Choice of Spectral Density Estimator in Ng-Perron Test: A Comparative Analysis

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ABSTRACT

Ng and Perron (2001) designed a unit root test, which incorporates the properties of DF-GLS and Phillips Perron test. Ng and Perron claim that the test performs exceptionally well especially in the presence of a negative moving average. However, the performance of the test depends heavily on the choice of the spectral density estimators used in the construction of the test. Various estimators for spectral density exist in the literature; each have a crucial impact on the output of test, however there is no clarity on which of these estimators gives the optimal size and power properties. This study aims to evaluate the performance of the Ng-Perron for different choices of spectral density estimators in the presence of a negative and positive moving average using Monte Carlo simulations. The results for large samples show that: (a) in the presence of a positive moving average, testing with the kernel based estimator gives good effective power and no size distortion, and (b) in the presence of a negative moving average, the autoregressive estimator gives better effective power, however, huge size distortion is observed in several specifications of the data-generating process.

Key words: *Ng-Perron Test, Monte Carlo, Spectral Density, Unit Root Testing*
JEL Classifications: C01, C15, C63

1. INTRODUCTION

Unit root testing is a well-known and one of most debated issues in econometrics. There are lots of economic and econometric implications of the existence of a unit root in time series data, including the incidence of spurious regression (Atiq-ur-Rehman, 2011; Libanio, 2005). Due to its importance, many tests and testing procedures were developed for testing for a unit root. However, the size and power properties of unit root tests have always been subject to debate.

In many economic time series models, errors may have heterogeneity and temporal dependence of unknown forms. This is the main source of size and power distortion of unit root tests. In order to draw more accurate inferences from estimates of parameters, constructing unit root tests based on long run variance (LRV) estimates has become important. LRV estimates take serial correlation and heterogeneity into account. The key to constructing an LRV is to estimate the spectral density (SD hereafter) at zero frequency. There are two main types of SD estimators: (1) autoregressive estimator of spectral density, (2) kernel based estimator of spectral density. However, literature does not provide any information about the relative performance of these estimators of spectral density. Many of existing tests for unit root, including the Ng-Perron test, use an estimator of spectral density at

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zero frequency. Ng and Perron (2001) have developed a new suit of tests, which according to them outperforms the other tests, especially in case of a negative moving average process. The output of the test is also affected critically by the choice of spectral density estimator, and the literature does not provide any guide in this regard. Ng and Perron (2001) do not discuss the effect of the choice of spectral density estimator and thus leave practitioners without guidance regarding the choice of an estimator of spectral density.

This study aims to investigate the properties of Ng-Perron test for different choices of SD estimators using Monte Carlo simulations. We examine the size distortion and effective power of the test, with both autoregressive (AR) estimator and kernel based (KB) estimators of spectral density, in the presence of a negative and positive moving average. The remainder of the paper is organized as follows: In Section 2 we discuss the Ng-Perron test and various estimators of spectral density. Section 3 consists of our Monte Carlo design. Section 4 explains the results. Section 5 provides details of detecting the sign of a moving average. Section 6 presents some concluding remarks.

2. EFFECT OF SPECTRAL DENSITY ESTIMATOR ON OUTPUT OF NG-PERRON TEST: A REAL DATA ILLUSTRATION

Like other tests, the output of Ng-Perron test depends crucially on the choice of the spectral density estimator, and the final decision may be quite contradictory for two different choices of the density. This fact is illustrated below with the help of a real data example.

We apply the Ng-Perron test on log GDP of UK from 1951-2007. Table 2.1 provides the outputs of Ng-Perron test with both estimators of spectral density estimators.

Spectral Density Estimator	Ng-Perron tests	
	With Drift	With Drift and Trend
AR Estimator	-8.25*	-25.25*
KB estimator (Parzen Kernel)	1.54	-3.81
Critical Value		
5% *Critical Value*	-8.1	-17.3

Table 2.1 Output of Ng-Perron test with AR and KB estimator for log UK GDP data
Notes: * 5% level of significance

According to the results in Table 2.1, the Ng-Perron test statistics is below the critical value for the autoregressive estimator of spectral density; hence, the unit root hypothesis should be considered rejected. On the other hand, for Kernel based estimator of spectral density, the Ng-Perron test statistics is far above the critical value; thus, the null of unit root could not be rejected even at a loose significance level. Therefore, the person applying the unit root test may be confused in the choice of result. In response to this ambiguity, we designed our study to compare the size and power properties of Ng-Perron test so that a practitioner may get some guidance on the selection of an optimal spectral density estimator.

3. COMPUTATION OF NG-PERRON TEST

In this section we discuss the Ng-Perron unit root test and different estimators of spectral density at zero frequency.

3.1. Test Statistics

Dufour and King (1991) and Elliott et al. (1996) found that local GLS detrending of the data yields significant power gains. Phillips and Perron (1987) found that use of SD could improve the performance of the test. Ng and Perron (2001) combine GLS detrending with SD to design a new test. The proposed test consists of a suite of four tests, namely MZ_a, MZ_t, MSB, and MPT. The four test statistics proposed by Ng-Perron are:

$$MZ_a = \frac{\left((T^{-1}\tilde{y}_t)^2 - \hat{f}(0)\right)}{2k}$$

$$MZ_t = MZ_a * MSB$$

$$MSB = \left(\frac{k}{\hat{f}(0)}\right)^{1/2}$$

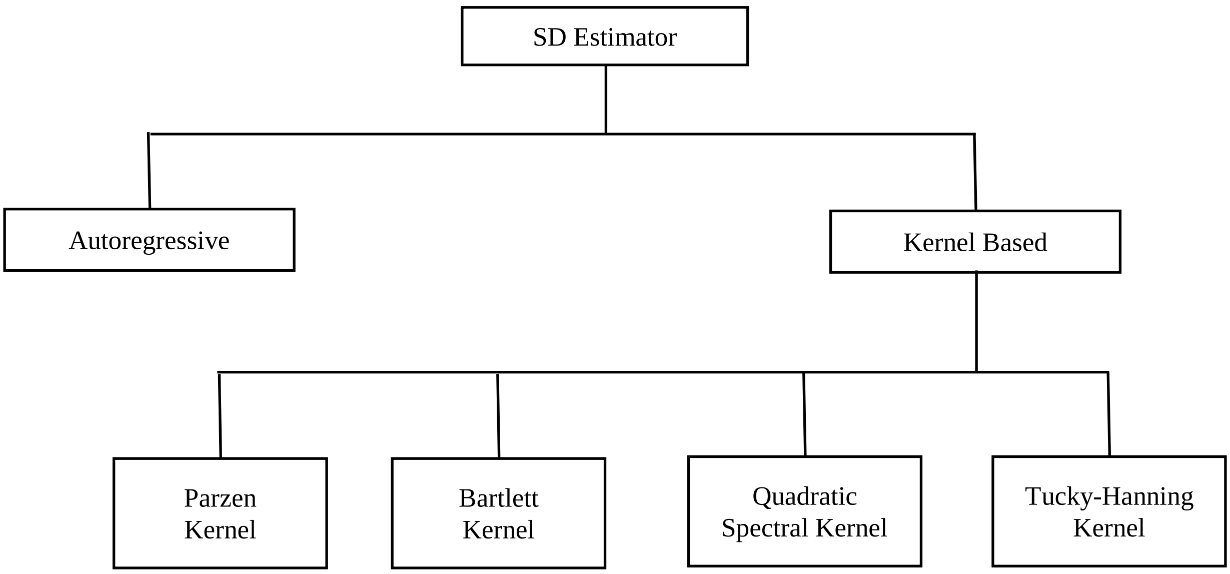
$$MPT = \begin{cases} \dfrac{\bar{c}^2 k - \bar{c}T^{-1}(\tilde{y}_t)^2}{\hat{f}(0)} & \text{when } d_t^0 \\ \dfrac{\bar{c}^2 k - (1-\bar{c})T^{-1}(\tilde{y}_t)^2}{\hat{f}(0)} & \text{when } d_t^1 \end{cases}$$

where d_t^0 represent drift and d_t^1 drift and trend in DGP, and $k = \sum_{t=1}^{T} \frac{(\tilde{y}_{t-1})^2}{T^2}$. The symbol $\hat{f}(0)$ indicates the estimate of spectral density at frequency zero.

3.2. Spectral Density at Frequency zero

The spectral density at frequency zero represent the heteroskedasticity and autocorrelated corrected (HAC) standard error. There are many ways to estimate the spectral density, which can be divided into two types: (a) autoregressive spectral density, and (b) kernel based spectral density, which can be further subdivided into four types. This hierarchy is summarized in the following Figure 3.1:

Figure 3.1 Summary of Spectral Density Hierarchy



The computational details of these estimators are as under:

3.2.1. Autoregressive (AR) Estimator of Spectral Density

Autoregressive estimator of spectral density was proposed by Stock (1990; see also Stock, 1994; Perron and Ng, 1998). This estimator, based on the estimation of parametric model, is identical to the equation of the ADF test equation.

After having GLS detrending series estimate the regression equation given below:

$$\Delta\tilde{y}_t = \rho\Delta\tilde{y}_{t-1} + \sum_{l=1}^{T}\Delta\tilde{y}_{t-l}\hat{\beta}_l + \varepsilon_t \tag{3.1}$$

Autoregressive estimator of spectral density is:

$$\hat{f}(0) = \frac{\hat{\sigma}_\varepsilon^2}{(1-\hat{\beta}(1))^2}$$

where $\widehat{\beta}(1) = \sum_{l=1}^{T}\widehat{\beta}_1$ and $\hat{\sigma}_\varepsilon^2 = \frac{\sum_{t=1}^{T-l}\hat{\varepsilon}_t^2}{T-l}$.

$\widehat{\beta}(1)$ is the sum of coefficients of lags of $\Delta\tilde{y}_t$. Here $\Delta\tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}$ and $\hat{\sigma}_\varepsilon^2$ represent the variance of residuals ($\hat{\varepsilon}_t$) from the equation (3.3).

3.2.2. Kernel Based (KB) Estimator of Spectral Density

Non parametric kernel based estimator of spectral density was proposed by Phillips (1987) and then restructured by Phillips and Perron (1988). Kernel based estimator of spectral density is the weighted sum of auto covariance, in which weights are decided by the kernel and bandwidth parameter.

Estimating the equation using GLS detrended series,

$$\Delta\tilde{y}_t = \rho\Delta\tilde{y}_{t-1} + \varepsilon_t \tag{3.2}$$

The kernel based estimator given as:

$$\hat{f}(0) = \sum_{j=-(T-l)}^{T-l}\hat{\gamma}(j)K(j/l) \tag{3.3}$$

$$\hat{\gamma}(j) = \frac{\sum_{t=1}^{T-j}\hat{\varepsilon}_t\hat{\varepsilon}_{t-j}}{T-j}$$

where l is bandwidth parameter, which act as a truncation lag in the covariance weighting, and K is the kernel function, which can be estimated in multiple ways listed below. $\hat{\gamma}(j)$ is j^{th} order auto covariance of residual from equation (3.2).

For the estimation of the kernel estimator of spectral density we consider the following kernels:

1. Bartlett Kernel

   $$K_{BT}(x) = \begin{cases} 1-|x| & \text{for } |x| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

2. Parzen Kernel

$$K_{PR}(x)=\begin{cases} 1\text{-}6x^2+6|x|^3 & \text{for } 0\leq|x|\leq 1/2 \\ 2(1\text{-}|x|)^3 & \text{for } 1/2\leq|x|\leq 1 \\ 0 & \text{otherwise} \end{cases}$$

3. Quadratic Spectral Kernel

$$K_{QS}(x)=\frac{25}{12\pi^2x^2}\left(\frac{\sin\left(\frac{6\pi x}{5}\right)}{\frac{6\pi x}{5}}-\cos\left(\frac{6\pi x}{5}\right)\right)$$

4. Tukey-Hanning Kernel

$$K_{TH}(x)=\begin{cases} \dfrac{1+\cos(\pi x)}{2} & \text{for } |x|\leq 1 \\ 0 & \text{otherwise} \end{cases}$$

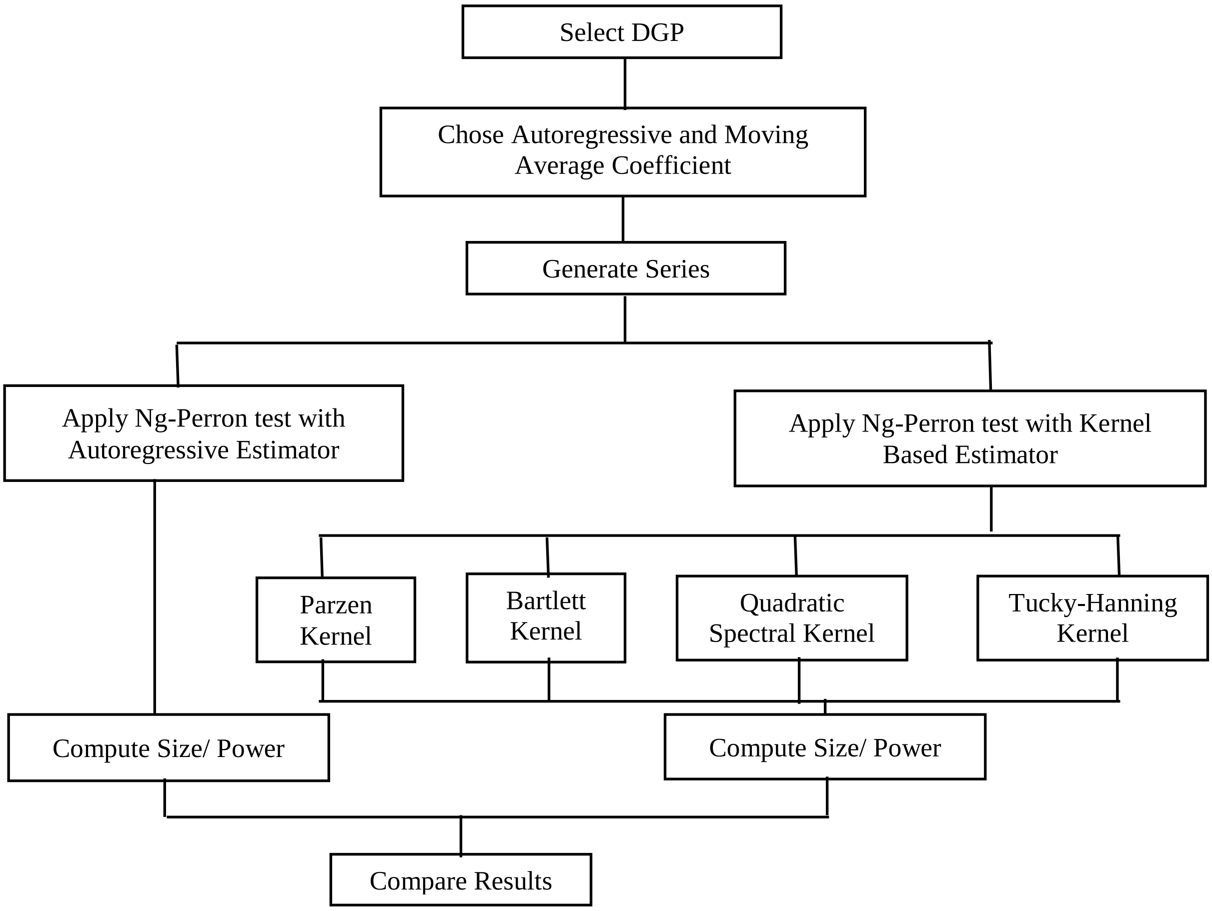
where $x = j/l$ for all kernels. Asymptotically, all of these kernels are equivalent (Andrews, 1991).

The computational details are given below. Ng and Perron point out that these four tests are equivalent in terms of size and power. Throughout our discussion, MZ_a is taken as representative of these four.

4. MONTE CARLO EXPERIMENT

In order to compare the performance of Ng-Perron test with *AR* and *KB* estimator of spectral density, we perform extensive Monte Carlo experiments, which is given in Figure 4.2 below.

Figure 4.2 Flow Chart of Monte Carlo Experiment



Every step of the above mentioned Monte Carlo experiment is summarized as under:

4.1. Data Generating Process

The following forms of the data generating process were used to conduct the Monte Carlo experiment:

DGP-I $y_t = \alpha + u_t$ $u_t = \rho u_{t-1} + \delta e_{t-1} + e_t,$

DGP-II $y_t = \alpha + \beta t + u_t$ $u_t = \rho u_{t-1} + \delta e_{t-1} + e_t,$

DGP-I resembles an ARMA process with an intercept but no trend, whereas *DGP-II* resembles ARMA with drift and trend, where $t = 1, 2,\ldots,T$.

4.1.1. Autoregressive Coefficient

Setting the autoregressive coefficient $\rho = 1$ will generate a unit series, which could be used to compute the power of Ng-Perron test, whereas setting Rho < 1 generates stationary series, which can be used to compute the power of the test. The following values of Rho were used for the Monte Carlo experiment: 0.99, 0.98, 0.95, 0.90, 0.85, 0.80, and 0.70.

4.1.2. Moving Average Coefficient

The aim of this study was to evaluate the performance of the Ng-Perron test both for positive and negative moving average processes. The following values were used in the experiment:
-0.80, -0.60, -0.40, -0.20, 0, 0.2, 0.4, 0.6, and 0.8

4.1.3. Calculating Size and Size Distortion

Ng and Perron provide a set of asymptotic critical values for their test. The test statistics calculated on the series generated under the null were compared with these critical values in order to calculate the actual size of the test. The size distortion is the difference between the actual size and nominal level of significance.

4.1.4. Calculating Power and Effective Power

The power of the test was computed by applying unit root tests to series generated with a stationary root. The probability of rejection of the null is the power of the test.

However, for several data generating processes, heavy size distortion was observed. Since it is not reasonable to compare the power of two tests with different sizes, we have used the effective power of the tests for comparison. The effective power was calculated as follows:
Effective Power for a DGP = Actual Power at Rho < 1 – Actual Size for Rho = 1.

5. MONTE CARLO RESULTS

This section illustrates the equivalence of KB estimators. There are four choices of kernels in this study whose computational details are given in Section 3. The figures below summarize the size and power of Ng-Perron test for different choice of kernels. The figures show that the power curves remains same of various choices of kernels. The experiment was repeated for a variety of DGP's and similar results were obtained. Analysis shows that choice of kernels does not significantly affect the size and power of test, therefore there is no need of summarizing the simulations for all four kernels. Only one of these kernels will be sufficient to observe the behavior of the remaining ones.

Figure 5.3 The Size and Power of MZ_a when Sample Size is 150 and $\delta = 0.4$ with *DGP-I*

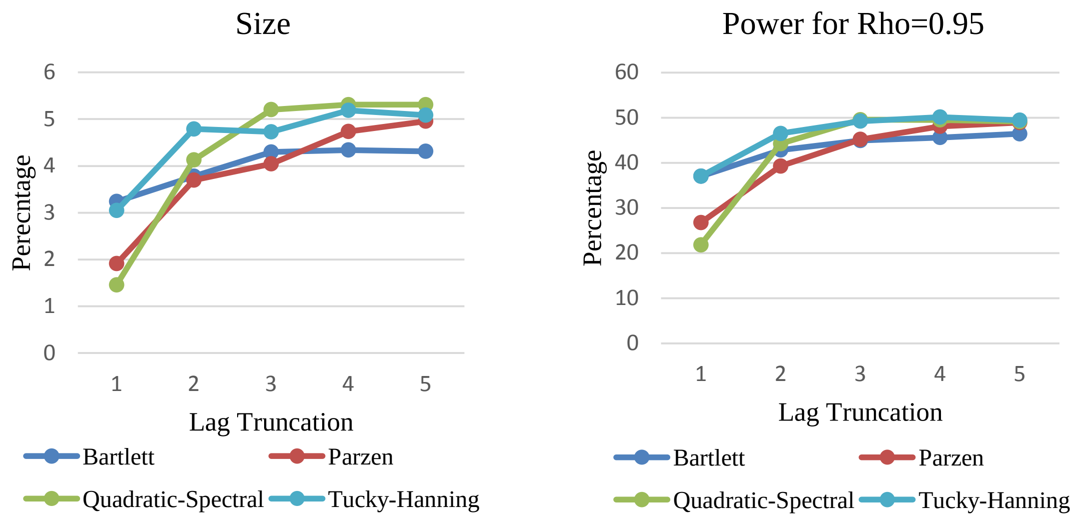
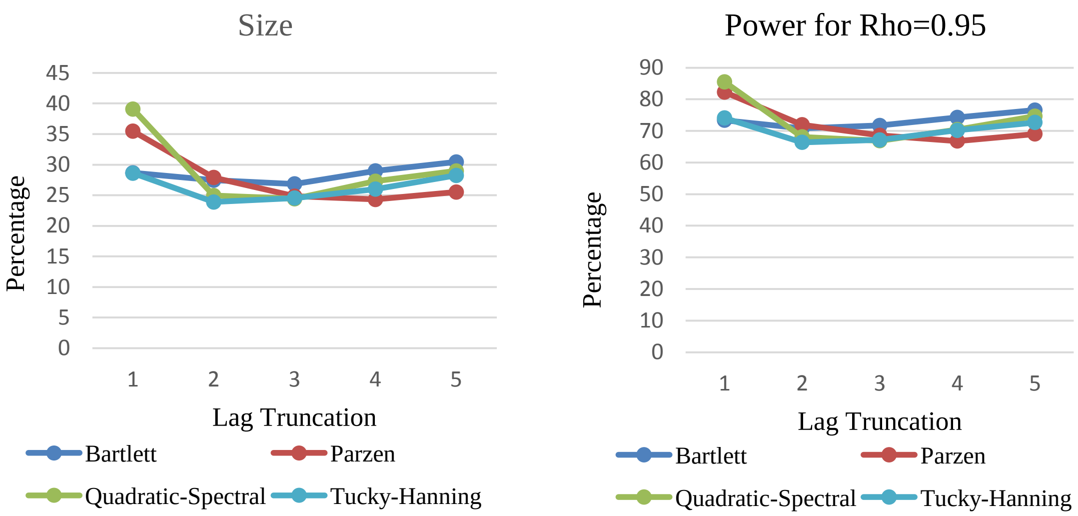


Figure 5.4 The Size and Power of MZ_a when Sample Size is 80, $\delta = -0.4$ with *DGP-I*



For the comparison of size and power of the test with AR estimator and KB estimator, we use the Parzen kernel as a representative for these four kernels.

5.2. Effective Power versus Power

Our results indicate that the size of the test is not stable, rendering comparison of the power meaningless. For a more meaningful comparison, we compare the distortion in size and the effective power of the test. Size distortion is the difference between the observed size and theoretical size (here 5%) of the test; effective power is defined as the difference between the empirical power and empirical size of the test.

5.3. Performance of Test with *DGP-I*

Both KB and AR estimators of spectral density are equivalent mathematically at zero lag length/ lag truncation for any data generating process. We discuss the performance of the test with KB and AR estimators of spectral density at nonzero lag length/ lag truncation. Our result shows that the test has very low effective power in a small sample size with both estimators. Figures 3 and 4 depict the size distortion and the effective power of the test with a positive value of moving average coefficient.

Figure 5.5 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = 0.2 for *DGP-I*

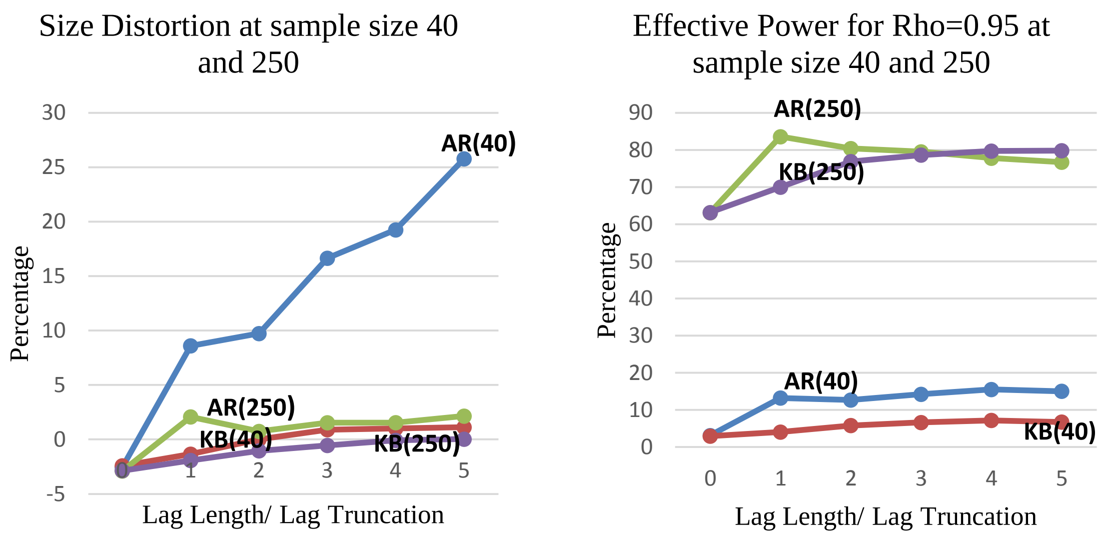
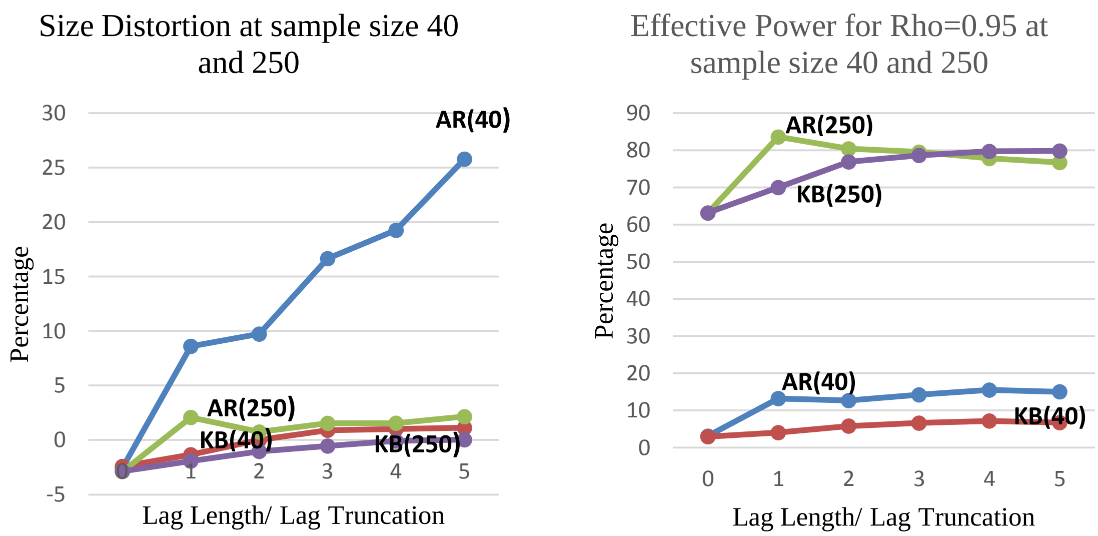


Figure 5.6 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = 0.6 for *DGP-I*



According to the figures above, the distortion in size and effective power of the test increases with lag length when we use an AR estimator of spectral density; on the other hand, when using a KB estimator, the effective power of the test improves with large lag truncation without any distortion in the size of the test. Therefore it could be deducted that in the case of a positive moving average, the KB estimator outperforms the AR estimator. The behavior of the effective power and distortion remains similar, for experiments with different values of MA and autoregressive parameters.

A different picture emerges when we have a negative moving average in the data generating process. Lag length selection has significant consequences on the performance of the test with an AR estimator. As evident in Figures 5 and 6, we observed that for a weaker negative moving average structure, the performance of the test was similar for both estimators of SD in large samples. As the negative moving average structure becomes stronger, distortion is high when using the effective power, demonstrating non-monotonic behavior and a decreasing KB estimator, regardless of the choice of the truncation lag and sample size. On the other hand, the size distortion with an AR estimator is smaller and reduces to zero when lag length is 5. The effective power shows non-monotonic behavior and starts decreasing after reaching its maximum value. Effect power is maximized when the lag length/ lag truncation is 2 and starts decreasing at a higher lag.

Figure 5.7 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = -0.20 for *DGP-I*

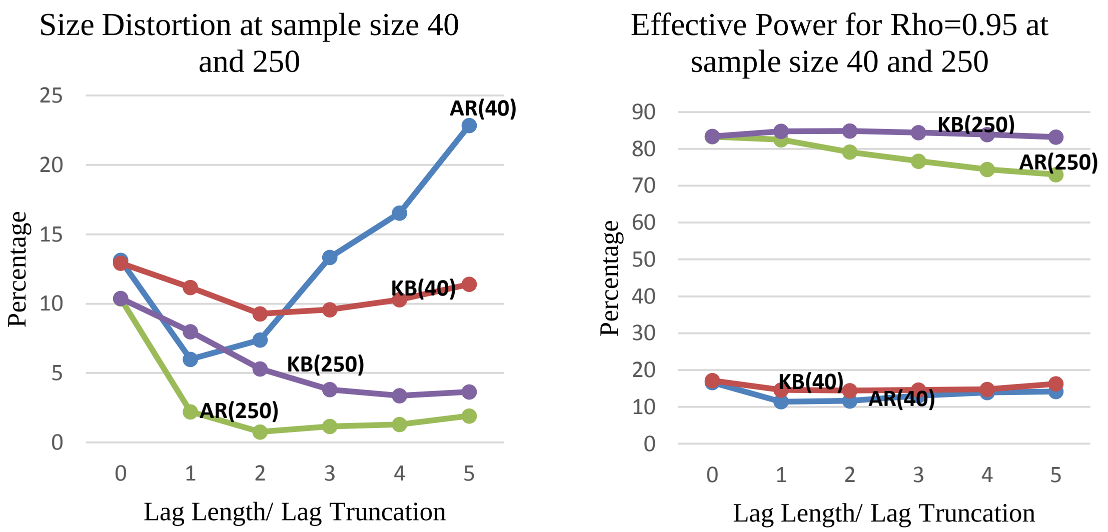
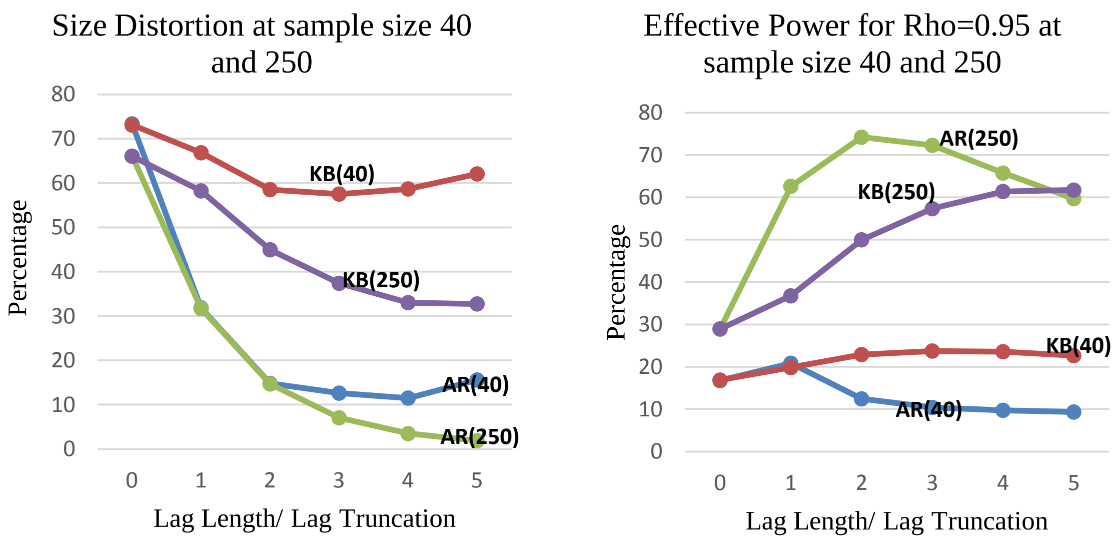


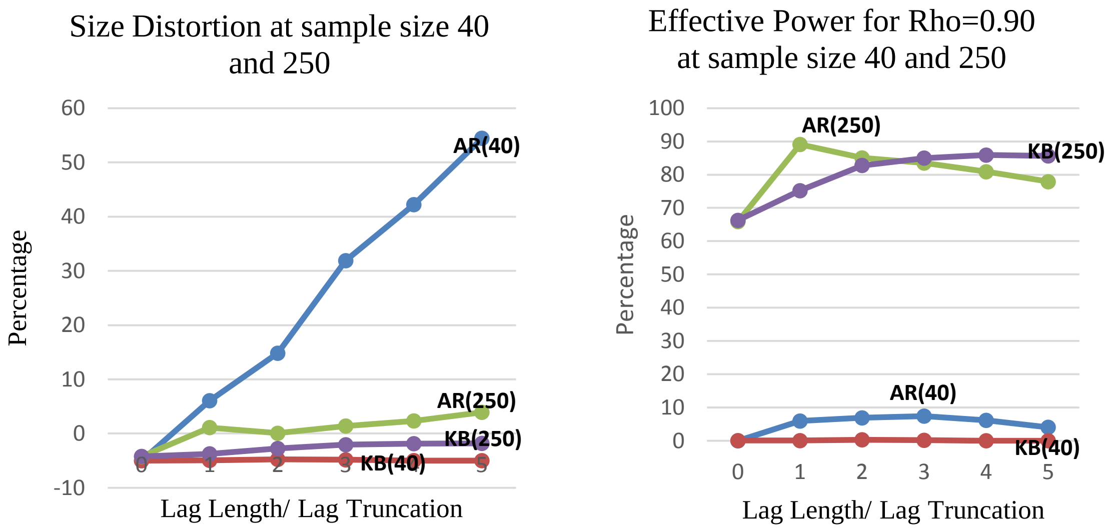
Figure 5.8 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = -0.60 for *DGP-I*



5.4. Performance of Test with *DGP-II*

In this part of the discussion we study the effective power and size distortion when the data generating process consists of a drift as well as a time trend. Figures 5.9 and 5.10, given below, depict the performance of the test when we have a positive moving average structure in DGP. Like the results for *DGP-I*, we observed that the *KB* estimator is a better choice for the Ng-Perron test when there is a positive moving average in DGP with nonzero lag truncations. The size distortion of the test is very small with the *KB* estimator even in small samples. On the other hand, the AR estimator gives huge size distortion for the small sample sizes, which could be as high as 60% in some cases. The effective power of the two estimators as shown in the right panel of Figure 5.9 is same for the smaller as well as for the larger sample size. Therefore, the estimator with a smaller size distortion should be preferred. Thus the *KB* estimator is preferred if there is a positive moving average. This conclusion matches with what we conclude for *DGP-I* in size is high with *AR* estimator with deterministic part consists both drift and time trend at the same time.

Figure 5.9 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = 0.20 for *DGP-II*



Figures 5.9 and 5.10 show the distortion in size and effective power of test for the AR and KB estimators at different lag length/ lag truncations. The Monte Carle experiment results are similar to DGP-1 for a negative moving average. In the large sample with a strong negative moving average structure size distortion is very high with both estimators at zero lag length/ lag truncation. There is a sharp decrease in the size distortion, approaching zero at lag 5 for the AR estimator with effective power at nearly 77%. On the other hand, the test with the KB estimator has a very low effective power i.e. maximum 35% power, and distortion in size is well above 40% for any sample size in the presence of a strong negative moving average.

Figures 5.11 and 5.12 shows that Monte Carlo the results obtained for the negative moving average also support our previous finding that the AR estimator is a better option in the presence of a negative moving average.

Based on our Monte Carlo results for the Ng-Perron test, we come to the conclusion that the nature of a moving average is important for the selection of an estimator for spectral density. Poor selection of an estimator may lead to incorrect inferences about the existence of a unit root.

Figure 5.10 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = 0.60 for *DGP-II*

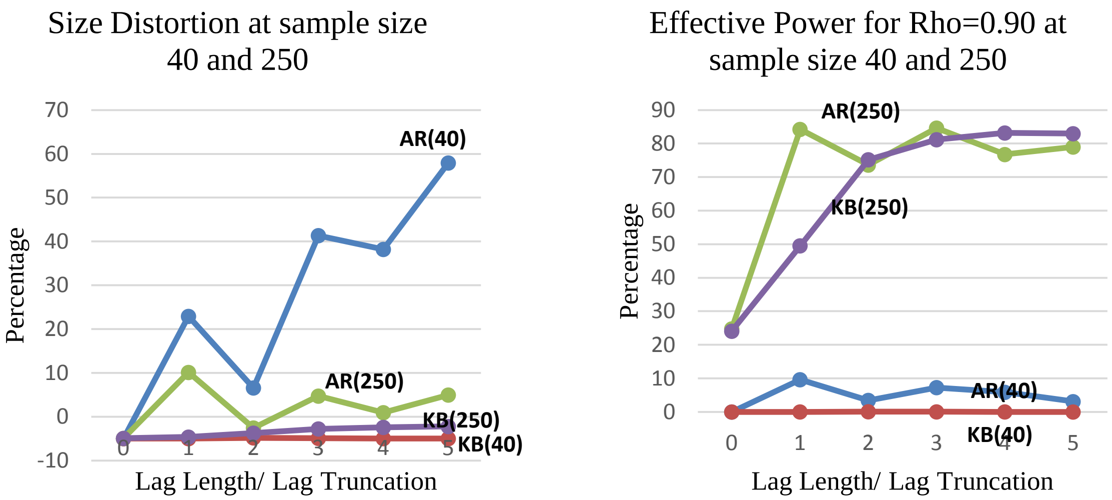


Figure 5.11 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = -0.20 for *DGP-II*

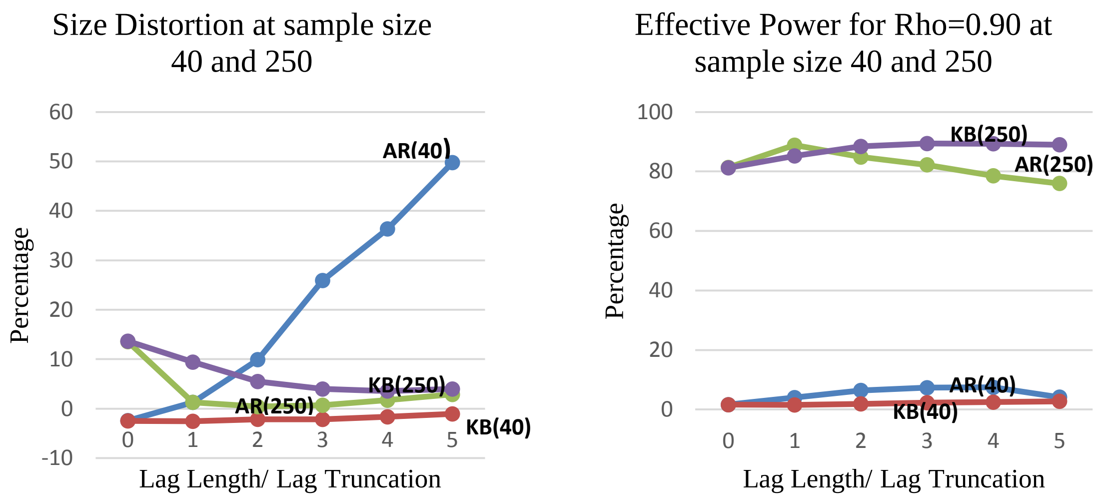
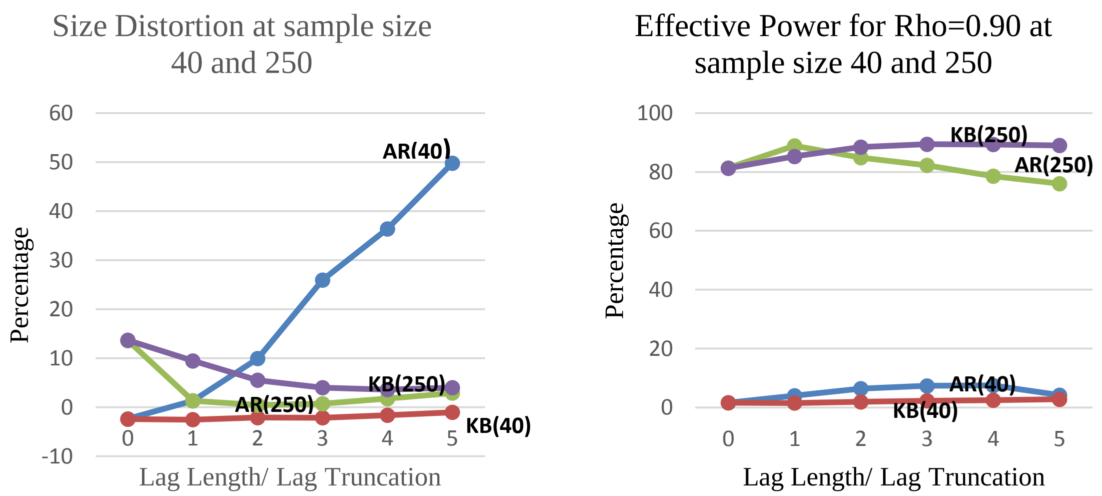


Figure 5.12 The Size Distortion and Effective Power Ng-Perron Test with *AR* and *KB* Estimator when *MA* = -0.60 for *DGP-II*



6. SUMMARY AND CONCLUSION

In this research we have evaluated the performance of the Ng-Perron test for the following choices of spectral density estimators:

1. Autoregressive estimator
2. Kernel based estimator with
    a. Bartlett Kernel
    b. Parzen Kernel
    c. Quadratic Spectral Kernel
    d. Tukey-Hanning Kernel

The simulation experiment was done on a variety of DGP's and for a wide range of parameter values. The simulation results reveal that the kernel based estimator with different kernels result in similar sizes and powers even in small samples. Hence we conclude that the choice of the kernel does not make any difference.

Further analysis reveals that if a data generating process contains a positive moving average, the kernel based estimator performs better, and for negative moving average, an autoregressive estimator performs better. In the preferred DPG's for the autoregressive estimator, huge size distortion occurs, whereas in the preferred range of DGP's for the KB estimator no size distortion was observed.

REFERENCES

Andrews, D. W. (1991). Heteroskedasticity and Autocorrelation Consistent Covariance Matrix estimation. *Econometrica*, 59 (3), 817-858.

Atiq-ur-Rehman (2011). Impact of Model Specification Decisions on Performance of Unit Root Tests. *International Econometric Review*, 3 (2), 22-33.

Dufour, J. M. and M. L. King (1991). Optimal Invariant Tests for the Autocorrelation Coefficient in Linear Regressions with Stationary or Nonstationary Errors. *Journal of Econometrics*, 47 (1), 115-143.

Elliott, G., T. J. Rothenberg and J. H. Stock (1996). Efficient Tests for an Autoregressive Unit Root. *Econometrica*, 64 (4), 813-836.

Libanio, G. A. (2005). Unit Roots in Macroeconomic Time Series: Theory, Implications, and Evidence. *Nova Economia Belo Horizonte*, 15 (3), 145-176.

Ng, S. and P. Perron (2001). Lag length Selection and the Construction of Unit Root Tests with Good Size and Power. *Econometrica*, 69 (6), 1519-1554.

Perron, P. and S. Ng (1998). An Autoregressive Spectral Density Estimator at Frequency Zero for Nonstationary Tests. *Econometric Theory*, 14, 560-603.

Phillips, P. C. (1987). Time Series Regression with a Unit Root. *Econometrica*, 55 (2), 277-301.

Phillips, P. C. and P. Perron (1988). Testing for a unit root in time series regression. *Biometrika*, 75 (2), 335-346.

Stock, J. H. (1990). A Class of Tests for Integration and Cointegration. Manuscript, Harvard University.

Stock, J. H. (1994). Unit Roots, Structural Breaks and Trends. *Handbook of Econometrics,* 4, 2739-2841.