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Research Article

A test for identification of math talent: developing a three-tier number sense test

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Abstract

Solving math problems requires inferential thinking skills, improved number sense, problem-solving strategies, deductive reasoning and computational skills. Mathematically talented students often use advanced number sense strategies to get the fastest and most accurate result. Although the existence of the number sense is known, it is difficult to describe it concretely. To materialize the intangible concept of number sense, a three-tier number sense test for secondary school students was developed and validated in this study. The developed test was carried out to 499 students studying in middle school in İzmir in Turkey. The first tier of the test consists of 25 multiple-choice mathematics questions. The second tier consists of the reason tier, which includes responses to the questions in the first tier (number sense-based, rule-based, misconception and guesswork). The third tier includes the confidence question, which measures the belief in the correctness of the response given to the question. The reliability of the test was calculated as .74 with the KR-20 formula. From the results of the analysis, it can be considered that the developed test is a valid and reliable measurement tool that can be used to determine the number sense levels of the students.

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Introduction

The specialists suggest that instead of the traditional approach in mathematics education, which has been adopted for a long time, which provides education by assuming the whole group equally, it is necessary to place the children in the right group in the classroom according to their individual characteristics and needs. The groups should be in a dynamic structure according to the learning speed of the young people and when necessary, the transition of the children between the groups should be ensured (New Trends in Mathematics: UNESCO, 1977, p. 96-97). The whole evaluation process with scales and tests is actually made for a definite and clear prediction about the student. Such a prediction is necessary in order to follow up more individuals with systematic observations and to be a guide in education and future studies (Hrich et al., 2019). The requirements for recognizing students' mathematical abilities by testing can be explained as follows; educational arrangement, identifying the areas of difficulty, preparing an individualized education program, following the development, assisting in future decisions such as career choice, comparing individuals according to certain norms and criteria, understanding the way of thinking mathematically (Chinn & Ashcroft, 1993, p. 18-19).

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According to researchers such as Gardner (1983) and Harling & Roberts (1988), some people are gifted in logical-mathematical thinking. Individuals with mathematical talented argue that they are rarely well understood in terms of mathematical thinking. Therefore, measuring mathematical ability is an important requirement (Gardner, 1983, p. 155).

In order to create the most appropriate goals and behaviors in a student, it is necessary to define the characteristics of the student instantly and in the process. A good educational assessment and evaluation tool should include questions about what the student knows and what he/she does not know, as well as how and why he/she learns (Whitfield, 1987, p. 151). Given these explanations, multi-tier tests are useful tools for assessing students' abilities.

Mathematical Talent and Number Sense

Wirtz (1974) noted long ago that number sense is difficult to define but easy to recognize. There are different definitions for number sense in the literature. Howden (1989) defines number sense as “the ability to grasp the meanings of numbers, develop multiple relationships between numbers, recognize the relative magnitudes of numbers, and know the relative effect of operations on numbers” (p. 6). McIntosh, Reys, and Reys (1992) defined number sense as “a person's general understanding of numbers and operations, and the ability to use this understanding in flexible ways to make mathematical judgments, and the tendency to develop useful strategies for dealing with numbers and operations” (p. 3).

On the other hand, Yang (2003) refers to “a person's general understanding of numbers and operations and their ability to handle everyday life situations involving numbers. This includes the ability to develop useful, flexible, and efficient strategies (i.e., mental calculation or estimation) for tackling numerical problems” (p. 116). Students with a good number sense can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions. They are able to invent their own strategies for executing digital operations. They can represent the same number in more than one way, depending on the context and purpose of this representation. They can recognize comparison numbers and number models, especially those that derive from the deep structure of the number system. They have a good sense of numerical magnitude and are able to recognize large numerical errors, that are inaccurate by order of magnitude. Finally, they can think or talk logically about the general properties of a numerical problem or expression without doing any calculations (Markovits & Sowder,1994).

Some researchers consider number sense as a skill or a kind of knowledge rather than an internal process and claim that it should be teachable (Robinson, Menchetti, & Torgesen, 2002). According to this claim, the number sense stems from our biological structure and its development can be supported. According to some hard-core theorists, number sense is fixed and unchanging because it is a specialized substratum component of the brain. It is thought that the emergence of the primitive components of the number sense in young children occurs spontaneously (Dehaene, 2001). There are some basic principles that support the development of numerical cognitive structures. It is thought that participating in numerical games and activities containing these principles will activate the number sense (Geary, 1995). Parallel to this point of view, it is stated that the development of number sense can be supported in both formal and informal teaching environments by using numerical facts effectively in board games starting from the pre-school period (Gersten & Chard, 1999).

Since number sense is an important topic in mathematics education, it has been a hot topic among mathematics educators, cognitive psychologists, researchers, teachers, and mathematics curriculum developers (Yang, 2005; Yang, 2003; Markovits & Sowder, 1994; McIntosh et al., 1992; Howden, 1989). Based on the definitions and studies on number sense, this study aims to develop a test to measure middle school students' number sense skills.

Giftedness and Number Sense

Mathematical talent consists of abstract thinking (algebraic thinking structure) and spatial thinking (geometrical mindset) skills and their combination. Although the ability to calculate quickly and memorize formulas are useful skills, they are not considered necessary conditions for mathematical ability (Krutetskii, 1976, p. 77). Mathematically talented students can see relationships between topics, concepts, and ideas without formal instructional interventions. They may intuitively understand mathematical functions and processes because of their developed sense of numbers, skip the steps of operations, and may not be able to explain how they arrived at the correct answer (Rotigel & Fello, 2004).

When the literature is examined, it has been seen that there are studies evaluating the number sense skills of gifted students. Artut & Er (2022) examined the strategies used by gifted fifth-grade students in solving number sense problems. The results of the analysis of the data collected by qualitative research techniques showed that even gifted students could not reach a sufficient level in the use of number sense-based strategies. In another study, the relationship between high school students' number sense and their mathematics performance was examined (Wang et al., 2017). According to the results of the study, the probability of successful performance in mathematics increases for individuals with a developed number sense. However, the precision of number sense is not directly related to performing at a high level in advanced mathematics. Earlier basic math skills were found to be more associated with number sense. If this situation is interpreted, it was stated that better basic math skills support better developed math skills. Erdoğan & Erben (2020) examined gifted students' use of predictive strategies while measuring. Predictive strategy development is accepted as one of the indicators of number sense (Jordan et al, 2007). According to the results of the study, gifted students do not differ from their typical peers in creating predictive strategies. Similarly, in a study conducted by Montague & van Garderen (2003), it was stated that gifted students had low predictive measurement skills. When Baroody & Gazke (1991) examined the use of estimation strategies by potentially gifted preschool children, it was found that these children were quite successful. The contrasts in the findings related to the number sense skills of gifted students in the literature show that more studies should be done in this context.

Problem of Study

Number sense and components is within the abilities that express mathematical talent (Davis & Rim, 2004). Also, using stratiges in contex of number sense is an important real-life skill (Montague & van Garderen, 2003). As a result of the literature review, it was seen that there were limited and contradictory findings on the number sense skills of gifted students. There is a need for studies that measure the number sense skills of students at different grade levels. This study aimed to develop a three-tier test that identify mathematical talent in the context of number sense skill. Within the scope of the study, the answer to the question "Would a three-tier number sense test be developed to identify mathematical ability?" was sought.

Method

Research Model

In this research, validity and reliability analysis is performed for the development of the "Three-Tier Number Sense Test". The study was designed by survey method. In survey research, it is a scientific method applied by critically analyzing, interpreting, generalizing and estimating source materials (Salaria, 2012).

Participants

A total of 499 secondary school students (143 fifth grade, 126 sixth grade, 117 seventh grade, 113 eight grade) in Izmir in Turkey volunrarily participated in this study. The age range of the participants varies between 11-14 ages. The participants contained from schools with various levels of socioeconomic status. Data from the participants were collected in the 2019-2020 academic year.

According to the standards of National Council of Teachers of Mathematics [NCTM] (2000), students from pre-school to the end of the secondary education period must have gained number systems, the relationship numbers and operations, the meaning of operations, multiple representations of quantities, appropriate estimation, reasoning for solving math problems. In addition, it is deduced from these standarts, students should have acquired advanced number sense skills (Reys, 1991). Since a student is expected to have developed achievements in the context of number sense by the end of primary education, it was thought that the most appropriate time to evaluate number sense as a predictor of mathematical talent was secondary school age. Therefore, secondary school students were chosen as the target sample.

Instrument

Studies with tests evaluating number sense were examined and a conceptual framework was created. The most detailed classification for number sense was made by McIntosh et al. (1992). In this classification, he created a conceptual framework for the sense of number. The conceptual framework has three main components for number sense: Numbers, operations, applications of numbers and operations. The authors stated that it would not be helpful to describe all possible components of number sense as number sense, because number sense develops and expands with age (p.5). [NCTM] (2000) grouped number sense into 5 sub-components: Having a good sense of numbers, Developing multiple relationships between numbers, understanding the relative magnitudes of numbers, knowing the relative effects of operations on numbers, being able to develop references for measurements of objects and situations in their environment.

Resnick (1989) grouped possible indicators of number sense into 7 categories: Using well-known number effects, judging whether a number plausibly satisfies the solution of the problem, approaching a numerical answer rather than calculating the exact result, decimal of the number system to parse and recombine numbers in simple operations, making sense of situations involving numbers and quantities, talking about numbers and their relationships, having an understanding of the relative magnitudes of numbers and quantities, switching flexibly between different possible representations of a quantity.

Based on the definitions and characteristics of number sense in the literature, Markovits & sowder (1994) compiled the behaviors that occur in the presence of number sense: Combining and separating numbers, moving flexibly between different representations, comprehending the relative size of numbers, dealing with the absolute size of numbers, using reference points, combining numbering, operation and relation symbols in a meaningful way, understanding the effects of operations on numbers, making mental operations with "discovered" strategies to take advantage of numerical and operational properties, using numbers flexibly to predict numerical answers of operations, interpreting of numbers.

Reys et al. (1998) used some definitions in the framework they created to develop the number sense test: Understanding the meaning and magnitude of the number, understanding and using the equivalent representations of the number, understanding the meaning and effect of operations, use and meaning of synonyms, mental operation, written operation and calculator, flexible operation strategies, measurement references.

According to the conceptual framework Yang (2019) created: Understanding the meaning of numbers, understanding the magnitude of numbers, using measurement references appropriately, understanding the relative effects of operations on numbers, developing different strategies appropriately, and judging the reasonableness of answers.

Although there are many studies on the sense of number, it has been seen that the boundaries of the concept cannot be drawn and a common terminology cannot be established for the components. Different nomenclatures are used for components covering the same skill. In this study, while determining the components of number sense, a conceptual framework was formed by considering the cognitive characteristics of the age group, the official mathematics program and the purpose of the study.

The designed Three-Tier Number Test incorporates the aforementioned five number sense components based on earlier studies (Resnick, 1989; McIntosh et al., 1992; Markovits & Sowder, 1994; Reys et al., 1998; [NCTM], 2000; Yang, 2019). Each component consists of five items; therefore, the Three-Tier Number Test contains 25 items.

Treatment of Data

According to previous studies on the number sense two-tier test (Yang & Lin, 2015) and three-tier test (Peşman & Eryilmaz, 2010), the scoring rules were defined following the criteria in Table 1. Scoring of the test is done in three stages. The first stage of the questions is true and false; The second stage, called the causality section, is in the form of gradual scoring; The ratio between the answer given in the self-confidence section and the level of confidence was examined.

Table 1. Scoring Rules for the Three-tier Number Sense Test

1st ve 2nd Stages					
Number sense test (1st tier)	Correct answer 4 Points				Wrong answer 0 Points
Reason options (2nd tier)	Number sense-based	Rule-based	Misconception	Guessing	
	4 Points	2 Points	1 Points	0 Points	0 Points
Score given	8 Points	6 Points	5 Points	0 Points	0 Points
			3rd Stages		
Confidence (3rd tier)	Very confident	Confident	Neutral	Unconfident	Very Unconfident
Score given	5	4	3	2	1

The participants' number sense performance (first two-tier test) was divided into the following four groups: (1) high number sense (NS), in which the average score was 6-8; (2) medium number sense, in which the average score is 4-6; (3) low number sense, in which the average score is less than 4.

To summarize, the correct answer to the test question the student gets 4 points from the first stage. In the reason section, he gets 4 points if he chooses an explanation for the sense of number as the reason for solving the question, and 2 points if he chooses a rule-based explanation. Even if the student gave the correct answer, if he chooses the answer related to a misconception arising from a confusion of information, the score he will get from the second stage will be 1, and if the guess result has reached the correct result, the score he will get will be 0.

To explain through the sample question item in Figure 1, if the student marks option C, that is, the correct answer, he gets 4 points from the first stage. If he chooses option C in the second stage, that is, the solution based on number sense, he gets 4 points from this stage. The first-second stage total score is 8. The highest score a student can get from a question item is 8. The highest score to get from the whole test is 200. While the confidence stage shows the belief that the student has solved the question correctly, it is not included in the scoring.

Problem 1: 71008 = (8 × 1) + (□ × 100) + (7 × 10000) ise □ = ?

A) 0
B) 1
C) 10
D) 100

I chose option **A** Because:	1.	□ symbol represents the hundreds digit, so it must be zero.
	2.	The next digit in the analysis must be zero.
	3.	I made a guess.
I chose option **B** Because:	1.	□ symbol represents the thousands digit, so it should be 1.
	2.	The next digit in the analysis should be 1.
	3.	I made a guess.
I chose option **C** Because:	1.	10 should be written instead of 1 because it is multiplied by 100 instead of 1000. 1 × 1000 = 10 × 100
	2.	(8 × 1) + (□ × 100) + (7 × 10000) = 71008 square must be 10. well 70000+ 1000+8
	3.	The digits to be resolved are 7, 10 and 8. Therefore, the answer is 10.
	4.	I made a guess.
I chose option **D** Because:	1.	□ symbol represents the hundreds digit, so it must be 100.
	2.	7, 100 and 8 resolved. The next number in the analysis should be 100.
	3.	I made a guess.

How sure are you that your answer is correct?				
Very Confident	Confident	Neutral	Unconfident	Very Unconfident

Figure 1. Three-Tier Number Sense Test Sample Item

Results

Cycles of the Action Research Process

Before the test questions were created, researches including number sense and its components were examined in chronological order. Then, tools developed to measure number sense skills were examined. In the first studies to determine number sense, researchers used written forms of information evaluation, aiming to save time. It was found appropriate for students to produce answers with their own formulas in terms of examining the variable to be measured (Berch, 2005). With this format, researchers were given the chance to observe students' misconceptions. However, over time, these forms began to be seen as insufficient to identify misconceptions (Whitacre, Henning, & Atabaş, 2020). It has been determined that students are reluctant to write full sentences and give detailed answers (Yang & Tsai, 2010; Yang, 2007). Due to time constraints, few open-ended questions can be asked to students. For such reasons, tests consisting of multiple choice questions have become popular among researchers over time (Çekirdekçi, Şengül, & Doğan, 2016). These research designs have been found to be very time efficient as they can cover a large number of topics and include many tasks (McIntosh et al., 1997; Singh, 2009). However, this format has been criticized for its high probability of predicting the correct answer (Yang, Li & Lin, 2008). For this reason, researchers have developed a new test form, namely tests consisting of two or three-tier tasks (Yang, 2019).

Two-tier tests consist of two parts. The first part contains content problems with multiple choice questions, while the second part contains a reasonable explanation of the problem presented in the first stage of the task. Multi-tier tests reduce the probability of students guessing the correct answer. The use of two-tier tests allows teachers and researchers not only to understand students' misconceptions, but also to discover the logic behind them (Peşman & Eryılmaz, 2010). In addition, these tests facilitate efficient and simple examination and assessment of misconceptions in a wide range of subjects, as they are practical and the time required to take place keeps the available teaching time to a minimum. Since the results obtained through two-tier tests were not considered sufficient to show the difference between misconceptions and lack of knowledge, and between understanding and lucky guesses, a new layer called the "confidence" stage was added in addition to the "content" and "reason" stages, and three-tier tests were added. (Caleon and Subraminiam, 2010; Yang and Lin, 2015).

Adding the third stage to the tests provides valuable information about students' self-confidence (Peşman & Eryılmaz, 2010). If the student is not sure about the answers given in the first and second tiers, it can be concluded that the correct answers are the result of guesswork. On the other hand, it is thought that a student who gives the correct answer at only one stage and states that he is sure of his answer may have a misconception (Milenković, Hrin, Segedinac, & Horvat, 2016). Three-tier tests greatly reduce students' misconceptions and lack of scientific knowledge and significantly increase the validity of study results (Stankov & Crawford, 1997; Sia, Treagust & Chandrasegaran, 2012). As a result of the theoretical information and literature review, it was decided to develop a number sense test suitable for the secondary school mathematics curriculum of Turkey, based on the three-tier number sense test developed by Yang (2019).

During the development of the test;

- Determining the sub-components of the three-stage number sense test,
- Determining the achievements in the middle school mathematics program of Turkey that are related to the number sense,
- Creating mathematical questions for the achievements found to be related to the number sense,
- The steps of making the reliability and validity of the test were followed.

A literature review was conducted to decide on the sub-components of the three-tier number sense test. Although number sense has been defined in various ways by researchers, it has been observed that there is a great deal of consensus on its sub-components (Sowder, 1992; Markovits and Sowder, 1994; McIntosh et al., 1997; Yang and Li, 2008; Faulkner and Cain, 2009; Yang, 2019). These subcomponents are,

- Ability to understand the basic meanings of numbers and operations,
- Ability to recognize number sizes,
- Ability to use multiple representations of numbers and operations,
- Ability to recognize the relative effects of operations on numbers,
- Ability to develop different strategies as appropriate and evaluate the reasonableness of an answer.

The process cycle after deciding on the number sense sub-components is given in the figure 2.

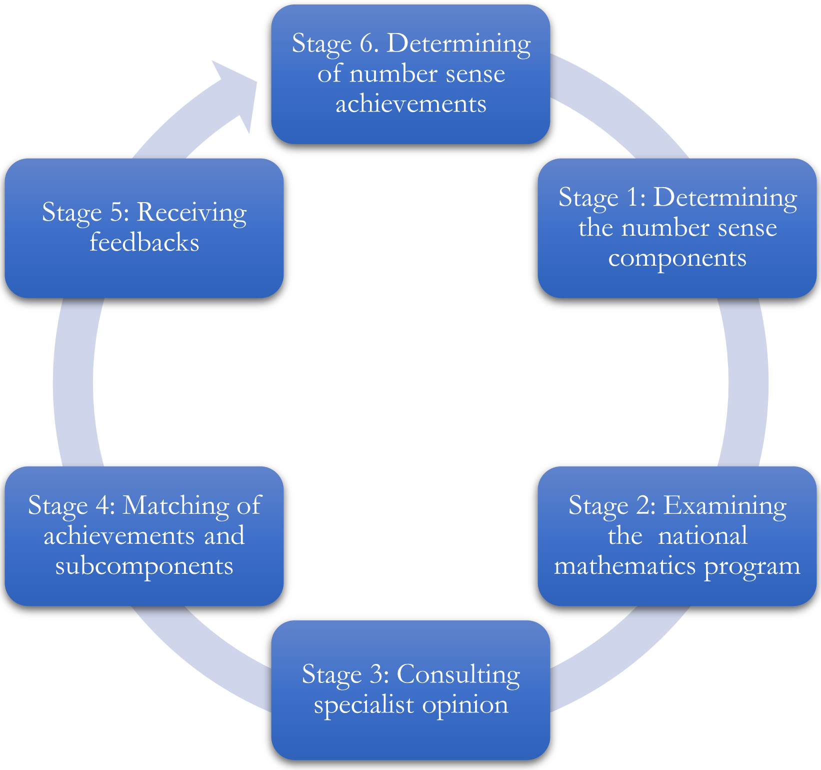

Figure 2. Determining the Achievements Related to Number Sense in the National Mathematics Program

After determining the number sense components, the achievements related to number sense in mathematics programs from the fifth to the eighth grade were examined. Opinions were received from a mathematician and a mathematics teacher for the compatibility between the achievements and the sub-components of number sense. Opinions were collected with a form in which the achievements and number sense components were included. In this form, compatible, incompatible and explanation boxes are included for each achievement and related number sense subcomponent. It was accepted that the common achievements found compatible by the researcher, mathematician and mathematics teacher were related to the sense of number. At the end of the process, it was concluded that in the middle school mathematics curriculum, 31 acquisitions of the fifth grade, 33 acquisitions of the sixth grade, 28 acquisitions of the seventh grade and 25 acquisitions of the eighth grade were related to the number sense.

Reliability and Validity

Number sense questions was collected in the question pool by the researcher. A total of 60 questions were selected from various mathematics books. Opinions were received from two mathematician on the appropriateness of the questions. In addition, opinions were received from two special education professionals, a mathematic teacher and a special education teacher for the number of questions expected to be included in the test. For the first tier, a total of 40 items, eight for each sub-component of number sense, were determined.

Opinions were received from three mathematics teachers, a special education professional and a mathematician for the validity of the test. Professionals and teachers were asked to evaluate the items in the draft test in terms of content validity. Professionals opinions on the validity of the questions were obtained using an professional evaluation form consisting of open-ended questions. The expert evaluation form was prepared using a two-response format as "appropriate" and "not appropriate". If the answer is not suitable, it is requested to write the reason in the explanation section. In accordance with the feedback of each, the questions in the test were reviewed and necessary corrections were made. Considering professionals opinions, it was decided that the test would consist of a total of 25 items, 5 for each sub-component. The distractors of the multiple-choice questions were created with three mathematics teachers by making use of the misconceptions in the literature.

The pilot application of the test was applied with a total of 106 students, including 32 students attending 5th grade, 28 students attending 6th grade, 29 students attending 7th grade and 17 students attending 8th grade. The results of the item difficulty and item discrimination index calculated through the Excel program after the pilot application are given in Table 2.

Table 2. Item Analysis of the Three-tier Number Sense Test

Item	Difficulty	Discrimination Values
1	0,64	0,46
2	0,28	0,33
3	0,51	0,36
4	0,64	0,49
5	0,51	0,50
6	0,76	0,50
7	0,58	0,63
8	0,77	0,45
9	0,29	0,42
10	0,46	0,39
11	0,65	0,54
12	0,74	0,40
13	0,45	0,55
14	0,72	0,43
15	0,65	0,45
16	0,52	0,49
17	0,35	0,37
18	0,46	0,49
19	0,72	0,51
20	0,41	0,39
21	0,30	0,41
22	0,44	0,45
23	0,25	0,53
24	0,78	0,35
25	0,59	0,33
$\bar{x}$	0,54	0,45

As the item difficulty value approaches zero, the question becomes more difficult, and when it approaches one, the question becomes easier. In order to strengthen the reliability of the test, it is expected that the item difficulty will be at the level of 0.5, that is, the questions of the test will generally consist of questions of medium difficulty. In addition, easy and difficult questions are also included in the tests (Büyüköztürk et al., 2010). The items in the test are grouped according to their difficulty levels in the table 3. The average difficulty of the three-tier number sense test was calculated .54. Since the value found was close to 0.50, it was considered as a medium difficulty test.

Table 3. Difficulty Levels of the Questions in the Three-tier Number Sense Test

	Easy	Medium	Hard
Items	1,4,6,8,11,12,14,15,19,24	3,5,7,10,13,16,18,20,22,25	2,9,17, 21,23

The fact that the discrimination indexes of the questions are between 0.30-0.40 indicate that they distinguish the students in the lower and upper groups at a good level with their answers to the test, and that the index scores of .40 and above indicate that the discrimination is at a very good level (Büyüköztürk et al., 2010). When the item discrimination indexes of the three-tier number sense test are examined, it is seen that all 25 questions have good and very good discrimination.

After the preliminary analysis of the test with the pilot study was completed, the three-tier number sense test was applied to 499 middle school students. The reliability coefficient of the first tier items of the test, which was calculated as .81 with the Kr-20 formula. The Kr-20 reliability coefficient of the test was calculated together with the second tier items was found .74.

Conclusion

In the study, item analysis, validity and reliability processes of the three-tier number sense test were included. The three-tier number sense test consists of 25 items. The first tier of the test consists of multiple-choice questions, the tier stage consists of items containing the reasons for the answers given in the first tier, and the items to measure the confidence in the answer given to the third tier question. The analysis of the data obtained in the study was made in the SPSS 21 program. The KR-20 confidence coefficient of the test was calculated as .74. From the results of the analysis, it is seen that the developed achievement test is a valid and reliable measurement tool that can be used to measure the number sense levels of middle school students.

Considering the findings obtained from the study, it is thought that, provide the necessary feedback to researchers to create a three-stage achievement test, the three-tier number sense test can provide teachers with necessary information about the number sense development of students, the introduction of the three-tier number sense test to the use of teachers will play an important role in increasing the mathematics achievement of middle school students.

Limitations of Study

This study was applied in several schools in Izmir in Turkey. Data collection from a single city is a limitation for the study. Similar studies can be diversified with data collected from different regions. Another limitation of the study was that the number sense test only included secondary school children. Since the sense of number is a developing and changing structure, studies on different age groups will contribute to the field. It is difficult to define the existence of the sense of number, so the tests created with different questions and structures will defeat the studies in this scope.

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Biodata of Author

Sıla Doğmaz Tunalı has been a research assistant of special education department at the Dokuz Eylul University since 2013. She graduated from the special education doctorate program in June 2022. She is interested in mathematics talent, mathematics learning disability and metacognitive skills. One of the future research goals is to investigate the ways gifted students use their number sense skills.

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Appendix 1. Turkish Version of Three Tier Test for Number Sense (for Secondary School Students)

Üç Aşamalı Sayı Hissi Testi-Ortaokul Düzeyi İçin

Sınıf: **Cinsiyet** : Kız ( ) Erkek ( )

Açıklama: Soruları çözün ve cevabınızı işaretleyin. Ardından açıklamalardan işaretlediğiniz cevabı seçme nedeninizi seçiniz. İşaretlediğiniz açıklama, sorunu çözmek için kullandığınız yöntemi içermelidir. Bu adımdan sonra cevabınızın doğru olduğundan ne kadar emin olduğunuzu işaretleyiniz.

Soru 1. 796 + 484 = 1280 olduğuna göre 7,96 + 4,84 işleminin sonucu kaçtır?

A) 1,28
B) 0,1280
C) 12,8
D) 1280

A şıkkını seçtim Çünkü:	1.	7,96 + 4,84 = 1,280 virgüllü olacaksa bu şekilde olur.
	2.	Bir tahminde bulundum.

B şıkkını seçtim Çünkü:	1.	7,96 + 4,84 = 0,1280 çünkü 4 basamak virgül ile sola ilerlenir.
	2.	Bir tahminde bulundum.

C şıkkını seçtim Çünkü:	1.	Soldaki işleme göre 2 basamak virgül atmak yeterli olacaktır. Yani 12,8 olur.
	2.	7,96 + 4,84 = 12,80 yani 12,8 olur.
	3.	Bir tahminde bulundum.

D şıkkını seçtim Çünkü:	1.	7,96 + 4,84 = 1280 olur.
	2.	Bir tahminde bulundum.

Cevabınızın doğruluğundan ne kadar eminsiniz?				
Çok eminim ( )	Eminim ( )	Kararsızım ( )	Emin değilim ( )	Hiç Emin Değilim ( )

Soru 2. 1234 ÷ 5 × 6 işlemine göre aşağıdakilerden hangisinin sonucu bu işlem ile aynıdır?

A) 1234 ÷ (5 × 6)
B) 1234 × 6 ÷ 5
C) 1234 ÷ 6 × 5
D) 5 × 6 ÷ 1234

A şıkkını seçtim Çünkü:	1.	İşlemde ki sayıların yerleri aynı olduğu için.
	2.	Sayıları paranteze almak sonucu değiştirmez.
	3.	Bir tahminde bulundum.

B şıkkını seçtim Çünkü:	1.	İşlemde, bölü 5 ve çarpı 6 'nın yerlerinin değişmesi sonucu etkilemez. Bölme ve çarpmanın birbirine göre işlem önceliği yoktur.
	2.	Öğretmenimiz çarpma ile bölmenin yer değiştirebileceğini söylemişti.

	3.	Bir tahminde bulundum.

C şıkkını şeçtim Çünkü:	1.	5 ve 6 rakamların yerlerinin değişmesi sonucu etkilemez.
	2.	Bir tahminde bulundum.

D şıkkını şeçtim Çünkü:	1.	Aynı işlem sadece rakamlarının yerleri değişmiş.
	2.	Bir tahminde bulundum.

Cevabınızın doğruluğundan ne kadar eminsiniz?				
Çok eminim ( )	Eminim ( )	Kararsızım ( )	Emin değilim ( )	Hiç Emin Değilim ( )

Soru 3. Aşağıdaki işlemlerde yer alan "□" şeklinin temsil ettiği sayı hangi işlemde en büyüktür?

A) □ + 39 = 195
B) □ – 39 = 195
C) □ x 39 = 195
D) □ ÷ 39 = 195

A şıkkını şeçtim Çünkü:	1.	Toplama işleminde artış olduğu için □ en büyüktür.
	2.	Bir tahminde bulundum.

B şıkkını şeçtim Çünkü:	1.	□ 'den 39 çıkarılmış yine 195 olmuş. O halde en büyük çıkarmada olmalıdır.
	2.	Bir tahminde bulundum.

C şıkkını şeçtim Çünkü:	1.	Çarpma işleminde sayılar hep büyüyor. O halde çarpmada □ en büyüktür.
	2.	Bir tahminde bulundum.

D şıkkını şeçtim Çünkü:	1.	□ 39 parçaya bölünmüş ve 195 çıkmış. □'nin baya büyük olması gerekir.
	2.	Bütün işlemleri yaptım. □ en büyük bölme işleminde çıktı.
	3.	Bir tahminde bulundum.

Cevabınızın doğruluğundan ne kadar eminsiniz?				
Çok eminim ( )	Eminim ( )	Kararsızım ( )	Emin değilim ( )	Hiç Emin Değilim ( )

Soru 4. 755 ◊ 5 = 151
36 □ 3 = 108
71 ○ 17 = 54
Verilen eşitliklerde dörtgen, kare ve daire şekillerinin yerine aşağıdakilerden hangisi gelmelidir?

	◊	□	○
A)	÷	×	-
B)	x	-	÷
C)	-	+	-
D)	÷	+	-

A şıkkını şeçtim Çünkü:	1.	İlk işlemde sayılar arasındaki fark fazla ancak bölme ile olur. İkinci işlemde sayı çok artmış çarpma olabilir. 3. Adımda az küçülmüş çarpma olabilir.
	2.	Bütün işlemleri deneyerek yaptım.
	3.	Bir tahminde bulundum.

B şıkkını şeçtim Çünkü:	1.	Sonuncusu bölme olacağı için.
	2.	Bir tahminde bulundum.

C şıkkını şeçtim Çünkü:	1.	1. ve 3. İşlemde azalma 2. İşlemde artma var o yüzden -,+,- olmalıdır.
	2.	Bir tahminde bulundum.

D şıkkını şeçtim Çünkü:	1.	Birinci işlem çok azalma yani bölme, ikinci işlem artma yani toplama, 3. İşlem azalma yani çıkarmadır.
	2.	Bir tahminde bulundum.

Cevabınızın doğruluğundan ne kadar eminsiniz?				
Çok eminim ( )	Eminim ( )	Kararsızım ( )	Emin değilim ( )	Hiç Emin Değilim ( )

Soru 5. İki basamaklı bir sayı ile üç basamaklı bir sayının çarpımının sonucu için aşağıdakilerden hangisi söylenebilir?

A) Daima 4 basamaklı bir sayıdır
B) 3 veya 4 basamaklı olabilir
C) Daima beş basamaklı bir sayıdır
D) 4 veya 5 basamaklı bir sayı olabilir

A şıkkını şeçtim Çünkü:	1.	10 x 100 =1000 yani dört basamaklı bir sayıdır.
	2.	Bir tahminde bulundum.

B şıkkını şeçtim Çünkü:	1.	İki basamaklı bir sayı üç basamaklı bir sayının çarpımının 3 basamaklı olması gerekir.
	2.	Bir tahminde bulundum.

C şıkkını şeçtim Çünkü:	1.	90 x 900 = 81000 yani beş basamaklıdır.
	2.	Bir tahminde bulundum.

D şıkkını şeçtim Çünkü:	1.	Sayılardan birisi 3 basamaklı olduğu ve diğeri 1 basamaklı olmadığı için 3 basamaktan fazla olacağı kesindir. 3 den daha büyük bir rakamla başlayan sayıların çarpım sonucu 1 basamak daha fazla olur. (30 x 300 =900, 40 x 300 = 12000) yani 4 veya 5 basamaklı olabilir.
	2.	10 X 100 = 1000 99 X 999 = 98 901 yani 4 veya 5 basamaklı olabilir.
	2.	Bir tahminde bulundum.

Cevabınızın doğruluğundan ne kadar eminsiniz?				
Çok eminim ( )	Eminim ( )	Kararsızım ( )	Emin değilim ( )	Hiç Emin Değilim ( )

Appendix 2. English Version of Three Tier Test for Number Sense (for Secondary School Students)

Three Tier Number Sense Test (for Secondary School Students)

Grade: **Gender :** Female ( ) Male ( )

Instruction: Solve the problems and mark your answer. Then, select the reason for choosing the answer you marked from the explanations. The description you mark should include the method you used to solve the problem. After this step, mark how sure you are that your answer is correct.

Q 1. Since 796 + 484 = 1280, what is the result of 7.96 + 4.84?

E) 1,28
F) 0,1280
G) 12,8
H) 1280

A Reason for choosing	1.	7,96 + 4,84 = 1,280 If there is a comma, it will be like this.
	2.	I guessed.

B Reason for choosing	1.	7,96 + 4,84 = 0,1280 because 4 digits move to the left with a comma.
	2.	I guessed.

C Reason for choosing	1.	According to the operation on the left, it will be sufficient to throw 2 digits of commas. So it would be 12.8.
	2.	7,96 + 4,84 = 12,80 so 12,8.
	3.	I guessed.

D Reason for choosing	1.	7,96 + 4,84 = 1280
	2.	I guessed.

How sure are you that your answer is correct?				
Very Confident	Confident	Neutral	Unconfident	Very Unconfident

Q 2. According to the operation 1234 ÷ 5 × 6, which of the following has the same result as this operation?

E) 1234 ÷ (5 × 6)
F) 1234 × 6 ÷ 5
G) 1234 ÷ 6 × 5
H) 5 × 6 ÷ 1234

A Reason for choosing	1.	Because the places of the numbers in the operation are the same.
	2.	Bracketing the numbers does not change the result.
	3.	I guessed.

B Reason for choosing	1.	In the operation, swapping the places over 5 and times 6 does not affect the result. Division and multiplication have no precedence over each other.
	2.	Our teacher said that multiplication and division can be replaced.
	3.	I guessed.

C Reason for choosing	1.	Changing the places of the 5 and 6 digits does not affect the result.
	2.	I guessed.

D Reason for choosing	1.	It's the same process, only the places of the digits have changed.
	2.	I guessed.

How sure are you that your answer is correct?				
Very Confident	Confident	Neutral	Unconfident	Very Unconfident

Q 3. In which operation is the number represented by the shape "□" in the following operations the greatest?

E) □ + 39 = 195
F) □ – 39 = 195
G) □ x 39 = 195
H) □ ÷ 39 = 195

A Reason for choosing	1.	Since there is an increase in addition, □ is the largest.
	2.	I guessed.

B Reason for choosing	1.	39 was subtracted from □ and it became 195 again. So it should be in the greatest subtraction.
	2.	I guessed.

C Reason for choosing	1.	In multiplication, numbers always get bigger. Then □ is the largest in multiplication.
	2.	I guessed.

D Reason for choosing	1.	□ It was divided into 39 parts and 195 came out. □ has to be pretty big.
	2.	I did all the operations. □ is output in the largest division operation.
	3.	I guessed.

How sure are you that your answer is correct?				
Very Confident	Confident	Neutral	Unconfident	Very Unconfident

Q 4. 755 ◊ 5 = 151
36 □ 3 = 108
71 ○ 17 = 54
Which of the following should replace the quadrilateral, square and circle shapes in the given equations?

	◊	□	○
A)	÷	×	-
B)	x	-	÷
C)	-	+	-
D)	÷	+	-

A Reason for choosing	1.	In the first operation, the difference between the numbers is large, but this happens with division. In the second operation, the number increased a lot, this multiplication may be possible. The number decreased in the third operation. The reason may be the multiplication.
	2.	I tried all the steps.
	3.	I guessed.

B Reason for choosing	1.	I think the last operation is division.
	2.	I guessed.

C Reason for choosing	1.	There is a decrease in the 1st and 3rd processes, there is an increase in the 2nd process, so it should be - ,+,-.
	2.	I guessed.

D Reason for choosing	1.	The first operation is much reduction, that is, division, the second operation is increase, that is, addition, the third operation is decrease, that is, subtraction.
	2.	I guessed.

How sure are you that your answer is correct?				
Very Confident	Confident	Neutral	Unconfident	Very Unconfident

Q5. What can be said about the result of multiplying a two-digit number with a three-digit number?

A) It is always a 4-digit number
B) It can be 3 or 4 digits
C) It is always a five-digit number
D) It can be a 4 or 5 digit number

A Reason for choosing	1.	10 x 100 =1000 that is a four digit number.
	2.	I guessed.

B Reason for choosing	1.	A two-digit number multiplied by a three-digit number must have 3 digits.
	2.	I guessed.

C Reason for choosing	1.	90 x 900 = 81000 5 digits.
	2.	I guessed.

D Reason for choosing	1.	Since one of the numbers has 3 digits and the other is not 1 digit, it is certain that it will be more than 3 digits. Multiplying numbers starting with a digit greater than 3 is 1 digit more. (30 x 300 = 900, 40 x 300 = 12000) so it can be 4 or 5 digits.
	2.	10 X 100 = 1000 99 X 999 = 98 901 four or five digits
	2.	I guessed.

How sure are you that your answer is correct?				
Very Confident	Confident	Neutral	Unconfident	Very Unconfident