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The Effect of Anisotropic Gaussian Schell-Model Sources in Generalized Phase Space Stokes Parameters

Serkan ŞAHİN *1

Abstract

Phase-space transforms describe spatial and angular information about light sources where one example is the Wigner functions in wave optics. Stokes parameters, on the other hand, supply information about the polarization of light beams. The generalized phase space Stokes parameters of 2D stochastic electromagnetic beams are already developed. In this article, the application of anisotropic light sources in generalized phase space Stokes parameters is theoretically investigated and numerically analyzed. There are several different ways of studying electromagnetic light beams depending on the spatial domain. But, most measure of the polarization of random light fields is carried out within the Stokes parameters context. In this account we study the electromagnetism, Stokes parameters, phase space, and the anisotropy properties of random light beams at once. We find here that when an anisotropy introduced in phase space then the cross terms of the Wigner matrix depart from the diagonal terms, which is not the same in configuration space. As a result, anisotropy has a different effect in Phase space, i.e. an anisotropic source introduces a phase and a variance change only in the cross terms of Wigner matrix. This is due to the use of anisotropy in the shifted kernel of Wigner transform.

Keywords: Optical coherence, phase space, Stokes parameters, Wigner function, physical optics

1. INTRODUCTION

A quasi-probability distribution is the Wigner distribution function (WDF) which was introduced in 1932 [1, 2] to attach quantum perspectives to classical statistical mechanics. Since then it is applied to many areas including optics. WDF has been extensively employed in both the classical and quantum regimes to characterize the spatial and temporal statistics of optical fields. The concept of the Wigner distribution function is not restricted to deterministic fields, it can be applied to stochastic fields as well [2]. The Wigner function has a number of noteworthy benefits for optical studies. One of them is its use in imaging where one can obtain a simplified WDF of image wave field in a partially coherent microscope [3]. Another one is its utilization in signal processing where one can obtain amplitude and phase retrieval, signal recognition and such easily via Wigner distribution function [4, 5]. Additional insight into the Wigner distribution can be one can achive wave field propagation through graded index media by means of Wigner distribution function [6]. In an experimental viewpoint, the Wigner

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distribution function helps in the measurements of optical field correlations [7]. Thus, the Wigner function is studied in optics in detail especially in connection with radiometry and partial coherence [8].

On the other hand, the use of random light beams can be favourable in graded index media where periodicity of the statistical properties of beams takes place [9]. As another option, it is known that the use of random light beams can improve the performance of free space laser communications across atmospheric turbulence [10, 11]. Main reason for these applications is the reduction in coherent interference, which is found to lower the intensity fluctuation (scintillation) at the receiver.

This paper focuses on anisotropic random light beams of spatially partially coherent, and we analyze its phase space through Wigner distribution function. The derivations pertain to the source plane. Following sections represent the organization of this paper's material. First we supply necessary information on the beam field and its propagation in the phase space context and then derive the Wigner transform of anisotropic Schell-model Gaussian beam. We obtain the generalized phase space Stokes parameters of anisotropic Gaussian Schell-model beam and finally compare it with configuration space generalized Stokes parameters.

2. MATERIAL AND METHOD

To develop the approach of anisotropic beams in phase space, we start with the two-point electric correlation matrix of the beam which is given in the form [12]

$$\overleftrightarrow{W}(\mathbf{r}_1,\mathbf{r}_2,\omega) = \begin{pmatrix} W_{xx}(\mathbf{r}_1,\mathbf{r}_2,\omega) & W_{xy}(\mathbf{r}_1,\mathbf{r}_2,\omega) \\ W_{yx}(\mathbf{r}_1,\mathbf{r}_2,\omega) & W_{yy}(\mathbf{r}_1,\mathbf{r}_2,\omega) \end{pmatrix}, \quad (1)$$

whose elements are $W_{ij}(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle E_i^*(\mathbf{r}_1,\omega)E_j(r_2,\omega)\rangle$, $(i,j = x,y)$. $E_x(\mathbf{r},\omega)$ and $E_y(\mathbf{r},\omega)$ are the elements of suitably constructed stationary, at least in the wide sense, statistical ensembles. $\mathbf{r}$ represents the two dimensional position vector on $\mathbf{r_1}$, $\mathbf{r_2}$ plane, ω is the temporal frequency and asterisk indicates the complex conjugate. We assume that the component of the electric field along z axis is neglected and only the two transverse field components $E_{x,y}(\mathbf{r},\omega)$ are necessary (and propagating on z axis).

The Wigner distribution function depends in conjunction on the canonical conjugate phase space variables, which for light beams are the transverse position vector $\mathbf{r}$ and the angular (spatial frequency) vector $\mathbf{k}$, and for optical pulses are the time variable t and temporal frequency ω. The Wigner function which depends on both $(\mathbf{r},\mathbf{k},t,\omega)$ can also be constructed [5]. We shall describe the Wigner phase space matrix by the integral of the shifted electric correlation matrix [2]

$$F_{ij}(\mathbf{r},\mathbf{k},\omega) = \int W_{ij}(\mathbf{r}+\frac{\mathbf{r}'}{2},\mathbf{r}-\frac{\mathbf{r}'}{2},\omega)e^{-i\mathbf{r}'\cdot\mathbf{k}}d\mathbf{r}', \quad (2)$$

which can be easily written in the matrix notation as

$$\overleftrightarrow{F}(\mathbf{r},\mathbf{k},\omega) = \begin{pmatrix} F_{xx}(\mathbf{r},\mathbf{k},\omega) & F_{xy}(\mathbf{r},\mathbf{k},\omega) \\ F_{yx}(\mathbf{r},\mathbf{k},\omega) & F_{yy}(\mathbf{r},\mathbf{k},\omega) \end{pmatrix}. \quad (3)$$

Equation [2] integrates the shifted electric correlation matrix Equation [1]. i throughout the paper represents the imaginary unit $\sqrt{-1}$. But in the subscripts $i,j = x,y$.

In the configuration (or parametric) space, the stokes parameters can be constructed by linear combinations of the electric correlation matrix [12]. In the same manner we build the phase space stokes parameters as given [13]

$$S_0(\mathbf{r},\mathbf{k},\omega) = F_{xx}(\mathbf{r},\mathbf{k},\omega) + F_{yy}(\mathbf{r},\mathbf{k},\omega), \quad (4)$$

$$S_1(\mathbf{r},\mathbf{k},\omega) = F_{xx}(\mathbf{r},\mathbf{k},\omega) - F_{yy}(\mathbf{r},\mathbf{k},\omega), \quad (5)$$

$$S_2(\mathbf{r},\mathbf{k},\omega) = F_{xy}(\mathbf{r},\mathbf{k},\omega) + F_{yx}(\mathbf{r},\mathbf{k},\omega), \quad (6)$$

$$S_3(\mathbf{r},\mathbf{k},\omega) = i[F_{yx}(\mathbf{r},\mathbf{k},\omega) - F_{xy}(\mathbf{r},\mathbf{k},\omega)], \quad (7)$$

Based on these equations, positional and directional intensities of the electromagnetic beams are the following (see [2] for scalar beams)

$$I_p(\mathbf{r},\mathbf{r},\omega) = \int S_0(\mathbf{r},\mathbf{k},\omega)d\mathbf{k}, \tag{8}$$

$$I_d(\mathbf{k},\mathbf{k},\omega) = \int S_0(\mathbf{r},\mathbf{k},\omega)d\mathbf{r}, \tag{9}$$

noting that they both depend on the first stokes parameter $S_0(\mathbf{r},\mathbf{k},\omega)$.

We will now consider the paraxial propagation of the WDF. The paraxial [2] propagation (see [14] for non-paraxial propagation), from an incident flat surface to a surface at z, corresponds to a linear transformation of the wigner function's initial argument, i.e. paraxial propagation reads

$$S_l(\mathbf{r},\mathbf{k},z,\omega) = S_l\left(\mathbf{r}-\frac{\lambda z}{2\pi}\mathbf{k},\mathbf{k},0,\omega\right), \tag{10}$$

for l ranges from 0 to 3. The constant involved is the wavelength λ. Another way of propagation analysis can be seen by assuming the medium is a first order optical system (with its ray transfer matrix), under this condition the WDF at plane z becomes [14]:

$$S_l(\mathbf{r},\mathbf{k},z,\omega) = S_l(A\mathbf{r}+B\mathbf{k},C\mathbf{r}+D\mathbf{k},0,\omega), \tag{11}$$

where the components of the ray transfer matrix ABCD are A, B, C, and D, and $A=1$, $B=-\frac{\lambda z}{2\pi}$, $C=0$, $D=1$ in free space as an example. WDF describes optical signals concurrently in spatial frequency and space. Geometrical optics' concepts of position and angle are analogous to this idea. This characteristic allows us to use the ray transfer matrix, often known as the ABCD matrix, to propagate light in phase space [15].

For an application of the theory we consider the general expression of the well known electromagnetic anisotropic Gaussian Schell-model beam (AGSMB) as given

$$W_{ij}(\mathbf{r_1},\mathbf{r_2},z=0,\omega) = \left(A_iA_j\right)^{\frac{1}{2}}B_{ij}\exp\left[-\frac{r_1^2}{4\sigma_i^2}\right]\exp\left[-\frac{r_2^2}{4\sigma_j^2}\right]\exp\left[-\frac{(r_2-r_1)^2}{2\delta_{ij}^2}\right] \tag{12}$$

In Equation [12] the parameters A_i, A_j, B_{ij}, σ_i, σ_j, and δ_{ij} are independent of location but they might depend on temporal frequency. The magnitudes of the electric field-vector components are A_i, A_j. σ_i^2 , σ_j^2 are the variances of the intensity dispersion through the source coordinates and the variances of the correlations between the elements of the electric field vector are δ_{ij}^2. Besides, the parameters Bij must satisfy the realizability conditions [16]: $B_{ii}=B_{jj}=1$, $B_{ij}=B_{ji}^*=B\exp[i\theta]$. The phase difference between two orthogonal electric field components is θ , and B is used for the modulus of correlation parameter.

Use of Equation [12] as a model beam first in Equation [2] then in Equations [4]-[7] constructs the phase space stokes parameters of anisotropic Gaussian Schell-model beams (we dropped the ω dependence for brevity):

$$S_0(\mathbf{r},\mathbf{k},0) = A_xB_{xx}\frac{\pi}{\Lambda_{xx}}\exp\left[-\left(\frac{\mathbf{r}^2}{2\sigma_x^2}+\frac{\mathbf{k}^2}{4\Lambda_{xx}}\right)\right] + A_yB_{yy}\frac{\pi}{\Lambda_{yy}}\exp\left[-\left(\frac{\mathbf{r}^2}{2\sigma_y^2}+\frac{\mathbf{k}^2}{4\Lambda_{yy}}\right)\right], \tag{13}$$

$$S_1(\mathbf{r},\mathbf{k},0) = A_xB_{xx}\frac{\pi}{\Lambda_{xx}}\exp\left[-\left(\frac{\mathbf{r}^2}{2\sigma_x^2}+\frac{\mathbf{k}^2}{4\Lambda_{xx}}\right)\right] - A_yB_{yy}\frac{\pi}{\Lambda_{yy}}\exp\left[-\left(\frac{\mathbf{r}^2}{2\sigma_y^2}+\frac{\mathbf{k}^2}{4\Lambda_{yy}}\right)\right], \tag{14}$$

$$S_2(\mathbf{r},\mathbf{k},0) = \left(A_xA_y\right)^{1/2}B_{xy}\frac{\pi}{\Lambda_{xy}}\exp\left[-\left(\frac{\mathbf{r}^2}{\Omega_{xy}}+\frac{2i}{4\Lambda_{xy}}\left(\frac{1}{4\sigma_y^2}-\frac{1}{4\sigma_x^2}\right)\mathbf{r}\cdot\mathbf{k}+\frac{\mathbf{k}^2}{4\Lambda_{xy}}\right)\right] + \left(A_xA_y\right)^{1/2}B_{yx}\frac{\pi}{\Lambda_{yx}}\exp\left[-\left(\frac{\mathbf{r}^2}{\Omega_{yx}}+\frac{2i}{4\Lambda_{yx}}\left(\frac{1}{4\sigma_x^2}-\frac{1}{4\sigma_y^2}\right)\mathbf{r}\cdot\mathbf{k}+\frac{\mathbf{k}^2}{4\Lambda_{yx}}\right)\right], \tag{15}$$

$$S_3(\mathbf{r},\mathbf{k},0) = i\left[\left(A_xA_y\right)^{1/2}B_{yx}\frac{\pi}{\Lambda_{yx}}\exp\left[-\left(\frac{\mathbf{r}^2}{\Omega_{yx}}+\frac{2i}{4\Lambda_{yx}}\left(\frac{1}{4\sigma_x^2}-\frac{1}{4\sigma_y^2}\right)\mathbf{r}\cdot\mathbf{k}+\frac{\mathbf{k}^2}{4\Lambda_{yx}}\right)\right] - \left(A_xA_y\right)^{1/2}B_{xy}\frac{\pi}{\Lambda_{xy}}\exp\left[-\left(\frac{\mathbf{r}^2}{\Omega_{xy}}+\frac{2i}{4\Lambda_{xy}}\left(c-\frac{1}{4\sigma_x^2}\right)\mathbf{r}\cdot\mathbf{k}+\frac{\mathbf{k}^2}{4\Lambda_{xy}}\right)\right]\right], \tag{16}$$

where Λ_{ij} and Ω_{ij} are

$$\Lambda_{ij} = \frac{1}{16\sigma_i^2}+\frac{1}{16\sigma_j^2}+\frac{1}{2\delta_{ij}^2}, \tag{17}$$

$$\frac{1}{\Omega_{ij}} = \frac{1}{4\sigma_i^2} + \frac{1}{4\sigma_j^2} - \left[\frac{1}{\left(4\sigma_i^2\right)^2} + \frac{1}{\left(4\sigma_j^2\right)^2} - \frac{1}{8\sigma_i^2\sigma_j^2}\right]\frac{1}{4\Lambda_{ij}}, \quad (18)$$

We note that, for isotropic ($\sigma_i = \sigma_j = \sigma$) Gaussian Schell-model beams our Equations [13-18] simplify to Equations [14-18] of refence [13].

This allows one to investigate the evolution of the generalized phase space Stokes vector $S_l(\mathbf{r}, \mathbf{k}, z, \omega)$ through out free space. We believe such methodologies developed for phase space in the optical coherence theory can be adopted to investigate the change of the statistical states of light beams in free space and atmospheric scenarios.

The study of the phase space characteristics of random electromagnetic beams is quite widespread recently. The statistical condition of an electromagnetic beam has long been satisfactorily described using the standard Stokes parameters. When used to characterize both the coherence and the polarization characteristics of electromagnetic fields, recently proposed generalized phase space Stokes parameters, which are thought of as a two-direction extension of the typical Stokes parameters, can be written in terms of the correlations between electric field components at two directions. The acquired results make it easier to handle the phase space transformation's propagation behavior.

3. THE RESEARCH FINDINGS AND RESULTS

We first notice that Equations [15]-[16] are different than Equations [13]-[14] in the sense that there is an additional phase introduced. And they are different from the parameters obtained in [13] for isotropic Gaussian-Schell model beams. A similar case is seen in the complex Gaussian representation of partially coherent sources on propagation [17]. Second, this is not the case in the configuration space, i.e.

$$S_0(\mathbf{r}_1, \mathbf{r}_2, \omega) = W_{xx}(\mathbf{r}_1, \mathbf{r}_2, \omega) + W_{yy}(\mathbf{r}_1, \mathbf{r}_2, \omega), \quad (20)$$

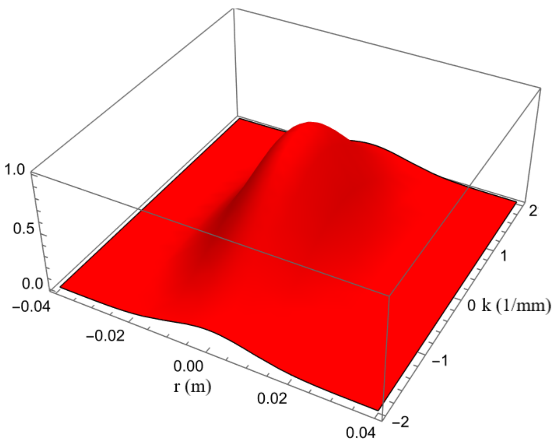
$$S_1(\mathbf{r}_1, \mathbf{r}_2, \omega) = W_{xx}(\mathbf{r}_1, \mathbf{r}_2, \omega) - W_{yy}(\mathbf{r}_1, \mathbf{r}_2, \omega), \quad (21)$$

$$S_2(\mathbf{r}_1, \mathbf{r}_2, \omega) = W_{xy}(\mathbf{r}_1, \mathbf{r}_2, \omega) + W_{yx}(\mathbf{r}_1, \mathbf{r}_2, \omega), \quad (22)$$

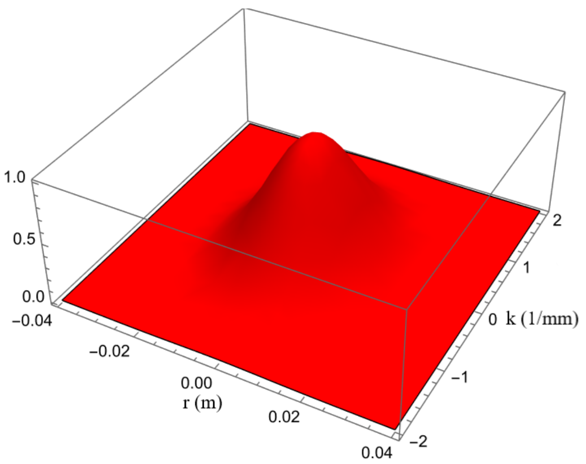
$$S_3(\mathbf{r}_1, \mathbf{r}_2, \omega) = i\left[W_{yx}(\mathbf{r}_1, \mathbf{r}_2, \omega) - W_{xy}(\mathbf{r}_1, \mathbf{r}_2, \omega)\right], \quad (23)$$

are all having the same functional forms (form of Equation [12] in this case) and same scalings [12]. Thirdly, Equations [15]-[16] have modified variances across the position vector r only, which is to say the position variance is rescaled. As a fourth notice, this rescaled variance has the imprint of the correlations δij. To compare $S_0(\mathbf{r}, \mathbf{k}, 0)$and $S_2(\mathbf{r}, \mathbf{k}, 0)$, we plot Equation [13] and Equation [15] in Figure 1.

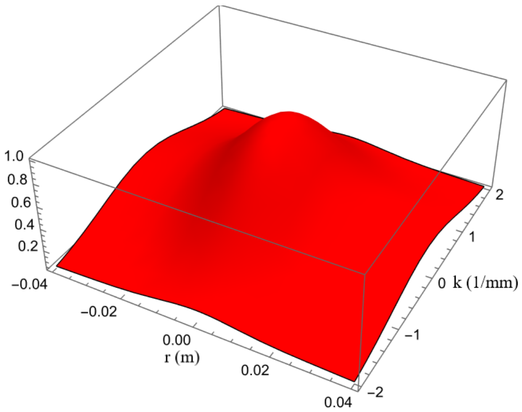
Since S_1 and S_3have the similar results their figuration is not presented for brevity. In the passage from Figure 1 (a) to Figure 1 (b) we see that the variance changes in **r** dimension. This is not very well pronounced in S_2 and S_3 which is because S_2 and S_3 are more symmetric in variances comparing to S_0 and S_1. In the analysis of the parts of Figure 1 we see that there is a rise in the variance with respect to the increment of anisotropy ratio α. In the numerical calculations it is worth mentioning that the phase space Stokes parameters might take negative values, since the Wigner functions are purely real, but they are not always positive [18]. The reason of the asymmetry between



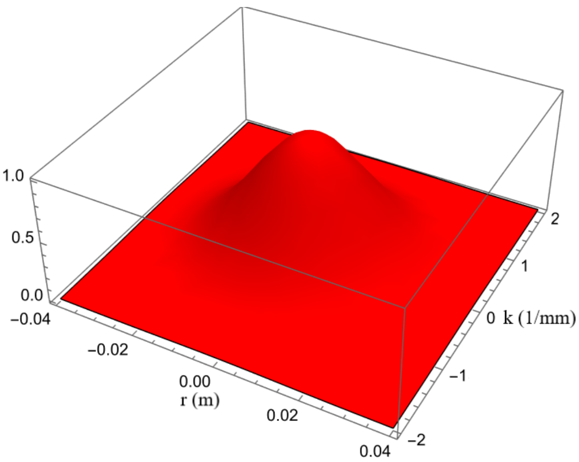
(a) S_0 for $\alpha = 1$.



(b) S_2 for $\alpha = 1$.



(c) S_0 for $\alpha = 4$.



(d) S_2 for $\alpha = 4$.

Figure 1 Changes in the generalized phase space Stokes parameters S_0 and S_2, calculated from Equation 14 and Equation 16, of anisotropic electromagnetic Gaussian Schell-model beams. The parameters are taken as: $A_x = A_y = 1$, $B_{xx} = B_{yy} = 1$, $B_{yx} = B_{xy} = 0.8$, $\theta = 0$, $\sigma_y = 1$ mm, $\sigma_x = \alpha\sigma_y$, $\delta_{xx} = \delta_{yy} = 1$ mm, $\delta_{xy} = \delta_{yx} = 1.5$ mm.

Equations [13]-[14] and Equations [15]-[16] is; we now introduced an anisotropy in the kernel (shifted electric correlation matrix) of Wigner transform which causes a phase and a rescaling in the cross terms F_{xy}, F_{yx} and so in S_2, S_3. Lastly, note that the coherence widths along x and y directions are taken as equal in this study, they can be taken differently as well and would be a place of another study. In such a case, we would expect an extra variance scalings on the direction of seperate fields.

4. DISCUSSION

With the help of derived analytics we have studied the spectral changes of anisotropic Gaussian Schell-model sources in generalized phase space Stokes parameters. On comparing Figure 1 (a) and Figure 1 (c) we see that the effect of anisotropy ratio on S_0 is more noticeable, however the comparison of Figure 1 (b) and 1 (d) shows us that the effect of anisotropy ratio on S_2 is dim. The main comparison, i.e., Figure 1 (a) vs Figure 1 (b), shows a profound change because of the phase and the variance in the obtained generalized phase space Stokes parameters. Therefore, the results demonstrate that the changes in the statistical properties can be influenced by the phase space transform.

Consideration of the complete statistics, i.e. amplitude, phase and polarization, is necessary for full field description of the light field. This is because the first Stokes parameter, S_0, is a measure of the total intensity of the light incident on the detectors. The second and third Stokes parameters, S_1 and S_2, provide information about the linear polarization state of the incident light. The fourth Stokes parameter, S_3, provides information about the circular polarization state of the incident light.

5. CONCLUSION

In this paper, generalized phase space Stokes parameters of anisotropic Gaussian Schell-model sources are derived. Effect of the anisotropy ratio α is plotted and analyzed. The Wigner transform results in different ways in the four Stokes parameters because of the shift in the transform. Based on the formulas obtained we see that the anisotropy of the source causes a modified variance in the last two Stokes parameters. The calculations show that the variance changes in r dimension in the Stokes parameters S_2 and S_3. And also, we have found that the modified variance has the imprint of correlations. Some areas of interest where this theory can be efficiently used are partially coherent imaging systems [3], optical heterodyne imaging [19], and tomography [20].

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The Declaration of Research and Publication Ethics
The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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