bow and arrow system was studied basing on the initial parameters: h = 0.68 m, l = 0.52 m, S = 1.62 m, $\varphi = 0.083$ rad, c = 129 Nm, l_a= 0.72 m.

A calculable experiment covered a wide range of parameters of the system taking into account instructions of producers and recommendations of sport archery coaches and sportsmen. A four-factor variation of parameters was done: a difference in the length of upper and lower branches of a string varied up to 100 mm, a height of a plunger – up to 40 mm, and a difference in distances of a string to the upper and lower ends of the handle (T) – up to 18 mm.

Parameters of the braced bow were determined using equations (22) – (25). Parameters of the drawn bow were determined using equations (5), (18) and (18) – (21). Corresponding calculations were done using Mathematica programs. The optimal height of a plunger was determined using equation (26).

Main results of modeling were collected in Table 1. A remarkable result of modeling is in the almost constant range of a nock point height, i.e. independence of this parameter from the difference in angles of limbs setup at the handle. In the studied range of difference in the length of the lower and upper string branches ($2\Delta S = 20 - 100$ mm), a height of the nock point was in the range from 2 to 11 mm, and a height of a plunger relatively to a centre of the handle – from 5 to 38 mm. For the common height of nock point 6–10 mm, the difference in length of upper and lower branches of a string was 60–80 mm. For the common difference of the bow asymmetry measured as tiller (3–12 mm), a difference of angles of limbs at the handle setup is recommended from 0.004 to 0.017 rad.

Table 1. Results of modeling (mm) for the optimal tuning of archery bow parameters and zero angle of attack of an arrow

$\Delta\varphi$, rad	0.005			0.010			0.015		
T, mm	3.7			7.4			11.1		
ΔS, mm	y_A	y_B	y_P	y_A	y_B	y_P	y_A	y_B	y_P
10	9.1	7.9	7.4	5.4	5.8	6.0	1.8	3.8	4.6
20	21.7	17.9	16.2	18.1	15.8	14.8	14.5	13.8	13.4

30	34.4	27.9	25.0	30.8	25.8	23.6	27.2	23.8	22.2
40	47.0	37.9	33.9	43.4	35.8	32.4	39.8	33.8	31.0
50	59.7	47.9	42.7	56.1	45.8	41.3	52.5	43.9	39.9

The necessary height of plunger that makes possible a free displacement of the arrow over the hand that holds a bow is approximately 20–30 mm. From the results of modeling (see Table 1), one can determined a lower value of difference in the length of upper and lower string branches as 60–80 mm. It is remarkably to notice, that this parameter is not depended on the difference of angles setup. However, this parameter significantly determines the direction of the arrow movement.

Two variants of bow testing regarding the tuning of a bow with an equal ratio of the string branches and equal height of the nock point, but a different ratio of the angles of limbs setup were done. In the first example, zero tiller was assumed ($T = 0$), i.e. the angles are equal: $\Delta\varphi = 0$, ΔS = 40 mm, y_A = 61 mm, y_B = 40 mm, y_P = 31 mm. In the second example, the recommended value of tiller was assumed as follow (Baudrillard, 2007): T = 11 mm, $\Delta\varphi$ = 0.015 rad, ΔS = 40 mm, y_A = 46 mm, y_B = 34 mm, y_P = 29 mm.

In the first example, the angle of the arrow launch direction with a normal to the handle $\left(\frac{y_A - y_P}{l}\right)$ was in 1.8 times greater than in the second example, i.e. increase of the difference in the angles of limbs setup causes deviation of the arrow off the normal direction of the arrow launch. This correlation is actual in all the rest practical combinations of bow parameters aimed a zero angle of attack of the arrow (see Table 1). Increase of this angle makes more comfortable of the conditions of holding of a bow because does not demand a significant difference in the height of handle and the nock point of a string. As a result, a setup of the lower limb with a greater angle that of the upper limb relatively a handle makes possible to decrease asymmetry of the drawn bow.

Correspondingly to the task of the archery shooting, an archer can use the methods of the bow and arrow tuning in different variants, but the initial parameter should be a height of the plunger. Modern archery bows are equipped with a special mechanism that is intended for tuning a position of the plunger. With other equal conditions, a minimum height of the position that obtains a free displacement of the arrow over the hand that holds a handle should be preferred. This is because increase of the height of the plunger causes increase of asymmetry of the bow in the vertical plane that complicates controlling of the bow and arrow system by the shooter (see Table 1).

After the plunger height was determined, a ratio of the string branches' length and a difference of the angles of limbs setup at the handle were determined. With other equal conditions, more comfortable for the archer is a smaller difference in length of the string branches. For the shooting on the long distances (70 – 90 m) azimuth angle of the arrow launch should be greater, therefore this difference should be minimal.

To decrease the difference in length of the string branches, the difference in the angles of limbs setup should be increased. But this increase causes decrease of the height of the plunger (see Table 1). The process of the bow and arrow tuning includes a selection of convenient variants and choosing among them the best regarding the difference in length of the string branches.

An imitation scheme describes the process of determination and selection of these convenient variants for the tuning of the bow and arrow system (Figure 4). For example, a height of the plunger was assumed $y_P = 20-30$ mm. A necessary difference between the length of the string branches (ΔS) and the difference in the angles of limbs setup $(\Delta\varphi)$ were determined from Table 1, and then a parameter of direction of the arrow launch $(y_A - y_P)$ was calculated (Table 2).

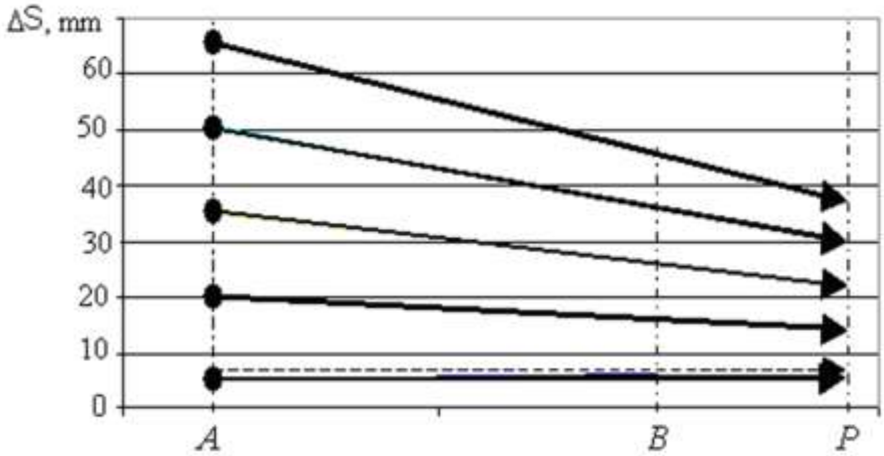

Figure 4. Results of virtual testing and tuning of the bow and arrow system to a zero angle of attack: *T*= 7.4 mm, *A* and *B* – situation of the nock point at the moments of string release and arrow launch correspondingly, *P* – plunger and rest.

Using a method of linear interpolation (i.e. calculation of intermeddle values between the values of the neighbor columns (see Table 1), the optimal values were determined. For example, corresponding formula for determination of the nock point coordinate in the braced position for $\Delta\varphi = 0.015$ and $y_P = 20.0$ mm was derived: $y_B = y_{B3} - (y_{P3} - y_P)(y_{B3} - y_{B2})/(y_{P3} - y_{P2})$, (27)

where sub-index figures signify ordinal numbers of lines in Table 1, $y_{P2} = 13.4$ mm, $y_{P3} = 22.2$ mm, $y_{B2} = 13.8$ mm, $y_{B3} = 23.8$ mm (see two last columns). Using equation (27) and analogous equations for a half difference in the length of string branches and nock point coordinate in the drawn position, these parameters were calculated: $y_B = 21.2$ mm, $\Delta S = 27.5$ mm, and $y_A = 23.9$ mm.

Table 2. Values of parameters of bow and arrow system tuning (plunger height: $y_P = 20-30$ mm)

$\Delta\varphi$, rad	0	0.005	0.010	0.015	0.020	0.025

ΔS, mm	22.7–34.0	24.3–35.6	25.9–37.2	27.5–38.8	29.7–41.0	31.4–42.8
$y_A - y_P$, mm	8.7–13.1	7.1–11.5	5.5–9.9	3.9–8.3	3.2–7.5	1.8–6.1

Accuracy of the linear interpolation was estimated as a comparison of the calculated y_A with solutions of the system of equations (19)–(21) using FindRoot function. In the studied example an error was near 0.2%. In the frames of the model, results of the extrapolation for y_B and calculation by equation (24) were equal. Increase of the difference in values of the angles of the limbs setup from zero up to the maximum (0.025 rad) needs really negligible increase of difference in the lengths of the string branches (near 12 mm) that causes a significant decrease of the nock point height relatively to the handle (about 20 mm).

An optimal height of the plunger and arrow rest (y_p) was determined using the mathematical model (23)–(26). For the medial difference of the lower and upper parts of a string ($2\Delta S = 60$ mm) and asymmetry of a bow (T = 18 mm, y_B = 24 mm), the height of the plunger and arrow rest should be equal 20 mm, for T = 11 mm, y_B = 26 mm – 21 mm, and for T = 4 mm, y_B = 29 mm – 22 mm (Figure 5).

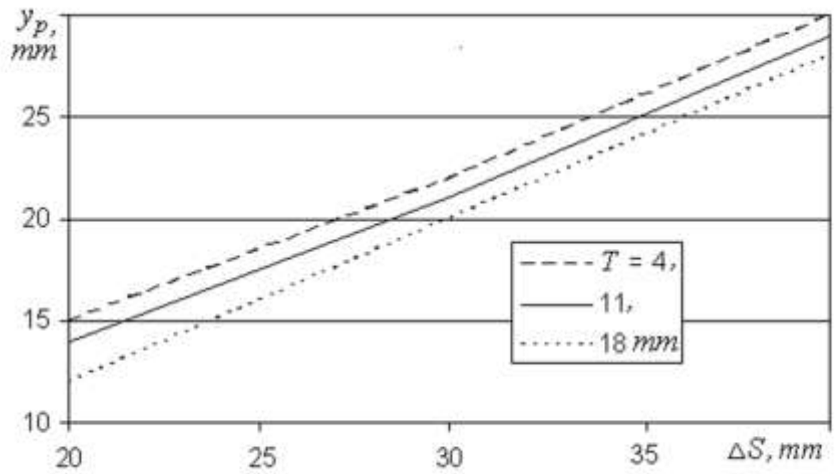

Figure 5. An optimal plunger height (y_P) vs. the difference of the string branches (ΔS) and the tiller difference (T).

For a greater difference of the lower and upper parts of a string ($2\Delta S = 70$ mm) and the same asymmetry of a bow, this height should be equal 24, 25, and 26 mm (y_B = 29, 31, 34 mm).

Discussions

The value of $y_A - y_P$ should be determined as minimum, but sufficient to obtain a free movement of the arrow above the arm that holds a bow. In the discussed example this value should be 18–20 mm that meets empirical recommendations for the tuning of the bow (Squadrone and Rodano, 1994).

Bow tuning in the vertical plane is a component of the whole process of the bow and arrow system (Park, 2011). Another component is adjusting of the bow and arrow movement in the lateral to the main plane. The aim of this adjustment is to avoid a stroke of the arrow tail to the handle that is known in theory and practice of archery as “archer paradox” (Peters, 2017). Corresponding mechanical and mathematical model of bow and arrow interaction in the lateral plane and the methods of tuning have been developed. Because amplitudes of bend vibrations of an arrow are a power smaller than a length of the arrow (Heller, 2012), linear model is suitable here. Using the linear model of shift vibration, one can assume non-significant correlation between modes of vibration in the two orthogonal planes, i.e. vertical and lateral. (Zanevskyy, 2001).

In the frames of the mechanical and mathematical model of the bow and arrow system that was derived and used in the adjusting of its parameters, it is reasonable to assume no influence of angle of attack on the lateral deflection of the arrow. Therefore parameters of the tuning of the system in the main plane and in the lateral plane are different, i.e. the process of tuning includes two different parts (Tiermas, 2017). As initial matter of the tuning are the main bow parameters: bow force, length of a handle, limbs, a string and the length of an arrow, which match anthropometrical parameters of an archer (Edelmann-Nusser et al., 2002).

A height of the nock point does not close depend on the difference the string branches length, but it is rather depended on the angles of the difference of the angles of limbs setup. The setup of the lower limb with a greater angle than the setup of the upper limb makes (Leroyer et al., 1993) possible to obtain smaller asymmetry of the drawn bow. The results of mathematical and mechanical modeling were presented in a simple form (see Tables 1,2 and Figure 5) that are suitable for coachers and shooters which are unready to use mathematical methods.

The analytical method of the virtual testing and tuning of the archery bow and arrow system developed in the research is recommended for the sport archery with a purpose of optimizing of the height of the plunger and arrow rest. An optimal regarding to the accuracy and the distance of shooting is a zero angle of attack of the arrow launch. To obtain this condition bow and arrow testing and tuning should be done. Well-known practical recommendations for the bow and arrow tuning are rather approximate and based on the empirical method of tests and mistakes. This method needs a long time and afford many efforts from archers during this testing. The bow and arrow system could not be symmetrical in the main plane because an arrow and a hand that holds a bow could not situated together at the same place. A zero angle of attack of the arrow could be obtained if a nock point of a string in the braced bow and the drawn bow, and a plunger are situated at the same straight line.

A modern sport recurve bow was described as a polygon with sides corresponding to the riser, the two limbs and the string. The string tension is generated by the torque in the limb joints with the torsion coefficient. The pivot point (where the bow is held) is fixed in the centre of the riser. The arrow length is equal the distance from the knocking point to the pivot point. A calculation method was prepared to assist in the task of tuning the bow. Parameters that are varied when tuning a bow are the location of the knocking point on the string, the location of the plunger on the riser where the arrow rests and the tiller as imbalance of the two limbs.

The main result was a determination of the unknown tuning parameters in terms of the given bow parameters, subject to one or more conditions that have to be met when the bow is tuned. The results of mathematical and mechanical modeling were presented in tables and graphs suitable for coachers and shooters which are not familiar with mathematical methods. The developed method is recommended for optimizing of the sport archery bow and arrow parameters.

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List of variables

c_U, c_L	Virtual stiffness of the upper and lower limbs
h	Length of a handle
h_U, h_L	Length of the upper and lower parts of a handle
l_U, l_L	Length of the upper and lower limbs

l_a	Length of an arrow
l_b	Distance of the pivot point to the braced string
A	Nock point in the drawn position
B	Nock point in the braced position
F_A, F_x, F_y	Force acting to a string by an archer's hand and its projections
F_U, F_L	Tensile forces in upper and lower string branches
F, c	Force parameters of the virtual symmetrical system
H	Pivot point
L	Sign of the lower part of a bow
P	Plunger and rest
S	String length
S_U, S_L	Length of the upper and lower branches of a string
x, y	Co-ordinates fixed to the handle
U	Sign of the upper part of a bow
α, β	Angles of the drawn symmetrical bow
γ_U, γ_L	Angles between upper and lower string branches and a handle
T	Tiller difference
ΔS	Half part of string branches length difference
$\Delta\zeta$	Small quantity
θ, γ, φ	Angles of the virtual symmetrical system
θ_U, θ_L	Angles between upper and lower limbs and a handle
φ_U, φ_L	Angles of the installed limbs to a handle
ϕ	Angle between the bow force vector and an arrow in the vertical plane