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Research Article

Contact Analysis of Elastic Layer Supported by a Wedge

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Abstract

In this paper the contact analysis of elastic layer supported by a wedge is considered in plane. The problem is formulated with closed formed integral equations. The length of the contact region, the pressure between the wedge and the layers, and the pressure between the layer and the punch are unknowns. Both the material and layer are elastic in this problem. This problem can be defined as a layer supported over two wedges with perpendicular angle. The upper surface of the layer is assumed as circle with a large radius. The thickness of the layer is finite. Singular integral equations are used in the formulation of the problem. The benefits of this formulation are the following: the problem can easily be generalized for the case of forces acting through many rigid punches. The solution gives the contact stress directly and the solution of the singular integral equations is an appropriate way in terms of numerical solution technique. The application of this problem is the train wheel that is contacted to the connection part of the rails. It is shown that the divergent terms at the kernels cancel each other by considering the equilibrium conditions. As a numerical example, the contact problem between the wheel and rails is investigated.

Keywords: Contact problem, wedge, elastic layer, pressure, applied mechanics

Kama ile Destekli Elastik Tabakanın Temas Analizi

Öz

Bu çalışmada düzlemde kama ile destekli elastik tabakanın temas analizi yapılmıştır. Bu problem kapalı formda integral denklemleri ile formüle edilmiştir. Temas bölgesinin uzunluğu, kama ve tabakalar arasındaki basınç ile tabaka ve panç arasındaki basınç problemin bilinmeyenleridir. Bu problemde hem malzeme hem de tabaka elastik kabul edilmiştir. Bu problem dik açılı iki kama tarafından destekli bir tabaka olarak tanımlanabilir. Tabakanın kalınlığı üst yüzeyin çok büyük yarıçaplı bir çember olduğu varsayımı ile sonlu olarak kabul edilmiştir. Problemin çözümü için tekil integral denklemleri kullanılmıştır. Bu formülasyonun yararları şöyle açıklanabilir ; problem basit bir şekilde kuvvetlerin bir çok panç tarafından etki ettirildiği durum için genelleştirilebilir. Çözüm temas gerilmesini direkt olarak verir ve tekil integral denklemlerinin çözümü sayısal çözüm tekniği bakımından uygun bir yoldur. Bu problemin uygulaması rayların birleşim bölgeleri ile temas eden tren tekerleği örneğidir. Denge koşullarını göz önüne alarak çekirdeklerdeki ıraksak terimlerin birbirini sadeleştirdiği gösterilmiştir. Sayısal örnek olarak teker ve ray arasındaki temas problemi rayların birbiri arasında aralıklı hali için incelenmiştir.

Anahtar Kelimeler: Temas Problemi, kama, elastik tabaka, basınç, uygulamalı mekanik

1. Introduction

In the late 19 th century [1-2] contact problem was first investigated. Boussinesq [1] investigated the contact of the punch to the elastic half plane and Hertz [2] investigated the contact of two elastic paraboloids. The results of Hertz problem obtained from experiments was presented in Dinnik [3]. In the following years Belyaev [4] found the points where the stress is maximum in elastic punch, and solved the problem of railways [5-6]. Various problems of half plane without friction forces can be found in Shtaerman [7]. The general solution methods of contact problem were investigated and various half plane problems with friction forces were solved in Muskhelishvili [8]. Frictional elastic contact with periodic loading was given by Barber et al. [9]. In addition frictional contact problem and anisotropic contact problem for layers were investigated by [10-12]. Sliding frictional contact at graded elastic medium was studied by Dag et al. [13]. Some of the thermal problems that are related with contact problem were analyzed by [14-15]. An anisotropic linear elastic layer and a rigid intender were investigated in terms of contact problem by Batra and Jiang [16]. The contact problem for layered medium supported by a wedge was presented in [17], and this reference is the companion study of this paper.

A general technique is developed for the solution of a layer in this study. The layer is supported by elastic wedges. The load is applied by means of a frictionless rigid or elastic punch Yanik and Bakioglu [17]. For the punch cases, punch profile is known at the beginning. However, the radius of profile is assumed to be very larger than the contact region. The displacements and stresses at the contact region between the layer and support which are perpendicular to the layer surface, are assumed to be equal to each other. In addition the friction forces are neglected. Moreover the dependence of the unknown boundaries of the contact region to external load is investigated. The kernels which are obtained from the solution of the general problem is investigated in a detailed way. And it is shown that the divergent part of the two kernels cancel each other. In addition, appropriate ways for separation of the divergent terms and integration of them are investigated. One of the rail road problems is solved as a numerical example.

2. Formulation of the Layer

If we take into account a layer that is supported by an elastic wedge as shown in Fig. 1 , the wedge constants are κ_2, μ_2 and the head angle is θ_0 . For the equilibrium condition of the system $0<t_0$ condition must be satisfied. Contact stresses are taken into account only on the top and the bottom surfaces. In other words there is not any load outside the contact regions. Let us consider *P* which is applied to the top surface of the layer by means of an elastic punch. In this case elastic constants are κ_3, μ_3 and the contact region's boundaries are ($a, \bar{a}$). Considering the elastic punch as an upper half plane, $v_3(x)$ displacement of the elastic punch profile can be expressed as

$$\frac{\partial v_3}{\partial x} = -\frac{1+\kappa_3}{4\pi\mu_3}\int_a^{\bar{a}} \frac{p_1(t)}{t-x}dt \tag{1}$$

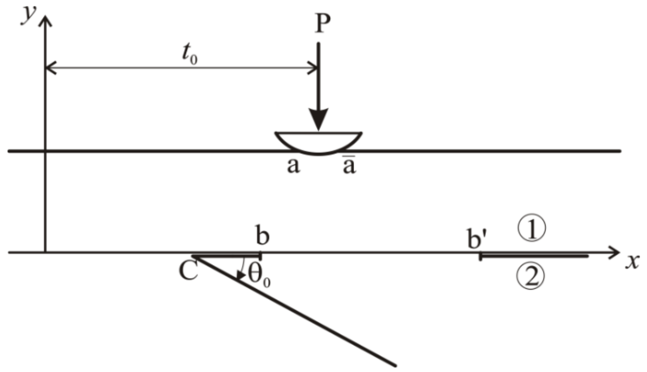

Fig. 1 A layer supported by an elastic wedge.

Where $p_1(t)$ is the unknown contact stress. If loads are applied directly to the top surface, neither there will be any contact on the top surface nor the problem will be a mixed boundary value problem. The elastic punch profile and the top surface of layer before the contact can be denoted as $f_3(x)$, $f_1(x)$ respectively. The vertical displacements of the layer is $v(x,h)$. *h* is the height of the layer. This yields to

$$v(x,h) + f_1(x) = v_3(x) + f_3(x) \tag{2}$$

taking the derivative of Eq. (2) by considering $f_1(x)$ as constant and using Eq. (1) one can obtain

$$\frac{\partial v}{\partial x}(x,h) = -\frac{1+\kappa_3}{4\pi\mu_3}\int_a^{\bar{a}} \frac{p_1(t)}{t-x}dt + \frac{df_3}{dx} \tag{3}$$

with necessary operations the following expression can be written

$$\frac{2}{1+\beta_1}\int_a^{\bar{a}} \frac{p_1(t)}{t-x}dt + \int_a^{\bar{a}} K_{22}(x,t)p_1(t)dt + \int_{L_{2v}} K_{21}(x,t)p_2(t)dt = \frac{4\pi\mu_1}{1+\kappa_1}\frac{df_3}{dx} \quad (a<x<\bar{a}) \tag{4}$$

here β_1 can be expressed as

$$\beta_1 = \left[(1+\kappa_1)\mu_3 - (1+\kappa_3)\mu_1\right] / \left[(1+\kappa_1)\mu_3 + (1+\kappa_3)\mu_1\right] \tag{5}$$

β_1 in the equation given above is defined as bi-elastic constant. This is a single constant which represents two elastic materials. In the rigid punch case, $\mu_3 \to \infty$ and $\beta_1 \to 1$. Singular behavior of $p_1(x)$ can be obtained by investigating the first term of Eq. (4). The first term is the dominant part of the singular integral equation. $p_1(x)$ can be presented as given below [8,18].

$$p_1(t) = g_1(t)(\bar{a}-t)^{\alpha}(t-a)^{\beta} \qquad (-1 < \mathrm{Re}(\alpha,\beta) < 1) \tag{6}$$

$$\alpha = 1/2 + N \qquad \beta = -1/2 + M \qquad \kappa = -(N+M) = -(\alpha+\beta) \tag{7}$$

In the equation given above, $g_1(t)$ is a continuous function with closed interval ($a \le t \le \bar{a}$). The solution of the integral equation which appears in Eq. (4) contains an arbitrary constant *C*. *M* and *N* are integers and κ is the index of the problem. It must be either -1,0 or +1. Index of the problem can't be determined by only considering mathematical point of view, in addition physical nature of the problem must be taken into account. Three cases of loading are considered as contact cases (see Fig. 2). In the first case (Fig. 2a), the load is applied by means of rigid punch. Derivatives of the contact curves have discontinuities. At the end points the stresses have singularities which are integrable. The index of the problem is κ=1. This yields to the following values like N=-1 and M=0. The unknown function can be defined by

$$p_1(t) = g_1(t)(\bar{a}-t)^{-1/2}(t-a)^{-1/2} \tag{8}$$

For the case given above an additional condition must be used to obtain the constant C. The constant C appears in the solution. The expression given below is used to determine C

$$\int_a^{\bar{a}} p_1(x)dx = P \tag{9}$$

In the second case, load is applied through a rigid or elastic punch (see Fig. 2b). There are no discontinuities in the derivatives of the contact curves. The stresses are bounded at the end points. The index of the problem is κ=-1. In addition N=0 and M=1. The unknown function is

$$p_1(t) = g_1(t)(\bar{a}-t)^{1/2}(t-a)^{1/2} \tag{10}$$

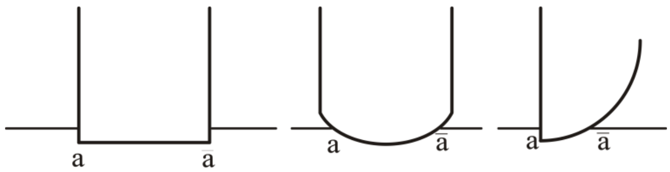

Fig. 2 Three cases of loading which are considered as contact cases

In this case the constant C is zero. This case can be identified as a receding contact problem. Therefore Eq. (9) is used to determine the contact boundaries. In the third case the load is applied with a rigid punch as shown in Fig. 2c. There are discontinuities at the derivatives of the contact curves at one end. And stresses are bounded at the other end. The index of the problem is κ=0. Therefore N=0 and M=0. The unknown function can be presented as

$$p_1(t) = g_1(t)(\bar{a}-t)^{1/2}(t-a)^{-1/2} \tag{11}$$

Again the constant C is defined as zero and this case is a receding contact problem as well. Thus Eq. (11) is used to determine the contact boundaries. Bottom surface of the layer contacts the elastic wedge. In this case the elastic constants are κ_2, μ_2 and the head angle is θ_0 as shown in Fig. 1. The surface curves of the layer and wedge are $\bar{f}_1(x)$ and $f_2(x)$ in a respective way. Moreover the vertical displacements are defined as $v(x,0)$ and $v_2(x)$. The equilibrium equation of the displacements can be presented as

$$v(x,0) + \bar{f}_1(x) = v_2(x) + f_2(x) \tag{12}$$

Following necessary operations which are not shown here for space constraints the field equation of the layer can be written as

$$\frac{2}{1+\beta_1}\int_a^{\bar{a}} \frac{p_1(t)}{t-x}dt + \int_a^{\bar{a}} K_{22}(x,t)p_1(t)dt + \int_b^{\bar{b}} K_{21}(x,t)p_2(t)dt = \frac{4\pi\mu_1}{1+\kappa_1}\frac{df_3}{dx} \qquad a<x<\bar{a}$$

$$\int_b^{\bar{b}} \frac{p_2(t)}{t-x}dt + \int_b^{\bar{b}} K_{22}(x,t)p_2(t)dt + \int_a^{\bar{a}} K_{21}(x,t)p_1(t)dt + \frac{1+\gamma}{1-\gamma}\int_b^{\bar{b}} K(x-c,t-c)p_2(t)dt = 0 \qquad b<x<\bar{b}$$

$$\gamma = \left[(1+\kappa_2)\mu_1 - (1+\kappa_1)\mu_2\right] / \left[(1+\kappa_2)\mu_1 + (1+\kappa_1)\mu_2\right]; \beta_1 = (1+\kappa_1)\mu_3 - (1+\kappa_3)\mu_1 / (1+\kappa_1)\mu_3 + (1+\kappa_3)\mu_1 \tag{13}$$

If the wedge is rigid then $\mu_2 \to \infty$ and $\gamma \to -1$. Therefore the last term of the second expression in Eq. (13) vanishes. If $\theta_0 = \pi$, then the problem is a layer problem which lies on the half plane. The solution of an elastic layer lying on half plane under the effect of a load applied by frictionless rigid punch is defined as [19]. This problem is symmetric.

$$\int_b^{\bar{b}} \frac{p_2(t)}{t-x}dt + \int_a^{\bar{a}} K_{22}(x,t)p_1(t)dt + \int_b^{\bar{b}} K_{21}(x,t)p_2(t)dt = \frac{4\pi\mu_1}{1+\kappa_1}\frac{df_3}{dx}$$

$$\int_b^{\bar{b}} \frac{p_2(t)}{t-x}dt + \frac{1-\gamma}{2}\int_b^{\bar{b}} K_{22}(x,t)p_2(t)dt + \frac{1-\gamma}{2}\int_a^{\bar{a}} K_{21}(x,t)p_1(t)dt = 0 \tag{14}$$

In Fig. 1, considering the symmetry with respect to y axis, one can write $\partial v_1 / \partial x$ as

$$-\frac{4\pi\mu_1}{1+\kappa_1}\frac{\partial v_1}{\partial x}(x,0) = \int_{-\bar{b}}^{-b} \frac{p_2(t)}{t-x}dt + \int_b^{\bar{b}} \frac{p_2(t)}{t-x}dt + \int_{-\bar{b}}^{-b} K_{22}(x,t)p_2(t)dt + \int_b^{\bar{b}} K_{22}(x,t)p_2(t)dt$$
$$+ \int_{-\bar{a}}^{-a} K_{21}(x,t)p_1(t)dt + \int_a^{\bar{a}} K_{21}(x,t)p_1(t)dt \tag{15}$$

In Eq. (15), after switching negative limits of the integral to positive limits, following equation can be obtained

$$-\frac{4\pi\mu_1}{1+\kappa_1}\frac{\partial v_1}{\partial x}(x,0) = \int_{-\bar{b}}^{-b}\left[\frac{1}{t-x} - \frac{1}{t+x}\right]p_2(t)dt + \int_{-\bar{b}}^{-b} \bar{K}_{22}(x,t)p_2(t)dt + \int_{-\bar{b}}^{-a} \bar{K}_{21}(x,t)p_1(t)dt \tag{16}$$

where

$$\bar{K}_{22}(x,t) = K_{22}(x,t) + K_{22}(x,-t) = -\int_0^{\infty} k_{22}(\alpha)[\sin\alpha(t+x) - \sin\alpha(t-x)]d\alpha$$

$$\bar{K}_{21}(x,t) = K_{21}(x,t) + K_{21}(x,-t) = -\int_0^{\infty} k_{21}(\alpha)[\sin\alpha(t+x) - \sin\alpha(t-x)]d\alpha \tag{17}$$

with necessary manipulations Eq. (16) yields to the following expression

$$\int_b^{\bar{b}}\left\{\frac{1-\gamma}{1+\gamma}\left[\frac{1}{t+x} - \frac{1}{t-x} - K_{22}(x,t)\right] - K\right\}p_2(t)dt = \frac{1-\gamma}{1+\gamma}\int_a^{\bar{a}} \bar{K}_{21}(x,t)p_1(t)dt \tag{18}$$

If the external forces are applied directly to the top surface then the hand side of Eq. (18) can be obtained. The solution for this symmetric case can be found in [20]. Above equation is the same with Eq. (12) which was given in [20]. The contact in this paper is an example of smooth contact. In addition $p_2(t)$ function has finite values at both ends. The index of the problem is (-1). Therefore $p_2(t)$ can be expressed as

$$p_2(t) = g_2(t)(b-t)^{-0.5}(t-\bar{b})^{0.5} \qquad (19)$$

eventually the boundaries of the contact region can be determined by the following equation

$$\int_b^{\bar{b}} p_2(t)dt = -P \qquad (20)$$

In the case when $b=c$ smooth contact does not occur at point b. For this case $p_2(t)$ function can be expressed as

$$p_2(t) = g_2(t)(\bar{b}-t)^{1/2}(t-b)^{\beta} \qquad \beta = p-1 \qquad (21)$$

Where $-\beta$ is the strength of singularity at point b. For the rigid wedge case $\bar{\alpha} = +1$, p=1/2 the strength of singularity at point b is β=-1/2. Consequently this is the expected result. Let us consider a single P force that is applied to the top surface of the layer at point t_0 without punch. In this case second expression in Eq. (13) can be rewritten as

$$\int_b^{\bar{b}} \frac{p_2(t)}{t-x}dt + \int_b^{\bar{b}} K_{22}(x,t)p_2(t)dt + \frac{1+\gamma}{1-\gamma}\int_b^{\bar{b}} K(x-c,t-c)p_2(t)dt = \int_a^{\bar{a}} K_{21}(x,t)p\delta(x-t_0)dt \qquad (22)$$

Some necessary operations in Eq. (22) yield to

$$\int_b^{\bar{b}} \frac{p_2(t)}{t-x}dt + \int_b^{\bar{b}} K_{22}(x,t)p_2(t)dt + \frac{1+\gamma}{1-\gamma}\int_b^{\bar{b}} K(x-c,t-c)p_2(t)dt = PK_{21}(x,t_0) \qquad (23)$$

For the unknown b and $\bar{b}$ values following equation can be used

$$\int_b^{\bar{b}} p_2(t)dt = -P \qquad (24)$$

If we divide both sides of Eqs. (22-23) with P the unknown function becomes $p_2(t)/P$ instead of $p_2(t)$. So the boundaries of the contact region becomes independent from P however dependent to the point t_0 where P is applied. When the loads are applied directly to the upper surface, loads can be expressed as

$$p_1(t) = r\varphi_1(t) \qquad (25)$$

Where r is a multiplier that characterizes the amplitude of the loads and $\varphi_1(t)$ is a function characterizing the distribution of the load. According to Eq.'s (24-25), if P is applied through a punch with flat bottom and parallel to layer surface, then the first equation in the right hand side of Eq. (13) will be zero. Therefore equation system can be defined as homogeneous equation system. The conditions for the solution of this equation system can be written as

$$\int_a^{\bar{a}} p_1(t)dt = -P \quad ; \quad \int_b^{\bar{b}} p_2(t)dt = -P \qquad (26)$$

The homogeneous equation system is solved with nonhomogeneous conditions. The first expression of Eq. (26) is the necessary condition for the first equation of Eq. (13) to have a single solution. The second expression in Eq. (26) is the condition to determine the boundaries $b,\bar{b}$ of the contact region. If we consider p_1/P and p_2/P as unknown functions, the boundaries of the contact region can be found independently from P force as seen from Eqs. (13 & 26).

3. Numerical Example

The numerical example consists of a railroad problem. This problem is numerically solved in this section. As stated in [17] the tip of the gap between two rails is the most crucial part of the rails (see Fig. 3). In comparison with the wheel the gap is smaller. In this case the head angels of the elastic wedges are 90 degrees and the wheel can be considered as upper half plane.

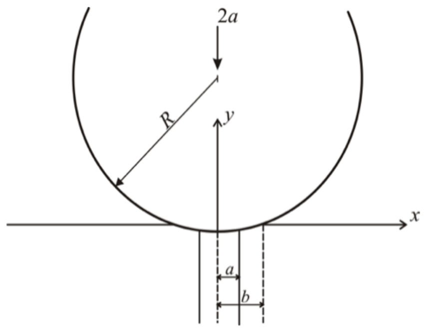

Fig. 3 Rail drawing [17]

The surface curves of wheel and rails before contact can be denoted as $f_1(x)$, $f_2(x)$ respectively. The vertical displacements of these contacting curves are expressed as v_1 and v_2. Considering this information following expression can be obtained.

$$\frac{\partial v_2}{\partial x}(x,0) + \frac{df_2}{dx} = \frac{\partial v_1}{\partial x}(x,0) + \frac{df_1}{dx} \quad ; \quad a < x < b \qquad (27)$$

The most critical stress for the rail is at $x=a$ point. This stress is called cleavage stress which is very important for fracture mechanics. Stress intensity factor is written as

$$k(a) = \lim_{x \to a}(x-a)^{-\beta} p(x) \qquad (28)$$

Making some necessary manipulations dimensionless form of $k(a)$ and g are presented as

$$\frac{k(a)(b-a)^{\beta}}{Q/R} = 2^{\beta+1/2}\frac{2R}{b-a}\frac{\bar{\phi}(-1)}{g} \qquad (29)$$

$$g = \int_{-1}^{1} \bar{\phi}(\tau)(1-\tau)^{1/2}(1+\tau)^{\beta}d\tau \qquad (30)$$

Fig. 4 shows the change of the dimensionless stress intensity factor related to the dimensionless contact length. The curves are composed considering constant a/R. For a constant system geometry Fig. 4 shows that when the contact length increases with respect to the increase of Q, stress intensity factor also increases. When R is constant the length of the contact region is constant as well, however the value of a increases. In this case the stress intensity factor also increases. $k(a)$ can be expressed by the help of Fig. 4 as

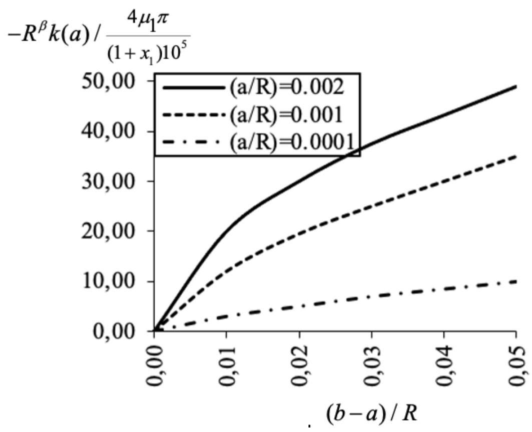

Fig. 4 Value of the dimensionless stress intensity factor related to dimensionless contact length for the constant a/R

$$k(a) = \frac{4\pi\mu_1}{(1+\kappa_1)10^5}\frac{1}{R^{\beta}}F_2\left(\frac{a}{R},\frac{b-a}{R}\right) \tag{31}$$

where $F_2(a/R,\ (b-a)/R)$ is the dimensionless function which is shown in Fig. 4. Taking β as -0.226, the stress at the tip point of the rail can be calculated as

$$p(x) \cong \frac{k(a)}{r^{0.226}} \quad ; \quad p(x) \cong \frac{4\pi\mu_1}{(1+\kappa_1)10^5}\left(\frac{R}{r}\right)^{0.226} F_2\left(\frac{a}{R},\frac{b-a}{R}\right) \quad ; \quad r = x-a \tag{32}$$

From From Eq. 32 when r→0, the stress goes to infinity. The singularity of this case is equal to the value -0.226. The singularity is equal to the value -0.5 for the most critical case. That case is the rigid wedge case.

4. Conclusions

It has been obtained for the case of directly acting forces that the contact region boundaries are independent from the amplitude of the load, however dependent to the distribution. This is also same for the force acting through a rigid punch. For the other cases the boundaries of the contact region are related to the distribution and amplitude of the load. The singularity is related to the bi-elastic constant. In addition the most critical singularity is found for the rigid wedge case and the value is -0.5. When the head angle of the wedge is π, the problem can be defined as contact problem of elastic layer lying in half plane. Although the kernels in the singular integral equation are divergent, the divergent terms in the integrals cancel each other. Accordingly, K_{21} and K_{22} kernels can be studied as Fredholm kernels. Another effective method for calculation of the kernels is the usage of contour integration. Using the residues in the integration of kernels is appropriate as well.

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To the loving memory of Prof Mehmet Bakioglu (1944-2015).

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