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COMPUTER-AIDED DESIGN, MANUFACTURE AND EXPERIMENTAL ANALYSIS OF A PAIR OF ELLIPTICAL SPUR GEARS

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ABSTRACT

In this study, geometrical equations of elliptical spur gears, which are too difficult to manufacture by traditional methods and which require specific machines equipped with special techniques, are developed using the methods in the literature. Using these equations, a LISP program on AutoLISP is created to model elliptical spur gears on AutoCAD with desired tooth number and modules. Elliptical spur gears are manufactured with *5* different modules by Wire EDM through the above-mentioned package program. The variations in the center distances of elliptical spur gears, the most important parameter for workability of gears, are experimentally determined by a simple test unit designed and manufactured within the context this study. In addition, the surface roughness and hardness of elliptical spur gears are obtained and hydraulic pump and noise analysis results are discussed. The experimental and computer-aided results show that the elliptical spur gears may widely be used in many industrial and mechanical applications in the future. In Turkey, a general analysis of elliptical teeth was conducted in order to shed light on future work in this area.

Keywords: Non-circular gears, Elliptical spur gears, CNC Wire EDM, Hydraulic pump

1. INTRODUCTION

Anon-circular gear (NCG) is a special gear design with special characteristics and purpose. While a regular gear is optimized to transmit torque to another engaged member with minimum noise and wear and with maximum efficiency, a non-circular gear's main objective might be ratio variations, axle displacement oscillations and more. Common applications include textile machines, potentiometers, CVTs (continuously variable transmissions), window shade panel drives, mechanical presses and high torque hydraulic engines.

As well as in comprehensive mathematical studies [1], non-circular gears have been historically used in clocks, musical instruments, automatic toys, key making machines, Geneva mechanisms and pumps [2]. A formulation based on envelope theory is also proposed to obtain involute tooth profiles [3].

Non-circular gears are generally preferred in multi-dimensional machines, such as paper bag machines, flow rate meters, typewriters, parceling machines, labeling machines, textile machines, in direct current meters, in robots and in calculators and cigarette machines [4]. Their other applications are in non-circular potentiometers for the control of electro-mechanical systems and in the transfer, design and measurement of continuously changing movement and force with low friction [5]. Non-circular gears have also been used in mechanisms with constant input but varying output, in the oscillators with various frequencies, in the ground-service equipments of space shuttles and in the shuttles themselves [6].

Some computer programs have been developed to draw the tooth profiles of rack cutters and elliptical spur gears. It has been observed that the under-cutting lines of elliptical spur gears have been examined, and that the tooth profiles of gears produced by rolling method and those of the elliptical spur gears cut on Wire EDM have been compared [7]. Some studies concerning the production of

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elliptical spur gears deal with the theoretical models with one cutter that makes rolling movement only on the pitch curve [8]. Recent developments in computer-based designs and in CNC technologies have made the production of elliptical spur gears more economical and more efficient [9].

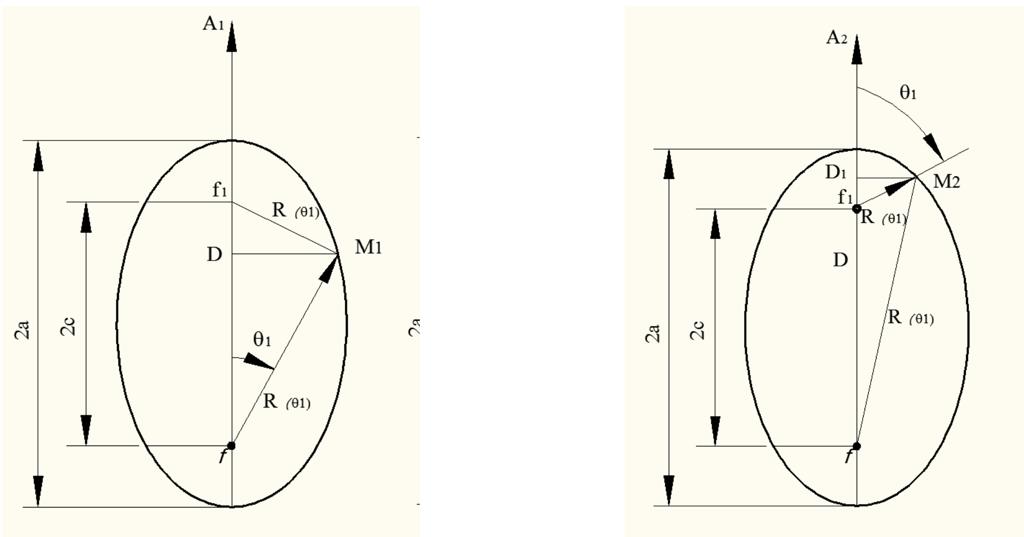
Non-circular gears are known to be used in various mechanisms in order to achieve unusual movements and velocities [10]. When compared to cam mechanisms, these mechanisms driven by non-circular gears provide simpler, more reliable and more sensitive solutions. Elliptical gears are produced more cheaply than cams [11] and they are very economical in comparison to servo systems that require expertise. Non-circular gears have been mainly studied to drive various bar mechanisms and their motion analyses are made when irregular movements are obtained [12]. The integration of five-bar linkages with non-circular gears has been proposed for the synthesis of precise path-generating mechanisms [13].

Solutions of geometrical equations on computers have been made in CVT (continuously variable transmission) systems, in addition to both theoretical [14] and empirical studies.

In this study, the design of the systems with elliptical spur gears are produced with *5* different modules and tooth numbers with the software on AutoCAD using the studies in the literature. *2-D* design data are transformed into cutting codes through MasterCAM, different CNC lines are developed for each pair of gears and *5* pairs of gears are manufactured on Wire EDM. The variations in the center distances of elliptical spur gears, which are not encountered in literary studies but enable gears work without interruptions, noise and with less wear, are measured by a simple test unit developed.

2. COMPUTER-AIDED DESIGN OF ELLIPTICAL SPUR GEARS

The curve circling the surfaces obtained by cutting inclined to the bottom of cylindrical and conical is called an ellipse. The points that create the distance from the two focal points are fixed and equal to the bigger axis. The geometric positions of these points are called elliptical coordinator. Polar equations of elliptical curves are calculated with the following equations (Figure 1):



a) b)

Figure 1. a) The polar axis is $\overline{fA_1}$ and the position vector $\overline{fM_1} = r_1(\theta_1)$ **b)** The polar axis is $\overline{f_1A_2}$ the position vector $\overline{f_1M_2} = r_1(\theta_1)$ [15]

"If Figure 1a. is selected polar equation of an ellipse is:"

$$R = \frac{P}{1 - e\cos(\phi)} \quad \text{P, e} \in \mathbf{R}, \tag{1}$$

"If Figure 1b. is selected polar equation of an ellipse is:"

$$R = \frac{P}{1 + e\cos(\phi)} \tag{2}$$

"where R=real numbers e value in this equation is:"

$$e = \frac{\sqrt{a^2 - b^2}}{a} \tag{3}$$

"f value in this equation is:"

$$f = \sqrt{a^2 - b^2} \tag{4}$$

"On the other hand;"

$$P = a.(1 - e^2) \tag{5}$$

In this study, elliptical spur gears are designed by rolling method and rack cutter. Since other methods of gear production are not dealt with, only the data of rack cutters on CAD are used.

Tooth surfaces of rack cutters are composed of two angular and symmetrical lines that constitute the *(α)* angle according to the main axis of *2-D* coordinate system. Complex numbers are used to obtain mathematically the coordinates of the cutter in *2-D* coordinate system (Figure 2) and the following equations are obtained:

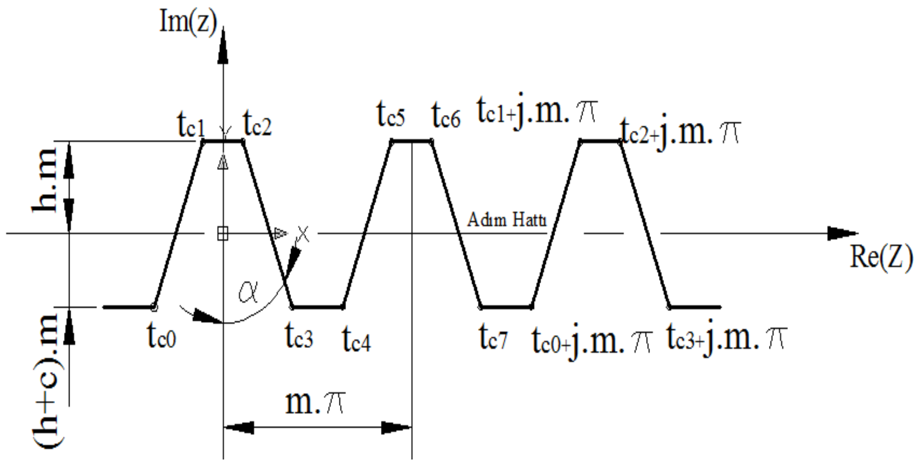


Figure 2. *2-D* model of cutter profile

$$\left.\begin{aligned}
t_{c0} &= m.\left(h_1 - i.\left(\frac{\pi}{4} + h_1.\tan(\alpha)\right)\right)\\
t_{c1} &= m.\left(-h_2 - i.\left(\frac{\pi}{4} - h_2.\tan(\alpha)\right)\right)\\
t_{c2} &= m.\left(-h_2 + i.\left(\frac{\pi}{4} - h_2.\tan(\alpha)\right)\right)\\
t_{c3} &= m.\left(h_2 + i.\left(\frac{\pi}{4} - h_2.\tan(\alpha)\right)\right)
\end{aligned}\right\} \tag{6}$$

where $i^2 = -1$, the pitch line of the cutter coincides with the complex axe. The fillet radiuses of standard tool are neglected in this case. “Complex co-ordinates of s^{th} corner point in the rack are:”

$$t_s = t_{s-4} + i.m.\pi \tag{7}$$

$s = 4,5,\ldots\ldots, 4z-1$ where $z = z+1$, is the total number of teeth of basic rack.

The complex coordinates at s^{th} corner points of the teeth of the pitch cutter:

$$R_M = (t_s - i.L).e^{(I\mu)} + X + i.Y - f \tag{8}$$

“The length of elliptical pitch curve is:”

$$L = \int_0^{2\pi} \sqrt{R^2 + \left(\frac{\partial}{\partial\phi}.R\right)^2}\, d\phi = m.\pi.Z \tag{9}$$

$$X = R.Cos\phi \tag{10}$$

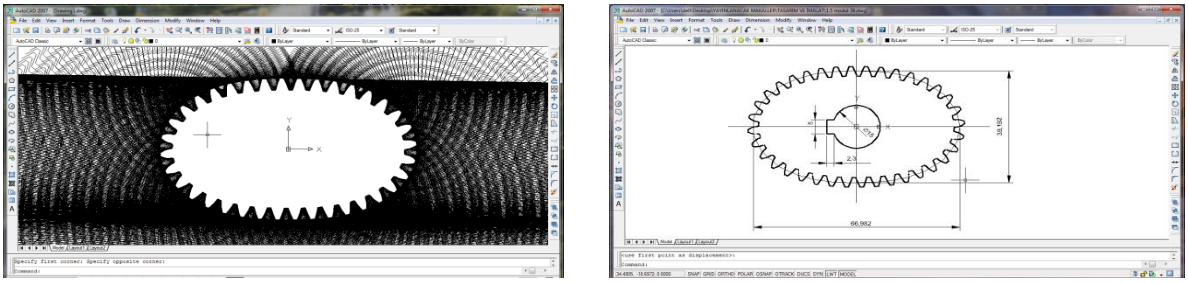
$$Y = R.Sin\phi \tag{11}$$

“and curve angle of tangential lines is:”

$$\mu = \frac{\pi}{N} \qquad [2, 15] \tag{12}$$

A module program is developed for *2-D* design of elliptical spur gears at required tooth number and modules on AutoCAD by transferring equations to AutoLISP.

In Figure 3. is the *2-D* image of the elliptical spur gear developed on AutoCAD according to the rolling method with AutoLISP program where a=*33,491* mm, b=*19,596* mm, z=*36* and m=*1,5*.



a) b)

Figure 3. a) Design of elliptical spur gear with rolling method on AutoCAD
b) 2D design of elliptical spur gear on AutoCAD

3. COMPUTER-AIDED MANUFACTURE OF ELLIPTICAL SPUR GEARS

In order to obtain standard modules, the tooth number and *"a"* and *"b"* values of the ellipse are determined. The reason for choosing standard modules is to compare the elliptical spur gears with those produced by the methods other than Wire EDM. The values of gear parameters from the LISP program that form the gear profile of elliptical spur gears according to the rolling method are given in Table 1 and the features of the materials from which gears are produced are given in Table 2. The codes are developed to process the elliptical spur gears whose design data are transferred onto MasterCAM and the pairs of the elliptical spur gears are produced from 8 mm thickness of *AISI 1040*, which is widely used in gear manufacture.

Table 1. Parameters that belong to elliptical spur gears cut for experimental studies

Module, ***m***	Tooth number, ***Z***	***a*** value of ellipse,	***b*** value of ellipse,	Rate, ***a***/***b***	Eccentricity, **e**
1.50	36	33,491	19,596	1,709	0,810
1.75	33	35.444	21.441	1,653	0,796
2.00	32	39.649	23.300	1,701	0,809
2.25	31	43,199	25.399	1,700	0,808
2.50	30	46,451	27.302	1,701	0,809

Table 2. Chemical compositions of carbon steels

	C	**Si**	**Mn**	**P**	**S**	Fields of Usage
AISI 1040	0.37 0.44	0.15 0.35	0.60 0.90	0.040	0.050	In the production of transmission pins, rails, gears etc. It can be hardened by induction and flame.

A *600 x 375 mm* plate made of *AISI 1040* is surface milled, grinded and hardened at *860* ^{0}*C*. The hardness is measured with the TIME Portable Hardness Tester (TH130) as *197 HV (kgf/mm2)* before processing at Wire EDM (Figure 4). These hardness values are compatible with the application values. The pairs of elliptical gears produced at five different modules and tooth number are given Figure 5.

Model	TIME Portable Hardness Tester (TH130)
Measuring range	HV (80-940)
Accuracy	± 0.5 %
Minimum thickness of hardened layers	0.8 mm

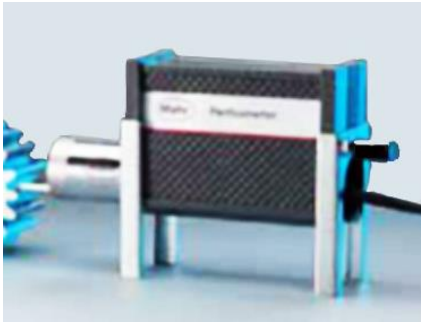
a) b)

Figure 4. a) Portable Hardness Tester **b)** Technical Specifications

Figure 5. The pairs of elliptical spur gears manufactured

In order to form a reference and convenience for future empirical studies, the elliptical spur gears are designed with the hub keyway at *5 mm* width and *2.3 mm* depth and the hubs with keyway are designed on AutoCAD. Cutting process is done on Fine Sodick A320D-EX21 Wire EDM. Taking industrial uses into account, C000 cutting codes are chosen. Dielectric liquid circulation pressure is $6\ kg/cm^2$, CuZn37 Master Brass wire is used with $\phi\, 0.25$ and $900\ N/mm^2$ tensile strength.

Firstly, the hub with keyway and then the gears that form along the elliptical pitch of each gear are manufactured on Wire EDM. In this manner, the axis passing through the rotation centre of the elliptical spur gears and the planes that form the side face of elliptical spur gears are made parallel. Surface roughness of the teeth are measured by Mahr Perthometer M1 from 3 points and the average surface roughness is *1,5 Ra (µm)* (Figure 6). It can be stated that a really smooth surface is produced in this way when compared with those of the grinned gears.



Model	Perthometer M1 (Mahr)
Measuring range	150 μm/s
Measuring Force	0,7mN
Contact point	0,8 mm
Traversing length	5,6 mm

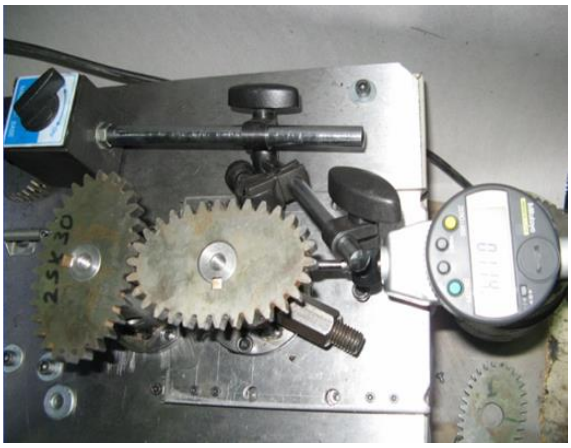
a) b)

Figure 6. a) Surface roughness tester **b)** Technical specifications

4. THE CENTER DISTANCE VARIATIONS OF ELLIPTICAL SPUR GEARS

An experimental apparatus is designed and produced in order to measure the variations in center distances of the elliptical spur gears manufactured (Figure 7a.).

For each module, the variation values of the pair of elliptical spur gears are measured by a digital dial indicator of *1/1000 mm* precision (Figure 7a.). The largest and the smallest deviation values of the elliptical spur gears manufactured are given in Figure 7b.



Modules	Minimum variation of center distance, A_{min}, μm	Maximum variation of center distance, A_{max}, μm	Total variation of center distance, A_t, μm
1.50	-2	129	131
1.75	-1	133	134
2.00	-6	135	141
2.25	-8	138	146
2.50	-10	140	150

a) b)

Figure 7. a) Test apparatus for the center distance variations of the elliptical spur gears, **b)** Limit values measured

In Figure 8, the experimental results of the elliptical spur gears with five different modules and with different tooth numbers show the systematic increase and decrease at certain angular positions.

Figure 8. The variations in center distances versus angular position for elliptical spur gears

For noise analysis of elliptical spur gears, *DT-2236* Digital Photo/Contact Tachometer is used to measure the number of revolutions. Measuring range of the device is *0,5-100.000 rev/min* and accuracy rate is *% 0,05* (Figure 9). Revolution is measured with digital tachometer on the gear shaft and CASELLA CEL-231 Digital Sound Level Meter is used to measure noise. Measuring range of the device is *30-135 decibel (dB).* Revolution and noise measurements are done three times, and noise values of each revolution are determined based on arithmetic average of these three results. This process is applied to the five pairs of gears and is shown in Figure 10. It can be seen from the figure that as the number of revolutions and modules increases, so does the value of the noise.

Figure 9. Noise test of elliptical spur gears

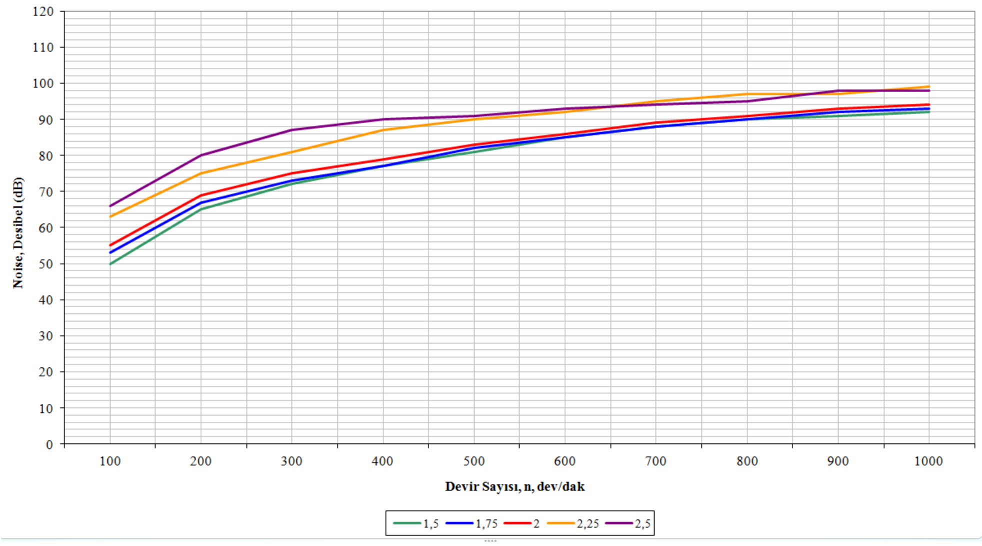


Figure 10. Noise graphic of elliptical spur gears

6. TEST OF SUPER GEARS USING A HYDRAULIC PUMP

Hydraulic pumps are used at various structures and features depending on the requirements of the hydraulic systems. It is known that pumps are widely used for transferring hydraulic power. Hydraulic spur gear pumps with high pressure can also be used as hydraulic motors due to thier cost-efficiency and efficiency.

The fact that spur gear pumps and motors have been widely used in industrial applications and that elliptical spur gears have gained priority in design and manufacture of fluxmeters have all led to the examination of the main characteristics of a pump comprising of a pair of elliptical spur gears in this section.

A pair of elliptical spur gears is produced on CNC wire EDM with *2* modules, z=*22*, *a= 28,02 mm*, *b=15,02 mm* and *10 mm* tooth width.

The stem design is made with the help of Solidworks in compliance with the gears produced axis work gap is given *c+0,01 mm* for elliptical spur gears in pump designs. In addition, axis numerical work gap is suggested to be *0,2xm* for gear pumps in literature. Aluminium stem is produced on CNC milling cutters according to the measurements of the pump produced (Figure 11).

Figure 11. Elliptical spur gear pump produced

The flow rates of the pump produced are made at *7* different speeds by using Festo hydraulic circuit elements, *DT-2236* Digital Photo/Contact tachometer and hydraulic motor oil. The equation and R^2 value obtained as a result of the measurements are given in Figure12.

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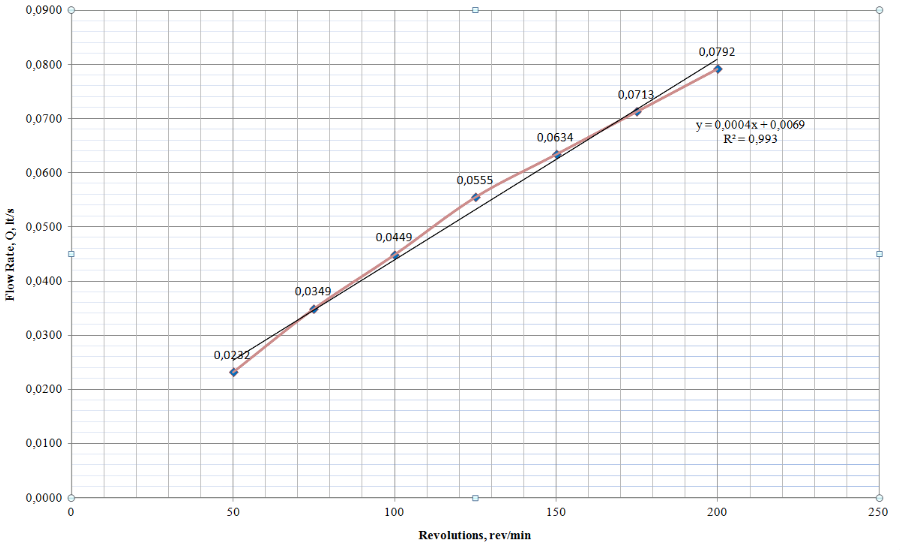

Figure 12. The flow rate graphics of elliptical spur gear pump

On the other hand, the pressure variations of the elliptical spur gear pump are made at *6* different speeds by using hydraulic circuit elements and pressure adjustment valve. The highest pressure is measured to be *10* bars at *175 rev/min*, which is the highest speed the experimental mechanism provided. The graphics of the values obtained at pressure experiment and the equation of the linear line and R^2 are given in Figure 13. and experimental study is shown in Figure 14.

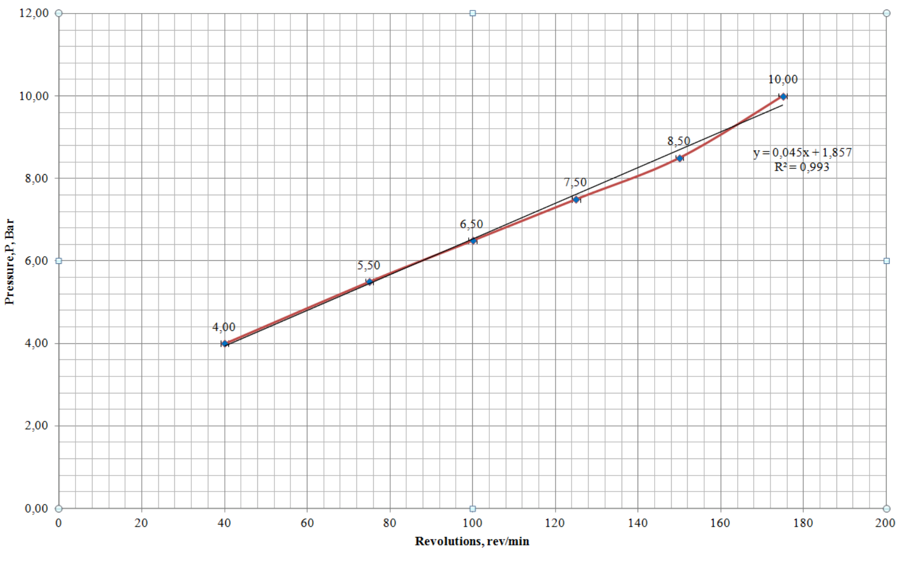

Figure 13. Pressure graphics of elliptical spur gear pump

Figure 14. Experiment of pressure variations at elliptical spur gear pump

Experiments on elliptical spur gear pumps produced are made at slow speeds between *50 rev/min* and *200 rev/min*. The reason for this is that the recent works on elliptical gear pumps have generally been on medicine, and especially on artificial hearts and kidney machines. On the other hand, elliptical gears have widely been used to measure higly sensitive and high-speed flow rates in hydraulic systems.

7. CONCLUSION

Instead of classical circular gears, which are inevitable for mechanical systems, the design, production and workability tests of elliptical spur gears, which are the most important elements of the non-circular gears, are carried out in this study.

An internationally-accepted method is used for the design and manufacture of elliptical spur gears. Using polar equations of elliptical curves together with the complex numbers, elliptical spur gear is designed and manufactured on CAD with "AutoLISP". The manufacture of elliptical spur gears with Wire EDM provides the opportunity to use some manufacturing methods such as moulding.

Apart from the general conclusions mentioned above, the results obtained from this study can be listed as below:

- The surface roughness of elliptical spur gears produced is remarkably low when compared to that of the gears produced by traditional methods (Ra=1,5 μm). This value proves that elliptical spur gears hardened before the CNC Wire EDM process can be used at direct movement and speed systems.
- Similar graphics of the variations in center distances are observed at all module values and tooth numbers of elliptical spur gears, which indicates that the design and manufacture faults are systematic. The fact that the difference between the highest and lowest variations in center distances is 146 μm shows that the elliptical spur gears can work.
- As the value of the modules increase, so do the value of the center distance variations.
- The reasons for the high values of center distance variations are as follows;
- Tooth profiles are designed with short lines
- 1^o angle is taken in radius calculation
- Removal of rough metal is done only once without sensitive cutting.

- It is thought that the high values of center distance variations will remarkably decrease if tooth profiles are obtained from tangential circles with 0.5^o angle.
- Considering that the accepted top limit of the backlash (maximum permissible variation in center distance) with mate for the cylindirical gears is 0,025.*m*, it is observed that the backlashes with mate of all the elliptical spur gears produced are under this limit.
- The module-based change of the biggest variation value of the center distance is linear. It is made possible to find, depending on the increase in the modules, the variations in center distance through e = 5,6 . *m* + 197.
- As the number of revolutions increases, so does the peripheral speed of gears, which will eventually cause an increase in the noise level. Therefore, future studies related to the surface roughness, hardness and friction of gears will be of great significance.
- Small pressure angle decreases radial forces, which makes the mechanism work more silently. The effect of different pressure angles may be the subject of another study.
- There will always be a little gap between gears even after a highly sensitive work and assembly. As this gap will be necessary for oiling, and since there will be some changes and replacements in the profiles of the teeth, this gap should be considered during design.
- Non-circular gears are not widely used at industry since their production and operation procedures are not well known. As a result, few international firms produce this type of gears. However, non-circular gears have a number of advantages with respect to eccentricity and linkages. Non-circular gears can transfer bigger forces in comparison to eccentricities and their links can be balanced more easily and accurately. This is significant especially to the high-speed and highly-sensitive machines.
- Elliptical spur gear with *2*-module, *22*-tooth and whose *a/b = 1,867* is placed into the hydraulic pump design. The main characteristic curves of the hydraulic pump is obtained. With this experiment, it is proved that elliptical gear pumps can work properly even at highly low speed rates and can be used at high flow rates in comparison with cylindrical spur gear pumps. This result is highly significant for artificial hearts and dialysis machines. It can be possible to produce more efficient elliptical spur gear pumps by using different materials and different gear dimensions.
- The increase in the number of revolution will lead to the increase in the circular speed of elliptical spur gears, which will also results in an increase in the sound level and abrasion. Therefore, future research and experiments will be important on the surface quality, hardness, and abrasion of elliptical spur gears that work in high speeds. In addition, elliptical spur gears can measure extremely sensitive and high speed flow rates.

Depending on the literary studies, it is expected that the kinematic analysis and computer-aided designs of the elliptical pitch curves will increase in the future. In Turkey, a general analysis of elliptical teeth was conducted in order to shed light on future work in this area.

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