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Research Paper

Best Member Detection and Using as Differential Evolution Crossover Operator in Decomposition-based Multiobjective Optimization Algorithm

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Abstract— Decomposition is a method to distributes a mutliobjective problems to the many single objective problems like scalarization. Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) is one of the many algorithms uses decomposition method. In MOEA/D algorithm genetic operators are preferred to alter the population. As one of the genetic operators, the crossover is an important element in the algorithm. Hence it is possible to propose new possible methods instead of well-known SBX method. Differential Evolution (DE) which is a single objective optimization algorithm can be used as crossover operator in MOEA/D. However, in DE the best member needed to be detected in the population. Even it is relatively easy in single objective, systematic methods are needed for this purpose. Therefore, in this research three different best member detection methodology will be compared in DE assist MOEA/D algorithm. These methods will be compared on benchmark problems with many objectives.

Keywords : *MOEA/D, decomposition, multiobjective optimization, crossover.*

1.Introduction

The aim of the multiobjective optimization problem is to succeeded many objectives at once therefore unlike single objective optimization instead of a single solution, a solution set is desired to obtain, hence this algorithm can be formulized as follows

$$\min \quad F(x) = [f_1(x), f_2(x), \dots, f_M(x)] \qquad (1)$$
$$subject\ to\ x \in \Omega$$

where Ω is given for the decision space and F:Ω→RM is a real valued objective space, where F has real valued objective function f. Unlike single objective optimization problems, instead of s single solution a set of solutions are needed for conflicted objective function values. Therefore, the best possible set of solutions are called Pareto set, since Pareto set composed from infinite number of solutions, the numerical sub-set of the Pareto set is called Pareto approximate set (generally it is also called Pareto set). For this reason, it is expected get solution as close as Pareto set.

There are many multiobjective optimization algorithms are proposed in literature. Among them Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) is an evolutionary multi objective optimization algorithm which is proposed by (Zhang, 2007), is an evolutionary algorithm uses decomposition idea to select best member. Decomposition is a scalarization idea so that the multiobjective optimization algorithm is converted to a single objective optimization algorithm with a set of weighing vector. Based on this vector set members are compared with each other and the best of them survived to the next generator. The conventional MOEA/D algorithm is proposed to use SBX crossover, polynomial mutation, and mostly PBI aggregation function. However, it is possible to use other formulations as crossover operator; like MOEA/D with Differential Evolution (MOEA/D-DE) (Li, 2009). In this variant instead of SBX crossover operator, Differential Evolution rules are applied to obtain offspring through the algorithm. In (Altinoz, 2022) different DE variants are applied to MOEA/D without considering best member and compared with MOEA/D algorithm. The results showed that SBX crossover method presents better performance when compared to DE formulations. However, on that study best member is not added to the DE formulations. In this research, best member included DE formulations are added to MOEA/D algorithm.

Also, since it is multiobjective optimization problem and it is not possible to get best member by just looking at the objective values, in this research three different best member detection methods are applied and compared with each other.

This study is organized as four main sections. At the first section and introduction of the study is proposed. Then in the second section, methods used in this study is given which are benchmark problems, algorithm, proposed method and performance measurement methods. Then implementation and their results are given in table. Finally, the conclusion of the study is presented.

2.Methods

This section begins with the definition of the benchmark problems. Then MOEA/D algorithm is explained. Next the proposed crossover ideas are given and explained. At last, the performance measurement methods, metrics, are defined for this research.

2.2. Benchmark Problems:

The empirical study will be made on this research. Therefore, it is needed to get some well prepared and solution known problems to compare the results. For this purpose, benchmark problems are proposed by researchers. On of thew possible set of the benchmark problems are proposed by Cheng and named as MaF (Cheng, 2017). In this research 15 benchmark problems with a variable objective dimension are preferred. The list of mathematical description of the benchmark problems are given below.

Table 1. Benchmark Problems

	Mathematical Formulations of Benchmark Problems
MaF1	$f_1 = (1 - x_1 \dots x_{M-1}) \dots (1 + g(x_M)) \dots f_M = (x_1)(1 + g(x_M)), g(x_M) = \sum_{i=1}^{M}\left(\left(x_i - \frac{1}{2}\right)^2\right)$
MaF2	$f_1 = (1 + g(x_M))cos\left(\frac{\pi}{2}\left(\frac{x_1}{2} + \frac{1}{4}\right)\right) \dots cos\left(\frac{\pi}{2}\left(\frac{x_{M-1}}{2} + \frac{1}{4}\right)\right) \dots f_M = (1 + g(x_M))x_1 sin\left(\frac{\pi}{2}\left(\frac{x_M}{2} + \frac{1}{4}\right)\right)\ g(x_M) = \sum_{i=1}^{M}\left(\left(\frac{\pi}{2}\left(\frac{x_i}{2} + \frac{1}{4}\right) - \frac{1}{2}\right)^2\right)$
MaF3	$f_1 = \left[(1 + g(x_M))cos\left(x_1\frac{\pi}{2}\right) \dots cos\left(x_{M-2}\frac{\pi}{2}\right) cos\left(x_{M-1}\frac{\pi}{2}\right) \dots cos\left(x_{M-2}\frac{\pi}{2}\right) sin\left(x_{M-1}\frac{\pi}{2}\right)\right]^4 \dots f_M = \left[(1 + g(x_M))sin\left(x_1\frac{\pi}{2}\right)\right]^2 g(x_M) = \left[100\lvert x_M\rvert + \sum_{i=1}^{M}\left(\left(x_i - \frac{1}{2}\right)^2 + cos\left(20\pi\left(x_i - \frac{1}{2}\right)\right)\right)\right]$
MaF4	$f_1 = \left(1 - cos\left(x_1\frac{\pi}{2}\right) \dots cos\left(x_{M-2}\frac{\pi}{2}\right) cos\left(x_{M-1}\frac{\pi}{2}\right) \dots cos\left(x_{M-2}\frac{\pi}{2}\right) sin\left(x_{M-1}\frac{\pi}{2}\right)\right) a(1 + g(x_M)) \dots f_M = a(1 + g(x_M))\left(1 - sin\left(x_1\frac{\pi}{2}\right)\right)$
MaF5	$f_1 = a^m\left[(1 + g(x_M))cos\left({x^a}_1\frac{\pi}{2}\right) \dots cos\left({x^a}_{M-2}\frac{\pi}{2}\right) cos\left({x^a}_{M-1}\frac{\pi}{2}\right) \dots cos\left({x^a}_{M-2}\frac{\pi}{2}\right) sin\left({x^a}_{M-1}\frac{\pi}{2}\right)\right]^4 \dots f_M = a\left[(1 + g(x_M))sin\left({x^a}_1\frac{\pi}{2}\right)\right]^4$
MaF6	$f_1 = (1 + g(x_M))cos(\theta_1) \dots cos(\theta_{M-2})cos(\theta_{M-1}) \dots cos(\theta_{M-2})sin(\theta_{M-1}) \dots f_M = (1 + g(x_M))sin(\theta_1)g(x_M) = \sum_{i=1}^{M}\left(\left(x_i - \frac{1}{2}\right)^2\right)$ $\theta_i = \begin{cases} \frac{\pi}{2}x_i, i = 1,2,\dots,l-1 \\ \frac{\pi}{4(1 + g(x_M))}(1 + 2g(x_M)x_i), i = l,\dots,M-1 \end{cases}$
MaF7	$f_1 = x_1, f_2 = x_2 \dots f_M = (1 + g(x_M))hg(x_M) = 1 + \frac{9}{\lvert x_M\rvert}\sum x_i, h = M - \sum_{i=1}^{M-1}\left(\frac{f_i}{1+g}(1 + sin(3\pi f_i))\right)$
MaF8	Multi-Point Distance Minimization Problem $f_1 = d(x, A_1), f_2 = d(x, A_2), \dots f_M = d(x, A_M)$
MaF9	Multi-Line Distance Minimization Problem $f_1 = d(x, A_1A_2), f_2 = d(x, A_2A_3), \dots f_M = d(x, A_1A_M)$
MaF10	$f_1 = y_M + 2\left(1 - cos\left(y_1\frac{\pi}{2}\right)\right) \dots \left(1 - cos\left(y_{M-1}\frac{\pi}{2}\right)\right) f_M = y_M + 2M\left(1 - y_1 - \frac{cos\left(10\pi y_1 + \frac{\pi}{2}\right)}{10\pi}\right), z_i = \frac{x_i}{2i} for\ i = 1,\dots,D$ ${t^1}_i = \begin{cases} z_i\ if\ i = 1,\dots,K \\ \frac{\lvert z_i - 0.35\rvert}{\lvert 0.35 - z_i\rvert + 0.35} if\ i = K+1,\dots,D \end{cases}, {t^2}_i = \begin{cases} {t^1}_i \\ 0.8 + \frac{0.8(0.75 - t_i^1)min(0,[t_i^1 - 0.75])}{0.75} - \frac{0.2(t_i^1 - 0.85)min(0,[0.85 - t_i^1])}{0.15} \end{cases}$ ${t^3}_i = {{t^2}_i}^{0.02}, {t^4}_i = \begin{cases} \frac{\sum 2j{t^3}_i}{\sum 2j} \\ \frac{\sum 2j{t^3}_i}{\sum 2j} \end{cases}, y_i = \begin{cases} ({t^4}_i - 0.5)max(1, {t^4}_i) + 0.5 \\ {t^4}_M \end{cases}$

MaF11	$f_1 = y_M + 2\left(1 - cos\left(y_1 \frac{\pi}{2}\right)\right) \dots \left(1 - cos\left(y_{M-1}\frac{\pi}{2}\right)\right) f_M = y_M + 2M(1 - y_1 cos^2(5\pi y_1)), z_i = \frac{x_i}{2i} for\ i = 1, \dots, D$ $t^1{}_i = \begin{cases} z_i\ if\ i = 1, \dots, K \\ \frac{\lvert z_i - 0.35\rvert}{\lvert 0.35 - z_i\rvert + 0.35} if\ i = K+1, \dots, D \end{cases}, t^2{}_i = \begin{cases} t^1{}_i \\ t^1{}_{K+2(i-K)-1} + t^1{}_{K+2(i-K)} + 2t^1{}_{K+2(i-K)-1} - t^1{}_{K+2(i-K)} \end{cases}$ $t^3{}_i = \begin{cases} \frac{\sum t^2{}_i}{K/M-1} \\ \frac{\sum t^2{}_i}{D-K/2} \end{cases}, y_i = \begin{cases} (t^3{}_i - 0.5)max(1, t^3{}_i) + 0.5 \\ t^3{}_M \end{cases}$
MaF12	$f_1 = y_M + 2\left(1 - cos\left(y_1 \frac{\pi}{2}\right)\right) \dots \left(1 - cos\left(y_{M-1}\frac{\pi}{2}\right)\right) f_M = y_M + 2M\left(1 - y_1 cos(\frac{\pi}{2} y_1)\right), z_i = \frac{x_i}{2i} for\ i = 1, \dots, D$ $y_i = \begin{cases} (t^3{}_i - 0.5)max(1, t^3{}_i) + 0.5 \\ t^3{}_M \end{cases}$
MaF13	$f_1 = sin\left(\frac{\pi}{2} x_1\right) + \frac{2}{J_1}\sum y_j^2, \dots, f_M = f_1^2 + f_2^{10} + f_3^{10} + \frac{2}{J_4}\sum y_j^2$
MaF14	$f_1 = x_1^f \dots x_{M-1}^f \left(1 + \sum c_{1,j} g_1\right), \dots, f_M = \left(1 - x_1^f\right)\left(1 + \sum c_{1,j} g_1\right)$
MaF15	$f_1 = \left(1 - cos\left(\frac{\pi}{2} x_1^f\right) \dots cos\left(\frac{\pi}{2} x_{M-1}^f\right)\right)\left(1 + \sum c_{1,j} g_1\right), \dots, f_M = 1 - sin\left(\frac{\pi}{2} x_1^f\right)\left(1 + \sum c_{1,j} g_1\right)$

2.1. Fundamentals of MOEA/D Algorithm:

The MOEA/D algorithm is proposed by (Zhang and Li, 2007). The algorithm is an evolutionary algorithm (EA) that means genetic operators are applied to the algorithm. The algorithm begins with randomly generated members and weight vectors. Also, each iteration based on the objective value of the members in population the current utopia point is updated and used in decomposition method. To generate offerings, crossover method is used. The original MOEA/D algorithm uses SBX crossover method. The SBX method is one of the important improvements on optimization society that makes it possible to update Genetic Algorithm from bit stream chromosome definition to real number chromosome definition (Deb and Agrawal, 1995). The SBX crossover is a mathematical calculation uses two parents (p) and generates offspring (c). The definition of the SBX crossover method is given below,

$$c_1 = x - \frac{1}{2}\beta(p_2 - p_1) \quad (2)$$

$$c_2 = x + \frac{1}{2}\beta(p_2 - p_1) \quad (3)$$

where, c is the offspring and p is the parents. In MOEA/D algorithm the parents are selected randomly from the neighbor of the current member. In this research, SBX and DE formulations are used to generate offspring. After offspring are generated, mutation operator is applied. The well-known polynomial mutation operator is applied to all of the algorithms in this research. Then as the final genetic operator the selection operator is applied. As the selection operator decomposition idea is used in MOEA/D algorithm. As decomposition method many possible methods are applied and in (Altinoz, 2022b) the author was compared them. The results indicates that Tchebycheff method gives good results in overall. Tchebycheff method presented as;

$$g(x) = max(w_i \lvert f_i(x) - z_i \rvert) \quad (4)$$

where w is the weight f is the objective value and z is the utopia point. By using the weight vector and objective value with utopia points the members with the neighbors are compared with each other and the best members survived to the next generation. Finally based on the Euclidean distance between survived members the neighborhood matrix is updated. Also, since the objective values are changed during each iteration the utopia point also updated. These steps are repeated until termination conditions are met.

2.2. Proposed Methods:

The DE member update rules are selected as crossover operator in MOEA/D algorithm. Previously the author applied these rules without considering the best member in (Altinoz,2022). However, in this research three methods for a possible detection of the best member is compared in this research and four DE rules are selected for this purpose, which are DE/best/1 (MOEADDE1), DE/best/2 (MOEADDE2), DE/current-to-best/1 (MOEADDE3), and DE/current-to-best/2 (MOEADDE4) methods will be compared in this research. The formulations are given below.

$$x_i = x_{best} + F(x_{r1} - x_{r2}) \tag{5}$$

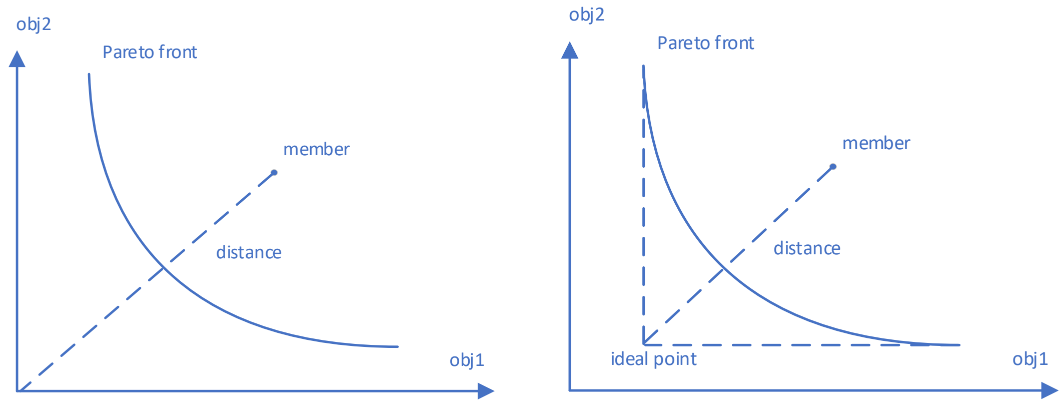
$$x_i = x_{best} + F(x_{r1} - x_{r2}) + F(x_{r3} - x_{r4}) \tag{6}$$

$$x_i = x_i + rand(x_{best} - x_i) + F(x_{r1} - x_{r2}) \tag{7}$$

$$x_i = x_i + rand(x_{best} - x_i) + F(x_{r1} - x_{r2}) + +F(x_{r3} - x_{r4}) \tag{8}$$

where r1, r2, r3, and r4 are the randomly selected index. Also, rand is the uniformly randomly number generator. These equations are used to generate population (offspring) in MOEA/D as crossover operator. For single objective optimization problems (for minimization) the smallest objective value gives the best result and corresponding best member. However, for multiobjective optimization algorithms it is not possible just look at a single objective value. For this reason, in this research three possible methods (cases) are used and compared with each other and the original MOEA/D algorithm with SBX crossover.

Case 1: The objective values are summed and the smallest are select as the best member. The objective values for each member are summed and sorted and the member with the smallest values is selected as the best member.
Case 2: The distance between the member and the origin is calculated and the member with the smallest distance is selected as the best member.
Case 3: The distance between member and the current nadir point is calculated. Also, the nadir point is updated through iterations. This distance is recorded and similarly the smallest is selected as the best member. In the following figure Case 2 and Case 3 are demonstrated.



(a) (b)

Figure 1. Proposed method for determine the best member a) Case 2 and b) Case 3

2.2. Performance Measurements:

Ion single objective optimization problems (for example minimization problem) the member who present the smallest objective value is the best solution. Therefore, it is relatively easy to compare members by only looking the objective value. If an algorithm using almost same resources and present smallest objective value indicates the superior performance of this algorithm. However, for multiobjective optimization problems, it is not possible to look only objective values. Since there are many objectives are needed to be minimized, it is not possible to get minimum value for all objectives. (The point which gives the utopic point or ideal point which is not possible to get that solution) Therefore instead of a single solution a set of solutions obtain from multiobjective optimization algorithm. For this reason, a toolset is needed to compare these set with another set that is produced from another optimization algorithm. This toolset is called metrics. Metrics are functions that evaluate this set and produce a single value. However, each metric is used for one purpose (like hypervolume metric it is possible to use two purposed); accuracy and distribution. The accuracy is the property of metric that gives hao the solution set is better in numerical number. The distribution is the property of the metric that gives how the solution set distributed on the objective space. It is expected that bath properties of any result are better. In this research IGD metric -for accuracy- and Spread metric -for distribution- are selected as metrics of the research. Inverted generalized distance (IGD) metric is given as (Ishibuchi, 2017)

$$f_{IGD} = \frac{\sum dist(a,P)}{|P|} \tag{9}$$

The IGD metric is based on computing the average distance between obtained solution candidates and the Pareto Front. The function $dist(a, P) = \sqrt{\sum(a_i - p_i)^2}$ is the distance calculation.

The second metric is defined in below (Ehregot 2000), where it is based on the calculation of the normalized squared sum of the distance between maximum and minimum difference between produced solutions and Pareto Front set (P).

$$f_{Spread} = \sqrt{\frac{1}{M}\sum\left(\frac{\max(a,P)-\min(a,P)}{P_{max}-P_{min}}\right)^2} \quad (10)$$

3.Implementation and Results

As a review, the aim of this research is to demonstrate the effect of the best member selection and possible selection methods (three methods), and also DE formulation (four different DE formulations) effects on the performance of the MOEA/D algorithm. Therefore 15 benchmark problems are selected and applied to the problem with 5, 10 and 15 objectives. Also, the results are compared with the non-best member DE formulation results in (Altinoz, 2022). The population size is selected as 125, 250 and 375 for 5 objective, 10 objective and 15 objective problems respectively. As the termination criteria maximum number of function evaluation is selected. Their values are 10^6, 2.10^6 and 3.10^6 respectively. Each implementation are repeated 10 times and their statistics are recorded as mean and standard deviation. Also rank sum test for each solution is calculated and recorded as statistical test result (at the end of each column and next to each result). The implementations are group with respect to the number of objectives and then as sub-group Case 1, Case 2, and Case 3 as proposed best member detection methods. At below, the numerical results are given as tables with respect to the number of objectives:

Objective 5:

Case 1:

Table 2. IGD metric values for 5 objective benchmark problems, Case 1

Prob	M	D	MOEADDE1C1	MOEADDE2C1	MOEADDE3C1	MOEADDE4C1	MOEAD
MaF1	5	14	3.0513e-1 (8.17e-2) -	2.6057e-1 (8.40e-2) =	2.5684e-1 (6.58e-2) =	**1.6842e-1 (1.82e-2) +**	2.2571e-1 (5.96e-7)
MaF2	5	14	1.6190e-1 (1.13e-2) -	1.5631e-1 (1.81e-2) -	1.6573e-1 (1.62e-2) -	1.4796e-1 (8.93e-3) -	**1.2509e-1 (6.05e-4)**
MaF3	5	14	1.5790e-1 (1.29e-2) -	1.8735e-1 (7.88e-3) -	1.3833e-1 (7.75e-3) -	1.8093e-1 (6.55e-3) -	**1.2238e-1 (1.04e-4)**
MaF4	5	14	7.5613e+0 (1.48e+0) =	**3.2977e+0 (2.54e-1) +**	8.1774e+0 (3.00e-1) +	3.2814e+2 (6.89e+2) -	8.6443e+0 (1.72e-1)
MaF5	5	14	**6.0338e+0 (2.88e-1) +**	6.5737e+0 (1.14e+0) +	6.9433e+0 (1.41e+0) =	6.8184e+0 (1.21e+0) =	8.0338e+0 (4.51e-2)
MaF6	5	14	2.7657e-1 (5.43e-2) -	1.7879e-1 (8.34e-2) -	2.4633e-1 (6.86e-2) -	5.4927e-2 (6.93e-2) -	**2.5046e-2 (2.41e-7)**
MaF7	5	24	1.1347e+0 (3.27e-1) =	9.4777e-1 (1.51e-1) =	9.5396e-1 (1.39e-1) =	**9.1863e-1 (8.75e-2) +**	1.0655e+0 (6.86e-2)
MaF8	5	2	**1.2577e-1 (2.10e-3) +**	1.3307e-1 (1.30e-3) +	1.5155e-1 (1.30e-2) +	1.3314e-1 (2.80e-3) +	2.6843e-1 (4.74e-3)
MaF9	5	2	1.5257e-1 (1.09e-2) -	1.4684e-1 (1.27e-3) -	1.4547e-1 (1.95e-3) =	1.5268e-1 (1.34e-3) -	**1.4469e-1 (2.12e-5)**
MaF10	5	14	1.1762e+0 (8.73e-2) -	1.1975e+0 (8.46e-2) -	2.0872e+0 (1.21e-1) -	2.2237e+0 (9.58e-2) -	**7.1573e-1 (9.73e-3)**
MaF11	5	14	8.7662e-1 (7.67e-2) -	1.1037e+0 (1.02e-1) -	8.9257e-1 (1.06e-1) -	1.0529e+0 (1.28e-1) -	**6.9300e-1 (2.29e-3)**
MaF12	5	14	1.9858e+0 (9.09e-2) -	1.9536e+0 (9.86e-2) -	2.0318e+0 (1.66e-1) -	1.8639e+0 (1.30e-1) -	**1.4027e+0 (4.01e-2)**
MaF13	5	5	4.1993e-1 (4.69e-2) -	3.0544e-1 (8.38e-2) -	3.6611e-1 (7.28e-2) -	2.2406e-1 (3.67e-2) -	**1.5518e-1 (4.12e-3)**
MaF14	5	100	8.7815e-1 (8.66e-2) =	9.1385e-1 (2.03e-2) -	2.1756e+0 (3.03e+0) -	9.0924e-1 (3.38e-2) -	**7.2503e-1 (2.61e-1)**
MaF15	5	100	7.6516e-1 (3.35e-2) -	7.8671e-1 (4.91e-2) -	8.1061e-1 (4.09e-2) -	8.3068e-1 (4.78e-2) -	**6.2647e-1 (7.94e-2)**
+/-/=			2/10/3	3/10/2	2/9/4	3/11/1	

Table 3. Spread metric values for 5 objective benchmark problems, Case 1

Problem	M	D	MOEADDE1C1	MOEADDE2C1	MOEADDE3C1	MOEADDE4C1	MOEAD
MaF1	5	14	8.6190e-1 (1.94e-1) +	7.1836e-1 (1.51e-1) +	7.6816e-1 (1.43e-1) +	**2.2988e-1 (8.05e-2) +**	1.8046e+0 (4.22e-2)
MaF2	5	14	1.0305e+0 (9.17e-2) -	7.1962e-1 (6.73e-2) -	9.8939e-1 (4.98e-2) -	6.3324e-1 (5.37e-2) -	**3.6317e-1 (3.18e-3)**
MaF3	5	14	1.6116e+0 (3.16e-1) -	1.3457e+0 (1.90e-1) -	1.6439e+0 (3.69e-1) -	1.3070e+0 (9.12e-2) -	**4.8908e-1 (1.14e-3)**
MaF4	5	14	1.1158e+0 (5.99e-2) -	1.0458e+0 (3.74e-1) =	1.1756e+0 (5.63e-2) -	1.1575e+0 (3.90e-1) =	**9.3785e-1 (1.38e-2)**
MaF5	5	14	1.7529e+0 (7.99e-2) -	1.6163e+0 (2.32e-1) -	1.5738e+0 (2.47e-1) -	1.4362e+0 (1.81e-1) -	**6.2711e-1 (5.24e-3)**
MaF6	5	14	1.0494e+0 (1.93e-2) +	1.1115e+0 (1.24e-1) +	**1.0478e+0 (1.95e-2) +**	1.5364e+0 (2.55e-1) +	1.7017e+0 (2.45e-2)
MaF7	5	24	1.1623e+0 (9.61e-2) -	1.0922e+0 (1.18e-1) -	1.1776e+0 (1.06e-1) -	9.7393e-1 (5.09e-2) -	**8.9376e-1 (1.81e-2)**
MaF8	5	2	8.5307e-1 (4.67e-2) -	9.1921e-1 (3.51e-2) -	8.8642e-1 (8.48e-2) -	9.0414e-1 (3.60e-2) -	**7.0644e-1 (3.49e-2)**
MaF9	5	2	1.6357e+0 (4.97e-1) -	1.7480e+0 (4.49e-1) -	1.7263e+0 (5.13e-1) -	1.6356e+0 (4.48e-1) -	**5.8811e-1 (7.37e-4)**
MaF10	5	14	1.2471e+0 (8.05e-2) -	1.2937e+0 (1.49e-1) -	1.2066e+0 (1.49e-1) -	1.2082e+0 (1.06e-1) -	**5.4408e-1 (1.58e-3)**
MaF11	5	14	1.0876e+0 (4.93e-2) -	1.2350e+0 (1.19e-1) -	1.0210e+0 (7.38e-2) -	1.1124e+0 (8.70e-2) -	**5.3979e-1 (6.71e-3)**
MaF12	5	14	1.2673e+0 (6.76e-2) -	1.0161e+0 (9.22e-2) -	9.7930e-1 (1.57e-1) -	1.0269e+0 (1.20e-1) -	**3.8443e-1 (1.11e-2)**
MaF13	5	5	1.5415e+0 (3.62e-1) =	1.6854e+0 (2.97e-1) -	1.5047e+0 (3.45e-1) =	1.4346e+0 (1.80e-1) =	**1.3438e+0 (3.60e-2)**
MaF14	5	100	1.3985e+0 (4.01e-1) =	1.6208e+0 (3.93e-1) -	1.5826e+0 (4.75e-1) -	1.7014e+0 (4.33e-1) -	**1.1127e+0 (2.57e-1)**
MaF15	5	100	1.6052e+0 (1.90e-1) -	1.6675e+0 (1.48e-1) -	1.6532e+0 (1.65e-1) -	1.5745e+0 (1.88e-1) -	**7.2992e-1 (6.76e-2)**
+/-/=			2/11/2	2/12/1	2/12/1	2/11/2	

Case 2:

Table 4. IGD metric values for 5 objective benchmark problems, Case 2

Problem	M	D	MOEADDE1C2	MOEADDE2C2	MOEADDE3C2	MOEADDE4C2	MOEAD
MaF1	5	14	1.7195e-1 (2.66e-2) +	1.7281e-1 (1.82e-2) +	1.9157e-1 (2.20e-2) +	**1.6290e-1 (3.76e-3) +**	2.2571e-1 (5.96e-7)
MaF2	5	14	1.3429e-1 (4.98e-3) -	1.3869e-1 (3.97e-3) -	1.5895e-1 (1.92e-2) -	1.4226e-1 (4.16e-3) -	**1.2509e-1 (6.05e-4)**
MaF3	5	14	1.3702e-1 (1.30e-2) -	1.8887e-1 (4.76e-3) -	1.3886e-1 (1.05e-2) -	1.8449e-1 (7.03e-3) -	**1.2238e-1 (1.04e-4)**
MaF4	5	14	5.7765e+0 (5.43e-1) +	**3.4713e+0 (2.44e-1) +**	7.4869e+0 (2.11e-1) +	3.6349e+2 (6.29e+2) =	8.6443e+0 (1.72e-1)
MaF5	5	14	6.3823e+0 (3.88e-1) +	**5.8859e+0 (5.07e-1) +**	7.2073e+0 (1.62e+0) =	6.9783e+0 (1.06e+0) =	8.3126e+0 (7.36e-1)
MaF6	5	14	1.1944e-1 (1.13e-1) -	4.5252e-2 (6.03e-2) -	9.3951e-2 (1.25e-1) -	2.5773e-2 (4.34e-5) -	**2.5046e-2 (2.45e-7)**
MaF7	5	24	1.3010e+0 (4.31e-1) =	1.0965e+0 (1.35e-1) =	9.7604e-1 (1.43e-1) =	**9.0951e-1 (4.35e-2) +**	1.0740e+0 (5.07e-2)
MaF8	5	2	**1.2530e-1 (3.53e-3) +**	1.3401e-1 (1.29e-3) +	1.4702e-1 (1.07e-2) +	1.3338e-1 (2.89e-3) +	2.6670e-1 (3.96e-3)
MaF9	5	2	**1.3789e-1 (2.77e-3) +**	1.5345e-1 (1.23e-3) -	1.4454e-1 (2.02e-3) =	1.5278e-1 (6.83e-4) -	1.4470e-1 (9.51e-6)
MaF10	5	14	1.0999e+0 (5.48e-2) -	1.1911e+0 (1.23e-1) -	1.9614e+0 (1.29e-1) -	1.9931e+0 (4.97e-2) -	**7.1339e-1 (9.39e-3)**
MaF11	5	14	9.1082e-1 (5.14e-2) -	1.0803e+0 (1.14e-1) -	8.2684e-1 (8.34e-2) -	1.0541e+0 (9.42e-2) -	**6.9547e-1 (3.51e-3)**
MaF12	5	14	1.9581e+0 (1.33e-1) -	1.9771e+0 (6.88e-2) -	2.0749e+0 (1.27e-1) -	1.8774e+0 (1.02e-1) -	**1.4002e+0 (4.25e-2)**
MaF13	5	5	2.8114e-1 (7.98e-2) -	2.4101e-1 (5.11e-2) -	3.0900e-1 (7.65e-2) -	2.2331e-1 (4.69e-2) -	**1.5350e-1 (4.19e-3)**
MaF14	5	100	8.0004e-1 (1.51e-1) =	8.5009e-1 (8.60e-2) =	1.4990e+0 (2.08e+0) =	1.0851e+0 (7.21e-1) =	**6.6160e-1 (2.75e-1)**
MaF15	5	100	8.0118e-1 (6.35e-2) -	8.1567e-1 (4.81e-2) -	8.6214e-1 (6.04e-2) -	8.3187e-1 (5.94e-2) -	**5.9892e-1 (5.25e-2)**
+/-/=			5/8/2	4/9/2	3/8/4	3/9/3	

Table 5. Spread metric values for 5 objective benchmark problems, Case 2

Problem	M	D	MOEADDE1C2	MOEADDE2C2	MOEADDE3C2	MOEADDE4C2	MOEAD
MaF1	5	14	3.9550e-1 (1.61e-1) +	2.6420e-1 (1.27e-1) +	6.2289e-1 (9.60e-2) +	**2.0510e-1 (4.48e-2) +**	1.8046e+0 (4.22e-2)
MaF2	5	14	6.8938e-1 (8.27e-2) -	6.2215e-1 (2.98e-2) -	8.3173e-1 (1.08e-1) -	6.1812e-1 (3.19e-2) -	**3.6317e-1 (3.18e-3)**
MaF3	5	14	1.8950e+0 (3.49e-1) -	1.6851e+0 (2.44e-1) -	1.9997e+0 (6.84e-2) -	1.4479e+0 (3.08e-1) -	**4.8908e-1 (1.14e-3)**
MaF4	5	14	1.1190e+0 (6.89e-2) -	1.1229e+0 (4.87e-1) =	1.2171e+0 (4.18e-2) -	1.2854e+0 (4.53e-1) -	**9.3785e-1 (1.38e-2)**
MaF5	5	14	1.7416e+0 (1.33e-1) -	1.7174e+0 (1.76e-1) -	1.4520e+0 (2.24e-1) -	1.5909e+0 (2.44e-1) -	**6.7672e-1 (1.52e-1)**
MaF6	5	14	1.4627e+0 (3.79e-1) =	1.5934e+0 (1.98e-1) =	**1.4178e+0 (3.36e-1) +**	1.6905e+0 (5.23e-2) =	1.7080e+0 (1.33e-2)
MaF7	5	24	1.0648e+0 (6.04e-2) -	9.7371e-1 (6.10e-2) -	1.1775e+0 (1.52e-1) -	9.7473e-1 (7.22e-2) -	**8.8669e-1 (1.15e-2)**
MaF8	5	2	8.6503e-1 (3.73e-2) -	9.2394e-1 (2.60e-2) -	8.9425e-1 (5.83e-2) -	9.0551e-1 (3.75e-2) -	**6.7568e-1 (3.84e-2)**
MaF9	5	2	1.6871e+0 (4.69e-1) -	1.6180e+0 (4.53e-1) -	1.8402e+0 (3.39e-1) -	1.1333e+0 (3.31e-1) -	**5.8811e-1 (4.47e-4)**
MaF10	5	14	1.1949e+0 (5.53e-2) -	1.2489e+0 (1.02e-1) -	1.0813e+0 (5.89e-2) -	1.1528e+0 (9.12e-2) -	**5.4457e-1 (8.05e-4)**
MaF11	5	14	1.0603e+0 (4.91e-2) -	1.1404e+0 (8.32e-2) -	1.0243e+0 (8.29e-2) -	1.0804e+0 (8.46e-2) -	**5.4799e-1 (1.12e-2)**
MaF12	5	14	1.2113e+0 (6.94e-2) -	9.8513e-1 (8.75e-2) -	9.5481e-1 (1.04e-1) -	9.2961e-1 (9.60e-2) -	**3.7540e-1 (1.90e-2)**
MaF13	5	5	**1.2823e+0 (1.13e-1) =**	1.5194e+0 (2.69e-1) =	1.5369e+0 (3.27e-1) =	1.4944e+0 (2.27e-1) -	1.3359e+0 (2.89e-2)
MaF14	5	100	1.5066e+0 (4.83e-1) =	1.8200e+0 (3.42e-1) -	1.6773e+0 (3.95e-1) -	1.8881e+0 (3.03e-1) -	**1.0681e+0 (2.93e-1)**
MaF15	5	100	1.5656e+0 (1.60e-1) -	1.5108e+0 (1.82e-1) -	1.6109e+0 (1.76e-1) -	1.5172e+0 (1.84e-1) -	**7.4350e-1 (4.64e-2)**
+/-/=			1/11/3	1/11/3	2/12/1	1/13/1	

Case 3:

Table 6. IGD metric values for 5 objective benchmark problems, Case 3

Problem	M	D	MOEADDE1C3	MOEADDE2C3	MOEADDE3C3	MOEADDE4C3	MOEAD
MaF1	5	14	4.1171e-1 (2.94e-2) -	2.4163e-1 (5.54e-2) =	2.7845e-1 (5.73e-2) -	**1.9298e-1 (3.38e-2) +**	2.2571e-1 (5.96e-7)
MaF2	5	14	1.4192e-1 (8.72e-3) -	1.4148e-1 (5.16e-3) -	1.5987e-1 (1.62e-2) -	1.4637e-1 (1.09e-2) -	**1.2509e-1 (6.05e-4)**
MaF3	5	14	1.3487e-1 (7.59e-3) -	1.8941e-1 (4.16e-3) -	1.3293e-1 (8.57e-3) -	1.8713e-1 (8.76e-3) -	**1.2238e-1 (1.04e-4)**
MaF4	5	14	7.0495e+0 (6.88e-1) +	**3.4984e+0 (3.05e-1) +**	8.4224e+0 (3.52e-1) =	7.1494e+2 (9.19e+2) =	8.6443e+0 (1.72e-1)
MaF5	5	14	**6.1835e+0 (3.34e-1) +**	6.5343e+0 (1.11e+0) +	6.9296e+0 (1.40e+0) +	6.5400e+0 (9.24e-1) +	8.0970e+0 (1.42e-1)
MaF6	5	14	2.4654e-1 (8.47e-2) -	6.6787e-2 (8.43e-2) -	1.3751e-1 (1.29e-1) -	2.5766e-2 (6.92e-5) -	**2.5046e-2 (4.60e-7)**
MaF7	5	24	1.0589e+0 (2.93e-1) =	9.4394e-1 (1.55e-1) =	**8.7218e-1 (1.05e-1) +**	8.8235e-1 (1.37e-1) +	1.0376e+0 (8.42e-2)
MaF8	5	2	**1.2760e-1 (3.64e-3) +**	1.3308e-1 (1.73e-3) +	1.4643e-1 (9.49e-3) +	1.3215e-1 (2.21e-3) +	2.6890e-1 (4.36e-3)
MaF9	5	2	**1.3625e-1 (2.01e-3) +**	1.5376e-1 (1.00e-3) -	1.4268e-1 (1.72e-3) +	1.5206e-1 (7.01e-4) -	1.4470e-1 (1.22e-5)
MaF10	5	14	1.1000e+0 (6.10e-2) -	1.1370e+0 (7.94e-2) -	1.8236e+0 (2.30e-1) -	1.9836e+0 (8.22e-2) -	**7.2040e-1 (2.24e-3)**
MaF11	5	14	9.0548e-1 (7.98e-2) -	1.0948e+0 (1.24e-1) -	9.2579e-1 (8.65e-2) -	9.7888e-1 (9.44e-2) -	**6.9355e-1 (3.40e-3)**
MaF12	5	14	1.9985e+0 (8.63e-2) -	1.9631e+0 (1.05e-1) -	2.0283e+0 (8.90e-2) -	1.9031e+0 (7.47e-2) -	**1.4029e+0 (3.03e-2)**
MaF13	5	5	3.2675e-1 (2.25e-1) -	1.8941e-1 (2.91e-2) -	2.6572e-1 (8.75e-2) -	2.0626e-1 (3.73e-2) -	**1.5427e-1 (3.15e-3)**
MaF14	5	100	7.6367e-1 (8.11e-2) =	8.4087e-1 (1.10e-1) =	2.7007e+0 (4.37e+0) =	8.9089e-1 (4.78e-2) =	**7.1624e-1 (2.69e-1)**
MaF15	5	100	7.6223e-1 (3.77e-2) -	7.9963e-1 (3.50e-2) -	8.1992e-1 (3.72e-2) -	8.1279e-1 (5.61e-2) -	**6.3106e-1 (7.83e-2)**
+/-/=			4/9/2	3/9/3	4/9/2	4/9/2	

Table 7. Spread metric values for 5 objective benchmark problems, Case 3

Problem	M	D	MOEADDE1C3	MOEADDE2C3	MOEADDE3C3	MOEADDE4C3	MOEAD
MaF1	5	14	9.6502e-1 (8.93e-2) +	5.9910e-1 (2.02e-1) +	7.9132e-1 (1.20e-1) +	**3.8834e-1 (1.96e-1) +**	1.8046e+0 (4.22e-2)
MaF2	5	14	8.1647e-1 (6.86e-2) -	6.4634e-1 (3.92e-2) -	9.0866e-1 (1.01e-1) -	6.2824e-1 (3.47e-2) -	**3.6317e-1 (3.18e-3)**
MaF3	5	14	1.9952e+0 (1.04e-1) -	1.3642e+0 (1.82e-1) -	1.9926e+0 (7.84e-2) -	1.4059e+0 (2.94e-1) -	**4.8908e-1 (1.14e-3)**
MaF4	5	14	1.1450e+0 (4.44e-2) -	1.0310e+0 (3.60e-1) =	1.2274e+0 (5.95e-2) -	1.2230e+0 (3.78e-1) =	**9.3785e-1 (1.38e-2)**
MaF5	5	14	1.7051e+0 (1.04e-1) -	1.6281e+0 (2.68e-1) -	1.4586e+0 (2.27e-1) -	1.5629e+0 (2.14e-1) -	**6.3386e-1 (8.22e-3)**
MaF6	5	14	**1.1663e+0 (2.98e-1) +**	1.4959e+0 (2.92e-1) =	1.2963e+0 (3.28e-1) +	1.6968e+0 (5.19e-2) =	1.7047e+0 (1.15e-2)
MaF7	5	24	1.1856e+0 (1.21e-1) -	1.0105e+0 (9.04e-2) -	1.2309e+0 (1.65e-1) -	1.0348e+0 (6.90e-2) -	**8.9025e-1 (1.73e-2)**
MaF8	5	2	8.7262e-1 (4.23e-2) -	9.1454e-1 (1.71e-2) -	9.4444e-1 (1.03e-1) -	9.2147e-1 (2.08e-2) -	**7.1959e-1 (4.43e-2)**
MaF9	5	2	1.7388e+0 (4.55e-1) -	1.1000e+0 (2.26e-1) -	1.7864e+0 (3.70e-1) -	1.3357e+0 (5.03e-1) -	**5.8824e-1 (3.69e-4)**
MaF10	5	14	1.2147e+0 (9.63e-2) -	1.1117e+0 (9.14e-2) -	1.1012e+0 (1.19e-1) -	1.1373e+0 (1.13e-1) -	**5.4410e-1 (1.07e-3)**
MaF11	5	14	1.0568e+0 (4.37e-2) -	1.1799e+0 (1.51e-1) -	1.0256e+0 (7.03e-2) -	1.0938e+0 (7.53e-2) -	**5.4407e-1 (1.08e-2)**
MaF12	5	14	1.2303e+0 (1.07e-1) -	1.0061e+0 (9.31e-2) -	8.9484e-1 (1.16e-1) -	9.1009e-1 (7.54e-2) -	**3.6826e-1 (2.58e-2)**
MaF13	5	5	1.5778e+0 (3.30e-1) =	1.4974e+0 (1.45e-1) -	1.6192e+0 (2.86e-1) -	1.5691e+0 (2.42e-1) -	**1.3541e+0 (4.80e-2)**
MaF14	5	100	1.6306e+0 (3.88e-1) -	1.7576e+0 (4.04e-1) -	1.5768e+0 (4.33e-1) -	1.7742e+0 (3.72e-1) -	**1.0499e+0 (1.97e-1)**
MaF15	5	100	1.5317e+0 (1.57e-1) -	1.5714e+0 (1.65e-1) -	1.6361e+0 (1.47e-1) -	1.4174e+0 (2.96e-1) -	**7.5840e-1 (2.44e-2)**
+/-/=			2/12/1	1/12/2	2/13/0	1/12/2	

Evaluation of the Results for Objective 5: In general, when case1, 2 and 3 is evaluated and compared with the original MOEAD algorithm, the results indicated the MOEA/D algorithm presents better results for both accuracy and distribution. For Case 1 MaF4,5,8,7 (which is only approximately 25% of all benchmark problems); for Case 2 MaF1,4,5,7,8,9 (approximately 40%); and Case 3 MaF1,4,5,7,8,9 (approximately 40%) the proposed methods gives better result, however still it is not possible to mention about the best proposed method. However, it is clear that Case 2 and Case 3 gives better result than Case 1. Therefore, it can be indicated from the results that summing the objectives and sorting the total value is not a good indicator for a best member.

When the results are compared with the results in (Altinoz, 2022) in general the best member proposal for the DE formulations reduce the performance of the algorithm's accuracy. It is expected from the addition of the best member to the crossover operator to drag the solutions to the Pareto front faster. However, that decreases the exploration property for the algorithm. When the results are investigated in detailed only for the benchmark problems MaF3,9,11the proposed methods gives better than the results in (Altinoz, 2022).

Objective 10:

Case 1:

Table 8. IGD metric values for 10 objective benchmark problems, Case 1

Problem	M	D	MOEADDE1C1	MOEADDE2C1	MOEADDE3C1	MOEADDE4C1	MOEAD
MaF1	10	19	4.2438e-1 (4.40e-2) =	4.4077e-1 (7.67e-2) =	4.7559e-1 (8.21e-2) =	**3.1916e-1 (3.40e-2) +**	4.6906e-1 (1.16e-4)
MaF2	10	19	4.1404e-1 (2.08e-2) -	6.0298e-1 (4.57e-2) -	4.9452e-1 (3.57e-2) -	3.8741e-1 (6.85e-2) -	**3.2263e-1 (1.06e-5)**
MaF3	10	19	**1.3070e-1 (3.04e-3) +**	1.4187e-1 (6.30e-3) =	1.3273e-1 (8.19e-3) =	1.4409e-1 (1.65e-2) =	1.3967e-1 (4.52e-5)
MaF4	10	19	2.8221e+2 (3.35e+1) +	**9.4310e+1 (1.99e+1) +**	2.9062e+2 (3.64e+1) +	5.9807e+3 (1.32e+4) =	4.0524e+2 (1.30e+1)
MaF5	10	19	3.0508e+2 (9.91e-1) -	3.0437e+2 (8.38e-1) -	**2.8970e+2 (2.76e+1) =**	3.0439e+2 (1.74e+0) -	2.9926e+2 (1.51e-1)
MaF6	10	19	2.0170e-1 (3.67e-2) -	2.6617e-2 (1.71e-4) -	2.6185e-2 (1.81e-4) -	2.6513e-2 (3.23e-5) -	**2.4645e-2 (3.24e-6)**
MaF7	10	29	1.6394e+0	1.4712e+0	1.5748e+0	**1.4600e+0**	2.6776e+0

			(1.43e-1) +	(7.67e-2) +	(1.90e-1) +	**(5.14e-2) +**	(3.30e-1)
MaF8	10	2	1.4369e-1 (2.77e-3) +	1.4612e-1 (6.13e-4) +	1.7544e-1 (1.13e-2) +	**1.4242e-1 (1.50e-3) +**	9.1366e-1 (4.07e-3)
MaF9	10	2	2.3153e-1 (2.12e-2) +	2.2741e-1 (7.67e-3) +	**2.2368e-1 (1.56e-2) +**	2.2662e-1 (9.21e-3) +	3.1497e-1 (2.87e-4)
MaF10	10	19	1.9113e+0 (2.89e-2) -	2.0337e+0 (7.42e-2) -	2.8198e+0 (2.65e-1) -	2.8910e+0 (3.59e-1) -	**1.6490e+0 (8.34e-3)**
MaF11	10	19	1.6495e+0 (1.44e-1) =	1.9933e+0 (1.21e-1) -	**1.5182e+0 (1.05e-1) +**	1.7556e+0 (1.56e-1) =	1.7833e+0 (1.26e-2)
MaF12	10	19	8.9568e+0 (1.02e+0) =	7.7479e+0 (1.44e+0) =	7.2210e+0 (1.14e+0) =	**6.8136e+0 (1.05e+0) =**	7.1852e+0 (1.33e+0)
MaF13	10	5	5.8756e-1 (4.05e-1) =	4.6976e-1 (5.65e-2) +	5.0890e-1 (1.08e-1) +	**3.5340e-1 (3.05e-2) +**	1.1805e+0 (1.59e-2)
MaF14	10	200	9.6076e-1 (6.34e-2) -	1.0045e+0 (6.76e-3) -	1.0130e+0 (5.75e-3) -	1.0091e+0 (4.94e-3) -	**4.4221e-1 (7.40e-2)**
MaF15	10	200	1.1306e+0 (1.66e-2) -	2.4969e+0 (6.90e-1) -	1.2684e+0 (1.84e-1) -	5.5471e+0 (1.56e+0) -	**9.9730e-1 (3.11e-2)**
+/-/=			5/6/4	5/7/3	6/5/4	5/6/4	

Table 9. Spread metric values for 10 objective benchmark problems, Case 1

Problem	M	D	MOEADDE1C1	MOEADDE2C1	MOEADDE3C1	MOEADDE4C1	MOEAD
MaF1	10	19	1.0644e+0 (5.39e-2) +	1.1077e+0 (7.46e-2) +	1.1703e+0 (1.38e-1) =	**5.2260e-1 (1.57e-1) +**	1.3020e+0 (2.41e-2)
MaF2	10	19	1.2057e+0 (2.03e-2) -	1.0215e+0 (2.37e-2) -	1.1542e+0 (2.01e-2) -	1.1489e+0 (2.48e-2) -	**6.8028e-1 (4.25e-3)**
MaF3	10	19	1.4963e+0 (2.31e-1) -	1.0037e+0 (6.77e-2) -	1.3109e+0 (2.29e-1) -	1.0456e+0 (4.80e-2) -	**6.4890e-1 (6.26e-4)**
MaF4	10	19	**1.1750e+0 (1.53e-2) +**	1.4730e+0 (5.45e-1) =	1.2617e+0 (6.30e-2) =	1.5009e+0 (4.94e-1) =	1.3283e+0 (1.19e-2)
MaF5	10	19	**1.0110e+0 (2.07e-3) +**	1.0155e+0 (4.75e-3) =	1.0900e+0 (1.63e-1) =	1.0150e+0 (4.62e-3) =	1.0257e+0 (9.70e-3)
MaF6	10	19	**1.0407e+0 (1.59e-2) +**	1.8744e+0 (1.49e-1) =	1.8133e+0 (1.99e-1) =	1.8472e+0 (5.76e-2) =	1.7803e+0 (4.32e-4)
MaF7	10	29	1.0094e+0 (8.28e-2) =	1.0882e+0 (7.49e-2) -	1.0435e+0 (3.12e-2) -	1.0392e+0 (3.26e-2) -	**9.8075e-1 (9.25e-3)**
MaF8	10	2	9.8234e-1 (7.25e-2) =	1.0903e+0 (2.75e-2) -	1.0224e+0 (3.14e-2) -	1.0175e+0 (4.26e-2) -	**9.2506e-1 (9.54e-3)**
MaF9	10	2	2.0347e+0 (1.31e-2) -	1.9900e+0 (6.31e-2) -	2.0220e+0 (2.72e-2) -	2.0347e+0 (2.99e-2) -	**1.0085e+0 (7.81e-4)**
MaF10	10	19	1.1635e+0 (5.99e-2) -	1.1281e+0 (4.73e-2) -	1.1304e+0 (9.12e-2) -	1.0973e+0 (5.27e-2) -	**7.5024e-1 (4.87e-3)**
MaF11	10	19	9.7174e-1 (2.96e-2) -	9.4393e-1 (3.30e-2) -	9.9059e-1 (5.37e-2) -	1.0376e+0 (7.18e-2) -	**7.4275e-1 (6.46e-4)**
MaF12	10	19	1.3031e+0 (3.02e-2) -	1.3027e+0 (4.21e-2) -	1.2288e+0 (5.69e-2) -	1.2946e+0 (8.84e-2) -	**1.0300e+0 (1.58e-1)**
MaF13	10	5	1.6775e+0 (3.65e-1) -	1.1861e+0 (1.77e-1) -	1.5917e+0 (2.65e-1) -	1.3993e+0 (6.69e-2) -	**1.0294e+0 (4.10e-3)**
MaF14	10	200	1.7064e+0 (3.84e-1) -	1.8852e+0 (1.62e-1) -	1.9510e+0 (4.37e-2) -	1.9962e+0 (3.83e-1) -	**1.0181e+0 (2.85e-1)**
MaF15	10	200	1.0355e+0 (3.28e-1) =	9.0706e-1 (6.04e-2) =	1.3784e+0 (2.80e-1) -	9.9498e-1 (1.42e-1) =	**8.3399e-1 (1.03e-1)**
+/-/=			4/8/3	1/10/4	0/11/4	1/10/4	

Case 2:

Table 10. IGD metric values for 10 objective benchmark problems, Case 2

Problem	M	D	MOEADDE1C2	MOEADDE2C2	MOEADDE3C2	MOEADDE4C2	MOEAD
MaF1	10	19	**2.6803e-1 (8.72e-3) +**	2.9278e-1 (2.91e-2) +	2.8092e-1 (1.28e-2) +	2.7649e-1 (8.29e-3) +	4.6906e-1 (1.16e-4)
MaF2	10	19	4.4023e-1 (3.47e-2) -	3.6386e-1 (1.26e-2) -	3.9007e-1 (6.49e-2) =	**3.2201e-1 (2.37e-2) =**	3.2262e-1 (1.28e-5)
MaF3	10	19	**1.2248e-1 (5.27e-3) +**	1.5426e-1 (1.70e-2) =	1.3190e-1 (8.83e-3) =	1.5995e-1 (1.29e-2) -	1.3966e-1 (1.98e-5)
MaF4	10	19	2.5567e+2 (1.47e+1) +	**9.1002e+1 (1.25e+1) +**	3.0609e+2 (8.41e+0) +	1.7825e+4 (2.62e+4) =	4.0460e+2 (4.62e+0)
MaF5	10	19	3.0472e+2 (6.03e-1) -	3.0536e+2 (7.17e-1) -	**2.9355e+2 (1.42e+1) =**	3.0154e+2 (8.53e+0) =	2.9930e+2 (2.62e-1)

MaF6	10	19	5.4134e-2 (3.38e-2) =	5.2700e-2 (5.79e-2) =	3.5767e-2 (2.23e-2) =	**2.6575e-2 (7.12e-5) =**	2.3449e-1 (2.90e-1)
MaF7	10	29	1.6151e+0 (1.05e-1) =	1.6612e+0 (9.44e-2) =	1.7183e+0 (1.72e-1) =	**1.4886e+0 (6.28e-2) =**	2.4116e+0 (6.34e-1)
MaF8	10	2	**1.4279e-1 (1.78e-3) +**	1.4612e-1 (6.53e-4) +	1.7340e-1 (6.42e-3) +	1.4299e-1 (1.10e-3) +	9.1394e-1 (3.08e-3)
MaF9	10	2	2.5329e-1 (1.91e-2) +	2.8442e-1 (1.96e-2) +	**2.1470e-1 (5.25e-3) +**	2.2355e-1 (1.00e-2) +	3.1496e-1 (1.62e-4)
MaF10	10	19	1.8650e+0 (4.36e-2) -	1.9659e+0 (8.95e-2) -	2.3922e+0 (2.91e-1) -	2.8914e+0 (2.39e-1) -	**1.6463e+0 (8.18e-3)**
MaF11	10	19	**1.4663e+0 (7.44e-2) +**	1.5590e+0 (6.15e-2) +	1.4724e+0 (9.36e-2) +	1.6329e+0 (1.33e-1) =	1.7870e+0 (6.03e-3)
MaF12	10	19	6.6217e+0 (2.54e-1) +	6.5651e+0 (3.02e-1) +	6.5785e+0 (5.06e-1) +	**6.2329e+0 (5.29e-1) +**	7.6520e+0 (2.65e-1)
MaF13	10	5	4.6448e-1 (9.64e-2) +	4.2276e-1 (4.41e-2) +	**4.0841e-1 (7.20e-2) +**	4.1580e-1 (4.63e-2) +	1.1735e+0 (8.69e-3)
MaF14	10	200	8.7082e-1 (1.55e-1) -	9.9599e-1 (1.39e-2) -	1.0046e+0 (6.33e-3) -	1.8696e+0 (1.23e+0) -	**4.7528e-1 (8.40e-2)**
MaF15	10	200	1.1257e+0 (2.45e-2) -	3.9960e+0 (1.46e+0) -	1.2617e+0 (1.19e-1) -	7.3658e+0 (6.26e-1) -	**9.5793e-1 (1.96e-2)**
+/-=			8/5/2	7/5/3	7/3/5	5/4/6	

Table 11. Spread metric values for 10 objective benchmark problems, Case 2

Problem	M	D	MOEADDE1C2	MOEADDE2C2	MOEADDE3C2	MOEADDE4C2	MOEAD
MaF1	10	19	3.5217e-1 (1.08e-1) +	4.2831e-1 (2.22e-1) +	6.0579e-1 (8.76e-2) +	**2.0155e-1 (1.76e-2) +**	1.3020e+0 (2.41e-2)
MaF2	10	19	1.1717e+0 (1.02e-2) -	1.1207e+0 (3.16e-2) -	1.1782e+0 (3.60e-2) -	1.1115e+0 (7.90e-3) -	**6.8016e-1 (8.41e-3)**
MaF3	10	19	1.4687e+0 (2.97e-1) -	1.0815e+0 (8.27e-2) -	1.8468e+0 (1.17e-1) -	1.0933e+0 (1.30e-1) -	**6.4955e-1 (9.65e-4)**
MaF4	10	19	**1.1626e+0 (8.21e-2) +**	1.8460e+0 (4.57e-1) =	1.2675e+0 (5.14e-2) =	1.6807e+0 (4.39e-1) =	1.3357e+0 (2.89e-3)
MaF5	10	19	**1.0110e+0 (1.44e-3) +**	1.0120e+0 (4.67e-3) +	1.0518e+0 (3.88e-2) =	1.0138e+0 (8.79e-3) =	1.0204e+0 (1.80e-3)
MaF6	10	19	**1.4700e+0 (3.33e-1) =**	1.8116e+0 (4.01e-1) =	1.9518e+0 (2.36e-1) =	1.8960e+0 (3.65e-2) -	1.4730e+0 (4.21e-1)
MaF7	10	29	**9.5766e-1 (1.05e-1) =**	1.0462e+0 (7.78e-2) =	9.6027e-1 (6.79e-2) =	1.0706e+0 (2.99e-2) -	9.9483e-1 (3.19e-2)
MaF8	10	2	9.8344e-1 (3.06e-2) -	1.0804e+0 (2.27e-2) -	1.0499e+0 (1.85e-2) -	1.0269e+0 (3.40e-2) -	**9.1973e-1 (5.51e-3)**
MaF9	10	2	1.9849e+0 (1.37e-2) -	2.0309e+0 (3.97e-2) -	2.0194e+0 (1.01e-2) -	1.8698e+0 (8.74e-2) -	**1.0094e+0 (3.06e-4)**
MaF10	10	19	1.1853e+0 (2.30e-2) -	1.1540e+0 (9.58e-2) -	1.0515e+0 (2.56e-2) -	1.0728e+0 (1.55e-2) -	**7.4746e-1 (7.28e-4)**
MaF11	10	19	9.9237e-1 (1.26e-2) -	1.0249e+0 (2.03e-2) -	9.6491e-1 (5.44e-2) -	1.1168e+0 (7.55e-2) -	**7.4217e-1 (1.17e-3)**
MaF12	10	19	1.4515e+0 (8.22e-2) -	1.3705e+0 (2.50e-2) -	1.2334e+0 (1.58e-1) =	1.3272e+0 (9.08e-2) -	**1.0738e+0 (2.82e-2)**
MaF13	10	5	1.2186e+0 (1.73e-1) -	1.5392e+0 (2.86e-1) -	1.5362e+0 (2.58e-1) -	1.3913e+0 (3.27e-1) -	**1.0289e+0 (1.08e-3)**
MaF14	10	200	1.9764e+0 (1.79e-1) -	2.0127e+0 (1.63e-1) -	1.8738e+0 (9.71e-2) -	2.1087e+0 (1.46e-1) -	**1.1057e+0 (2.95e-1)**
MaF15	10	200	1.0632e+0 (3.23e-1) =	9.3339e-1 (6.04e-2) -	1.1625e+0 (1.66e-1) -	1.0224e+0 (9.56e-2) -	**8.5083e-1 (3.99e-2)**
+/-=			3/9/3	2/10/3	1/9/5	1/12/2	

Case 3:

Table 12. IGD metric values for 10 objective benchmark problems, Case 3

Problem	M	D	MOEADDE1C3	MOEADDE2C3	MOEADDE3C3	MOEADDE4C3	MOEAD
MaF1	10	19	5.5535e-1 (2.45e-2) -	4.5248e-1 (5.85e-2) =	5.5167e-1 (3.21e-2) -	**3.4447e-1 (6.72e-2) +**	4.6906e-1 (1.16e-4)
MaF2	10	19	4.1564e-1 (1.88e-2) -	5.4589e-1 (3.58e-2) -	4.6433e-1 (5.16e-2) -	3.6301e-1 (5.91e-2) =	**3.2263e-1 (1.52e-5)**
MaF3	10	19	**1.2596e-1 (1.12e-2) +**	1.4475e-1 (5.97e-3) =	1.3124e-1 (1.01e-2) =	1.6797e+3 (3.76e+3) =	1.3966e-1 (2.88e-5)
MaF4	10	19	3.3644e+2 (5.13e+0) +	**9.7998e+1 (9.84e+0) +**	3.3788e+2 (1.33e+1) +	7.4023e+3 (1.02e+4) =	4.0071e+2 (7.72e+0)

MaF5	10	19	3.0418e+2 (1.31e+0) -	3.0459e+2 (2.09e+0) -	**2.8281e+2 (4.44e+1) =**	3.0307e+2 (3.34e+0) =	2.9934e+2 (3.45e-1)
MaF6	10	19	2.1735e-1 (4.24e-2) -	1.0548e-1 (1.09e-1) -	1.0332e-1 (9.74e-2) =	2.6627e-2 (2.45e-4) -	**2.4645e-2 (2.45e-6)**
MaF7	10	29	1.5279e+0 (8.85e-2) +	1.4644e+0 (8.68e-2) +	1.5486e+0 (1.08e-1) +	**1.3980e+0 (1.01e-1) +**	2.4232e+0 (3.01e-1)
MaF8	10	2	1.4301e-1 (3.11e-3) +	1.4597e-1 (1.35e-3) +	1.8484e-1 (1.42e-2) +	**1.4298e-1 (1.14e-3) +**	9.1153e-1 (4.62e-3)
MaF9	10	2	2.7965e-1 (1.73e-2) +	3.0353e-1 (3.59e-3) +	**2.2281e-1 (1.75e-2) +**	2.2786e-1 (1.19e-2) +	3.1480e-1 (2.27e-4)
MaF10	10	19	1.8310e+0 (7.03e-2) -	1.9769e+0 (8.01e-2) -	2.5481e+0 (2.72e-1) -	3.0145e+0 (1.48e-1) -	**1.6432e+0 (4.82e-3)**
MaF11	10	19	1.5333e+0 (8.46e-2) +	1.5389e+0 (9.48e-2) +	**1.4365e+0 (3.89e-2) +**	1.5302e+0 (1.02e-1) +	1.7870e+0 (1.12e-2)
MaF12	10	19	6.4464e+0 (2.88e-1) +	**5.9962e+0 (4.59e-1) +**	6.0426e+0 (2.24e-1) +	6.2512e+0 (3.61e-1) +	7.6425e+0 (2.06e-1)
MaF13	10	5	6.2758e-1 (4.70e-1) =	4.4941e-1 (5.56e-2) +	4.8135e-1 (7.39e-2) +	**3.7193e-1 (2.82e-2) +**	1.1744e+0 (1.99e-3)
MaF14	10	200	9.9070e-1 (9.68e-3) -	1.1692e+0 (5.54e-1) -	1.0105e+0 (5.96e-3) -	2.3535e+0 (2.85e+0) -	**4.3634e-1 (5.80e-2)**
MaF15	10	200	1.0771e+0 (2.61e-2) -	6.1295e+0 (2.42e+0) -	1.2620e+0 (4.17e-2) -	8.1933e+0 (1.32e+0) -	**9.7279e-1 (2.79e-2)**
+/-/=			7/7/1	7/6/2	7/5/3	7/4/4	

Table 13. Spread metric values for 10 objective benchmark problems, Case 3

Problem	M	D	MOEADDE1C3	MOEADDE2C3	MOEADDE3C3	MOEADDE4C3	MOEAD
MaF1	10	19	1.3313e+0 (7.92e-2) =	1.0057e+0 (1.54e-1) +	1.2976e+0 (6.18e-2) =	**6.1648e-1 (3.25e-1) +**	1.3020e+0 (2.41e-2)
MaF2	10	19	1.1845e+0 (2.32e-2) -	1.0282e+0 (7.32e-3) -	1.1546e+0 (1.71e-2) -	1.1194e+0 (3.24e-2) -	**6.7635e-1 (1.05e-2)**
MaF3	10	19	1.6152e+0 (3.64e-1) -	1.0216e+0 (3.69e-2) -	1.8571e+0 (1.50e-1) -	1.0241e+0 (6.24e-2) -	**6.4918e-1 (1.21e-3)**
MaF4	10	19	1.2246e+0 (2.24e-2) +	1.2873e+0 (4.39e-1) =	**1.1889e+0 (8.09e-2) +**	1.8068e+0 (3.49e-1) =	1.3327e+0 (7.94e-3)
MaF5	10	19	**1.0143e+0 (3.74e-3) =**	1.0157e+0 (6.79e-3) =	1.0466e+0 (6.39e-2) =	1.0209e+0 (1.14e-2) =	1.0193e+0 (1.59e-3)
MaF6	10	19	**1.2793e+0 (5.10e-1) =**	1.6288e+0 (3.63e-1) =	1.8371e+0 (3.77e-1) =	2.0057e+0 (1.77e-1) -	1.7726e+0 (1.70e-2)
MaF7	10	29	1.0347e+0 (6.79e-2) =	1.0992e+0 (3.97e-2) -	1.0350e+0 (7.93e-2) =	1.0317e+0 (2.62e-2) -	**9.8501e-1 (5.74e-3)**
MaF8	10	2	9.8396e-1 (2.40e-2) -	1.0891e+0 (3.30e-2) -	1.0448e+0 (5.02e-2) -	1.0264e+0 (3.05e-2) -	**9.2155e-1 (5.69e-3)**
MaF9	10	2	1.9980e+0 (6.02e-3) -	2.0473e+0 (1.69e-2) -	2.0272e+0 (1.07e-2) -	2.0130e+0 (3.83e-2) -	**1.0088e+0 (6.45e-4)**
MaF10	10	19	1.2100e+0 (6.18e-2) -	1.1616e+0 (7.62e-2) -	1.1213e+0 (1.09e-1) -	1.0970e+0 (5.45e-2) -	**7.4627e-1 (1.05e-3)**
MaF11	10	19	9.9258e-1 (2.66e-2) -	1.0716e+0 (2.18e-2) -	9.5251e-1 (1.84e-2) -	1.1073e+0 (3.00e-2) -	**7.4200e-1 (4.50e-4)**
MaF12	10	19	1.4131e+0 (5.21e-2) -	1.2443e+0 (3.03e-2) -	**1.0685e+0 (2.96e-2) =**	1.3166e+0 (5.20e-2) -	1.0774e+0 (1.75e-2)
MaF13	10	5	1.5878e+0 (4.84e-1) -	1.5844e+0 (3.75e-1) -	1.6802e+0 (4.06e-1) -	1.6091e+0 (2.33e-1) -	**1.0306e+0 (2.79e-3)**
MaF14	10	200	1.9592e+0 (1.25e-1) -	2.0090e+0 (1.02e-1) -	1.9085e+0 (7.33e-2) -	2.2106e+0 (6.05e-1) -	**8.6436e-1 (3.21e-1)**
MaF15	10	200	**8.7030e-1 (7.93e-2) =**	1.0179e+0 (6.23e-2) -	1.2233e+0 (2.58e-1) -	1.0160e+0 (3.87e-2) -	8.9083e-1 (6.07e-2)
+/-/=			1/9/5	1/11/3	1/9/5	1/12/2	

Evaluation of the Results for Objective 10: As the number of objectives increases in number the dimension of the objective space and also the complexity of the problem increases. When compared to 5 objective problems, it is relatively harder problems are considered for 10 objective benchmark problems. In general, the proposed methods give better results when compared with MOEA/D algorithm in accuracy. However, when the distribution of the solutions is considered and compared, still MOEA/D gives best performance. The aim of the best member in the crossover is to drag the solution candidates to the Pareto front faster. Also, since the new generated offspring follow the best member, that causes grouping inside the population which reduces the distribution property of the population. That şis the main reason why the proposals are failed when distribution considered.

For Case 1, MOEA/D gives better results for only 5 benchmark problems which is approximately 35% of overall. However, still it is not possible to mention the best formulation. Still among them MOEADDE4 gives the best results.

For Case 2, MOEA/D gives the best results for only 3 benchmark problems (20%). MOEADDE1 and MOEADDE4 gives best performance among 4 formulations with the best result on 4 benchmark problems.

For Case 3, similar performance is obtained in Case 1; MOEA/D gives 35% performance wit MOEADDE4 gives the best result among the formulations.

When the three best member selection methods are compared, at the fist sight Case 2 gives the best performance than Case 1 and Case 3. However, when Case 2 is investigated statistically, for MaF2,5,6,7, MOEA/D gives almost same results with the DE formulations. Therefore, it is possible to comment that all three proposals give almost same performance in accuracy and distribution. In accuracy, they improve the performance of the algorithm however they reduce the performance in distribution. When three cases are compared with each other Case 2 gives the best results overall.

It is possible to compare the results obtained in (Altinoz, 2022). For Case 2 the proposed methods give better results at MaF1,3,9,11,12 benchmark problems when compared the results in (Altinoz, 2022). When compared with 5 objective problems, the results improved however still falls behind the3 expected level.

Objective 15:

Case 1:

Table 14. IGD metric values for 15 objective benchmark problems, Case 1

Problem	M	D	MOEADDE1C1	MOEADDE2C1	MOEADDE3C1	MOEADDE4C1	MOEAD
MaF1	15	24	4.2271e-1 (2.43e-2) =	4.4229e-1 (3.61e-2) =	**3.8666e-1 (4.04e-3) =**	3.9568e-1 (6.30e-3) =	5.7597e-1 (4.07e-5)
MaF2	15	24	5.3451e-1 (3.46e-2) =	6.0612e-1 (1.32e-1) =	5.3913e-1 (2.56e-2) =	**3.4291e-1 (2.39e-2) =**	3.6377e-1 (5.86e-5)
MaF3	15	24	**1.2601e-1 (4.64e-3) =**	1.3385e-1 (3.81e-3) =	1.3114e-1 (3.20e-3) =	1.3306e-1 (5.41e-4) =	1.3007e-1 (3.08e-5)
MaF4	15	24	1.0928e+4 (8.39e+2) =	**3.0255e+3 (8.55e+2) =**	1.2207e+4 (1.92e+2) =	4.5292e+5 (7.78e+5) =	1.5912e+4 (1.91e+2)
MaF5	15	24	7.3256e+3 (4.65e-1) =	7.3259e+3 (1.66e-1) =	7.3236e+3 (3.34e+0) =	7.3254e+3 (5.15e-1) =	**7.3225e+3 (7.89e-2)**
MaF6	15	24	2.3059e-1 (3.50e-2) =	3.6162e-2 (1.30e-2) =	3.8095e-2 (1.35e-2) =	**2.1561e-2 (2.51e-5) =**	5.2212e-2 (8.65e-8)
MaF7	15	34	2.3145e+0 (1.72e-1) =	**1.8402e+0 (1.19e-1) =**	2.2282e+0 (1.21e-1) =	2.0632e+0 (3.71e-2) =	3.7825e+0 (3.05e-1)
MaF8	15	2	**1.7333e-1 (1.46e-3) =**	1.7625e-1 (3.69e-4) =	1.9079e-1 (6.76e-3) =	1.7624e-1 (6.51e-4) =	1.3433e+0 (2.00e-4)
MaF9	15	2	5.3584e-1 (2.45e-2) =	5.4746e-1 (1.02e-3) =	**5.3209e-1 (2.71e-2) =**	5.4756e-1 (9.60e-4) =	9.2701e-1 (4.76e-4)
MaF10	15	24	2.3421e+0 (9.51e-3) =	2.4272e+0 (5.60e-2) =	2.8018e+0 (2.08e-1) =	2.9706e+0 (5.98e-1) =	**2.1430e+0 (9.45e-4)**
MaF11	15	24	2.4543e+0 (1.53e-1) =	2.5140e+0 (1.69e-1) =	2.3941e+0 (2.12e-1) =	2.3902e+0 (8.95e-2) =	**2.3098e+0 (1.56e-2)**
MaF12	15	24	1.7229e+1 (1.62e+0) =	1.2173e+1 (1.19e+0) =	1.2791e+1 (2.12e+0) =	**1.1779e+1 (3.60e-1) =**	1.3780e+1 (4.86e-1)
MaF13	15	5	6.5561e-1 (6.97e-2) =	5.0399e-1 (8.91e-2) =	5.1790e-1 (9.63e-2) =	**5.0265e-1 (2.58e-2) =**	1.5278e+0 (6.75e-2)
MaF14	15	300	1.0334e+0 (9.76e-3) =	1.0034e+0 (3.28e-2) =	9.6280e-1 (6.89e-2) =	9.7458e-1 (7.52e-2) =	**7.7473e-1 (3.57e-1)**
MaF15	15	300	1.3656e+0 (4.07e-2) =	6.9924e+0 (2.16e+0) =	2.1560e+0 (1.32e-1) =	1.2592e+1 (3.05e+0) =	**1.0787e+0 (3.12e-2)**
+/-/=			0/0/15	0/0/15	0/0/15	0/0/15	

Table 15. Spread metric values for 15 objective benchmark problems, Case 1

Problem	M	D	MOEADDE1C1	MOEADDE2C1	MOEADDE3C1	MOEADDE4C1	MOEAD
MaF1	15	24	**1.0325e+0 (6.40e-2) =**	1.2053e+0 (2.91e-1) =	1.0423e+0 (8.56e-2) =	1.0949e+0 (9.35e-2) =	1.1855e+0 (3.48e-3)
MaF2	15	24	1.1431e+0 (2.54e-2) =	1.0596e+0 (2.41e-2) =	1.0998e+0 (2.03e-2) =	1.0599e+0 (1.49e-2) =	**9.7390e-1 (6.99e-3)**
MaF3	15	24	1.3592e+0 (3.18e-1) =	9.8593e-1 (2.45e-2) =	1.5952e+0 (3.85e-1) =	1.1559e+0 (2.51e-1) =	**8.9062e-1 (5.62e-3)**
MaF4	15	24	1.4110e+0 (7.40e-2) =	1.3957e+0 (3.93e-2) =	1.4476e+0 (4.63e-2) =	1.7902e+0 (5.46e-1) =	**1.1739e+0 (1.55e-2)**
MaF5	15	24	1.0003e+0 (7.79e-5) =	**1.0002e+0 (1.46e-4) =**	1.0005e+0 (5.25e-4) =	1.0003e+0 (1.07e-4) =	1.0007e+0 (4.17e-5)
MaF6	15	24	**1.0127e+0 (2.36e-3) =**	1.1313e+0 (1.37e-1) =	1.1347e+0 (1.53e-1) =	1.9787e+0 (4.63e-2) =	1.5950e+0 (4.43e-5)
MaF7	15	34	9.9757e-1 (8.39e-2) =	**8.9825e-1 (8.97e-2) =**	1.0376e+0 (3.64e-2) =	9.0759e-1 (2.36e-2) =	1.0010e+0 (5.20e-3)
MaF8	15	2	1.0714e+0 (4.25e-2) =	9.8676e-1 (2.45e-2) =	1.2182e+0 (4.15e-2) =	1.0340e+0 (3.31e-2) =	**9.6435e-1 (6.41e-3)**
MaF9	15	2	2.0754e+0 (4.80e-2) =	1.7633e+0 (4.97e-1) =	2.0731e+0 (1.80e-2) =	1.7134e+0 (4.66e-1) =	**9.6228e-1 (3.78e-5)**
MaF10	15	24	1.2135e+0 (6.26e-2) =	1.0606e+0 (5.01e-2) =	1.0583e+0 (3.95e-2) =	1.0292e+0 (2.59e-2) =	**9.7380e-1 (4.91e-3)**
MaF11	15	24	9.7848e-1 (1.17e-2) =	9.8957e-1 (1.69e-2) =	9.7737e-1 (2.80e-2) =	9.9325e-1 (3.46e-2) =	**9.7406e-1 (3.86e-3)**
MaF12	15	24	1.2145e+0 (5.81e-2) =	1.1330e+0 (1.23e-1) =	1.1680e+0 (9.06e-2) =	**1.1260e+0 (3.19e-2) =**	1.2688e+0 (3.26e-2)
MaF13	15	5	1.3756e+0 (3.76e-1) =	1.5196e+0 (2.96e-1) =	1.3185e+0 (5.30e-2) =	1.6543e+0 (2.01e-1) =	**1.0145e+0 (1.77e-3)**
MaF14	15	300	**1.0205e+0 (2.97e-2) =**	1.4279e+0 (6.23e-1) =	2.1571e+0 (1.16e-1) =	1.8610e+0 (5.98e-1) =	1.1943e+0 (2.34e-1)
MaF15	15	300	**9.0349e-1 (2.44e-2) =**	1.0789e+0 (3.31e-2) =	1.1245e+0 (4.59e-2) =	1.0747e+0 (4.13e-2) =	1.0382e+0 (6.99e-3)
+/-/=			0/0/15	0/0/15	0/0/15	0/0/15	

Case 2:

Table 16. IGD metric values for 15 objective benchmark problems, Case 2

Problem	M	D	MOEADDE1C2	MOEADDE2C2	MOEADDE3C2	MOEADDE4C2	MOEAD
MaF1	15	24	3.4759e-1 (2.89e-2) =	3.6932e-1 (2.87e-2) =	**3.4535e-1 (2.25e-2) =**	3.6057e-1 (3.01e-2) =	5.7597e-1 (4.07e-5)
MaF2	15	24	4.4587e-1 (3.41e-2) =	3.5196e-1 (3.04e-2) =	3.9882e-1 (1.29e-2) =	**3.4978e-1 (1.21e-2) =**	3.6378e-1 (5.06e-5)
MaF3	15	24	1.3231e-1 (3.72e-3) =	1.3376e-1 (2.68e-3) =	1.3339e-1 (2.98e-3) =	1.3309e-1 (1.73e-3) =	**1.3007e-1 (5.60e-5)**
MaF4	15	24	1.0337e+4 (1.42e+3) =	**2.6060e+3 (2.24e+2) =**	1.1751e+4 (5.36e+1) =	2.9159e+3 (5.62e+2) =	1.5936e+4 (3.35e+1)
MaF5	15	24	7.3256e+3 (4.14e-1) =	7.3257e+3 (5.77e-1) =	**7.0819e+3 (4.22e+2) =**	7.3248e+3 (4.31e-1) =	7.3226e+3 (1.53e-1)
MaF6	15	24	2.9631e-2 (1.61e-2) =	3.0344e-2 (1.49e-2) =	6.3401e-2 (7.23e-2) =	**2.1544e-2 (1.11e-4) =**	2.5366e-1 (3.49e-1)
MaF7	15	34	2.5181e+0 (5.26e-1) =	**2.0359e+0 (3.35e-2) =**	2.3757e+0 (3.63e-1) =	2.0785e+0 (4.83e-2) =	3.5726e+0 (3.30e-1)
MaF8	15	2	**1.7334e-1 (1.61e-3) =**	1.7739e-1 (1.93e-3) =	1.9012e-1 (6.23e-3) =	1.7868e-1 (1.47e-3) =	1.3463e+0 (2.70e-3)
MaF9	15	2	5.5010e-1 (2.11e-3) =	5.4826e-1 (7.71e-4) =	**5.4758e-1 (5.47e-4) =**	5.4934e-1 (9.91e-4) =	9.2699e-1 (3.24e-4)
MaF10	15	24	2.2614e+0 (5.50e-2) =	2.3668e+0 (7.33e-2) =	2.3930e+0 (1.35e-1) =	2.6539e+0 (8.47e-2) =	**2.1451e+0 (2.29e-3)**
MaF11	15	24	2.3450e+0 (2.24e-1) =	2.3339e+0 (9.51e-2) =	**2.2589e+0 (4.90e-2) =**	2.3282e+0 (8.26e-2) =	2.2968e+0 (1.81e-3)
MaF12	15	24	1.1470e+1 (1.43e+0) =	1.1190e+1 (7.53e-1) =	**1.0801e+1 (3.04e-1) =**	1.1406e+1 (5.96e-1) =	1.4529e+1 (7.98e-2)
MaF13	15	5	6.7436e-1 (4.79e-2) =	**5.3724e-1 (1.69e-2) =**	5.8080e-1 (9.77e-2) =	5.6502e-1 (2.44e-2) =	1.4273e+0 (1.47e-2)
MaF14	15	300	8.6519e-1 (1.47e-1) =	1.0023e+0 (5.31e-2) =	1.0360e+0 (3.14e-3) =	9.7447e-1 (8.92e-2) =	**6.6127e-1 (3.38e-1)**
MaF15	15	300	1.3438e+0 (4.96e-3) =	7.5747e+0 (2.93e+0) =	1.9841e+0 (1.92e-1) =	1.3286e+1 (3.92e+0) =	**1.0611e+0 (3.95e-2)**
+/-/=			0/0/15	0/0/15	0/0/15	0/0/15	

Table 17. Spread metric values for 15 objective benchmark problems, Case 2

Problem	M	D	MOEADDE1C2	MOEADDE2C2	MOEADDE3C2	MOEADDE4C2	MOEAD
MaF1	15	24	9.4888e-1 (1.39e-1) =	9.4559e-1 (5.75e-2) =	**9.0350e-1 (1.27e-1) =**	9.8859e-1 (9.86e-2) =	1.1855e+0 (3.48e-3)
MaF2	15	24	1.1216e+0 (2.31e-2) =	1.1019e+0 (5.17e-2) =	1.1331e+0 (3.84e-2) =	1.0329e+0 (4.37e-2) =	**9.7096e-1 (2.95e-2)**
MaF3	15	24	1.2142e+0 (4.21e-1) =	1.0191e+0 (4.01e-2) =	1.6568e+0 (1.02e-1) =	9.8471e-1 (1.78e-2) =	**8.8052e-1 (3.28e-3)**
MaF4	15	24	1.4239e+0 (2.20e-2) =	1.3759e+0 (7.84e-2) =	1.4813e+0 (1.92e-2) =	1.4643e+0 (6.51e-2) =	**1.1649e+0 (2.86e-3)**
MaF5	15	24	**1.0001e+0 (1.97e-5) =**	1.0002e+0 (1.42e-4) =	1.1821e+0 (3.14e-1) =	1.0002e+0 (1.08e-4) =	1.0007e+0 (3.53e-6)
MaF6	15	24	1.8481e+0 (2.08e-1) =	1.9351e+0 (1.89e-1) =	1.5614e+0 (3.95e-1) =	1.9041e+0 (3.06e-2) =	**1.3979e+0 (3.41e-1)**
MaF7	15	34	8.9948e-1 (1.78e-2) =	8.4305e-1 (1.58e-2) =	9.2120e-1 (4.39e-2) =	**8.0800e-1 (9.08e-2) =**	1.0038e+0 (4.26e-3)
MaF8	15	2	1.0976e+0 (2.33e-2) =	1.0358e+0 (5.77e-2) =	1.2524e+0 (7.51e-2) =	1.0773e+0 (6.38e-2) =	**9.6872e-1 (7.75e-3)**
MaF9	15	2	2.0958e+0 (1.82e-2) =	1.9921e+0 (8.19e-2) =	2.0801e+0 (2.18e-2) =	1.7756e+0 (5.22e-1) =	**9.6203e-1 (1.61e-4)**
MaF10	15	24	1.2238e+0 (9.21e-2) =	1.1034e+0 (5.13e-2) =	1.0682e+0 (4.73e-2) =	1.1079e+0 (7.59e-2) =	**9.7465e-1 (9.41e-3)**
MaF11	15	24	9.7676e-1 (4.12e-2) =	9.8654e-1 (1.51e-2) =	**9.4505e-1 (2.27e-3) =**	9.7051e-1 (1.97e-2) =	9.7408e-1 (3.15e-3)
MaF12	15	24	1.1111e+0 (5.56e-2) =	1.0482e+0 (2.58e-2) =	**9.6772e-1 (1.70e-2) =**	1.0260e+0 (7.42e-2) =	1.2094e+0 (6.20e-3)
MaF13	15	5	1.6664e+0 (5.11e-1) =	1.7241e+0 (3.08e-1) =	1.2896e+0 (1.99e-1) =	1.6870e+0 (3.58e-1) =	**1.0221e+0 (1.41e-3)**
MaF14	15	300	1.6036e+0 (6.24e-1) =	1.4526e+0 (6.05e-1) =	1.4738e+0 (6.79e-1) =	1.5736e+0 (4.98e-1) =	**1.0887e+0 (9.33e-2)**
MaF15	15	300	**8.0066e-1 (6.38e-2) =**	1.0338e+0 (5.75e-2) =	1.0997e+0 (6.98e-2) =	1.0961e+0 (4.01e-2) =	1.0542e+0 (3.00e-2)
+/-/=			0/0/15	0/0/15	0/0/15	0/0/15	

Case 3:

Table 18. IGD metric values for 15 objective benchmark problems, Case 3

Problem	M	D	MOEADDE1C3	MOEADDE2C3	MOEADDE3C3	MOEADDE4C3	MOEAD
MaF1	15	24	5.8066e-1 (3.88e-2) =	4.6113e-1 (2.02e-2) =	5.4422e-1 (2.79e-3) =	**4.4595e-1 (3.53e-2) =**	5.7597e-1 (4.07e-5)
MaF2	15	24	5.1477e-1 (5.26e-2) =	5.0414e-1 (7.77e-2) =	4.4638e-1 (9.26e-3) =	**3.3916e-1 (1.46e-2) =**	3.6377e-1 (5.71e-5)
MaF3	15	24	**1.2769e-1 (2.09e-3) =**	1.3316e-1 (2.63e-3) =	1.2980e-1 (1.59e-3) =	1.3462e-1 (1.41e-3) =	1.3011e-1 (4.76e-5)
MaF4	15	24	1.2863e+4 (7.18e+1) =	**3.0883e+3 (2.02e+2) =**	1.2708e+4 (2.22e+2) =	4.1056e+5 (7.06e+5) =	1.5700e+4 (4.42e+2)
MaF5	15	24	7.3258e+3 (3.36e-1) =	7.3260e+3 (4.22e-2) =	7.3244e+3 (2.09e+0) =	7.3251e+3 (1.21e-1) =	**7.3224e+3 (1.85e-2)**
MaF6	15	24	1.7175e-1 (3.42e-2) =	2.2224e-2 (1.77e-3) =	7.9264e-2 (7.78e-2) =	**2.1478e-2 (3.24e-6) =**	2.5389e-1 (3.49e-1)
MaF7	15	34	2.2410e+0 (2.35e-1) =	1.9753e+0 (4.19e-2) =	2.5511e+0 (3.51e-1) =	**1.9739e+0 (1.11e-1) =**	3.5293e+0 (2.08e-1)
MaF8	15	2	1.7642e-1 (2.67e-3) =	1.7693e-1 (8.07e-4) =	1.9333e-1 (7.52e-3) =	**1.7587e-1 (5.30e-4) =**	1.3477e+0 (7.02e-4)
MaF9	15	2	5.4898e-1 (2.34e-3) =	5.4849e-1 (9.24e-4) =	**5.4610e-1 (2.28e-4) =**	5.4730e-1 (8.79e-4) =	4.4809e+0 (6.16e+0)
MaF10	15	24	2.2746e+0 (1.47e-2) =	2.3465e+0 (6.51e-2) =	2.5455e+0 (3.62e-1) =	2.6148e+0 (2.15e-1) =	**2.1431e+0 (1.04e-3)**
MaF11	15	24	2.2926e+0 (1.55e-1) =	2.3133e+0 (6.78e-2) =	**2.2889e+0 (3.53e-2) =**	2.3197e+0 (1.94e-1) =	2.2961e+0 (5.90e-4)
MaF12	15	24	**1.0759e+1 (2.09e-1) =**	1.1475e+1 (8.42e-1) =	1.0964e+1 (4.19e-1) =	1.1437e+1 (1.01e+0) =	1.4898e+1 (5.85e-1)
MaF13	15	5	4.9599e-1 (1.56e-1) =	4.9302e-1 (2.92e-2) =	5.1224e-1 (4.54e-2) =	**4.5776e-1 (7.94e-2) =**	1.5233e+0 (6.08e-2)
MaF14	15	300	9.4823e-1 (1.19e-1) =	9.5904e-1 (2.28e-1) =	1.0202e+0 (1.22e-2) =	9.9585e-1 (1.98e-2) =	**6.7068e-1 (3.25e-1)**
MaF15	15	300	1.4131e+0 (2.17e-1) =	1.2527e+1 (5.70e-1) =	5.0055e+0 (5.51e+0) =	1.1156e+1 (2.42e+0) =	**1.0723e+0 (3.74e-2)**
+/-/=			0/0/15	0/0/15	0/0/15	0/0/15	

Table 19. Spread metric values for 15 objective benchmark problems, Case 3

Problem	M	D	MOEADDE1C3	MOEADDE2C3	MOEADDE3C3	MOEADDE4C3	MOEAD
MaF1	15	24	1.2893e+0 (4.46e-2) =	1.1977e+0 (1.38e-1) =	1.2337e+0 (4.16e-2) =	1.3282e+0 (3.43e-2) =	**1.1855e+0 (3.48e-3)**
MaF2	15	24	1.1058e+0 (2.25e-2) =	1.0861e+0 (3.30e-2) =	1.0884e+0 (5.01e-2) =	1.0512e+0 (1.73e-2) =	**9.6792e-1 (3.93e-3)**
MaF3	15	24	1.4038e+0 (2.65e-1) =	9.6657e-1 (1.80e-2) =	1.3904e+0 (2.54e-1) =	1.0343e+0 (6.43e-2) =	**8.8441e-1 (6.51e-3)**
MaF4	15	24	1.4359e+0 (4.63e-2) =	1.4522e+0 (5.55e-2) =	1.5018e+0 (2.91e-2) =	1.3729e+0 (9.34e-2) =	**1.1841e+0 (2.35e-2)**
MaF5	15	24	1.0002e+0 (7.70e-5) =	**1.0001e+0 (6.81e-5) =**	1.0032e+0 (5.09e-3) =	1.0002e+0 (7.49e-5) =	1.0007e+0 (8.84e-5)
MaF6	15	24	**1.1116e+0 (9.35e-2) =**	1.8397e+0 (1.77e-1) =	2.0275e+0 (2.59e-1) =	1.9796e+0 (1.95e-2) =	1.3979e+0 (3.42e-1)
MaF7	15	34	9.4923e-1 (3.66e-2) =	8.8683e-1 (7.26e-2) =	9.4940e-1 (4.12e-2) =	**8.8241e-1 (3.25e-2) =**	1.0047e+0 (5.29e-3)
MaF8	15	2	1.1345e+0 (9.11e-2) =	1.0309e+0 (5.70e-2) =	1.3067e+0 (8.70e-2) =	9.7357e-1 (6.37e-2) =	**9.6888e-1 (2.40e-3)**
MaF9	15	2	1.7590e+0 (5.46e-1) =	1.9128e+0 (1.46e-1) =	2.0890e+0 (1.73e-2) =	1.6274e+0 (4.08e-1) =	**9.7499e-1 (2.25e-2)**
MaF10	15	24	1.2211e+0 (9.26e-2) =	1.0957e+0 (8.61e-2) =	1.0742e+0 (3.89e-2) =	1.0701e+0 (5.65e-2) =	**9.7360e-1 (6.36e-3)**
MaF11	15	24	**9.6633e-1 (1.24e-2) =**	1.0143e+0 (4.70e-3) =	9.8168e-1 (4.82e-2) =	9.9181e-1 (2.29e-2) =	9.7309e-1 (2.89e-3)
MaF12	15	24	9.9020e-1 (6.93e-2) =	1.0820e+0 (4.77e-2) =	**9.5270e-1 (1.21e-1) =**	1.0739e+0 (5.48e-2) =	1.1917e+0 (2.97e-2)
MaF13	15	5	1.4869e+0 (3.51e-1) =	1.5577e+0 (2.27e-1) =	1.5974e+0 (3.17e-1) =	1.9742e+0 (1.57e-1) =	**1.0168e+0 (1.84e-3)**
MaF14	15	300	1.9258e+0 (6.65e-1) =	1.7842e+0 (6.15e-1) =	1.5629e+0 (5.79e-1) =	1.5320e+0 (5.58e-1) =	**1.1452e+0 (1.30e-1)**
MaF15	15	300	**9.9858e-1 (9.39e-2) =**	1.0369e+0 (4.57e-2) =	1.1876e+0 (1.13e-1) =	1.0538e+0 (4.68e-2) =	1.0184e+0 (1.09e-2)
+/-/=			0/0/15	0/0/15	0/0/15	0/0/15	

Evaluation of the Results for Objective 15: The results from 15 objective benchmark problems gives promising values when compared with other results at 5 and 10 objective benchmark problems. In general, for all three cases MOEA/D could not dominate the results with respect to the statistical tests. It is clear from the results that as the number of objectives are increased in number in other words as the objective space dimension and decision space dimension; therefore, the complexity of the problem is increased the effect of best member also increases with respect to the accuracy of the solutions. It is the indication that by using the conventional methods, it will relatively hard to find members their own way at the objective space. The best member become the anchor for each of the other solution and that helps to increase the performance of the algorithm. For Case 1; only four benchmark problem results is better than proposals. However, three of them are almost same performance with other formulations. That mean the DE formulations gives better or even the same results when compared with MOEA/D algorithm. Still, it is not possible to mention about the best DE formulation for this case, still MOEADDE4C1 looks better than others. For Case 2, almost same results can be concluded. However statistically MOEA/D gives worse or same results with DE formulations, and MOEADDE3C2 gives better performance than others. Finally for Case 3, it is not possible to mention about MOEA/D algorithm. The DE formulations gives better results especially MOEADDE4C3. When three cases are compared with each other Case 1 gives the worst performance, Case 2 and Case 3 gives relatively similar results. However, as the decision space dimension increases Case 2 gives better results than Case 3. Also, it is possible to compare results with (Altinoz, 2022). For all benchmark problems the best member proposal gives better results (comparing with Case 2) and it is clear that as the number of objectives increases the improvement also increases.

4. Conclusion

In this research the best members in the population detected by using three methods. The detected best members are integrated in DE formulations and they are used as crossover operator. The performance of these DE formulations and best member selection operators are compared on 15 benchmark problems. The results not only compared with each other but also from the previous study which is given for DE formulations without best members. From the results it is inferred that best member could not help to improve the distribution property of the solutions. On contrary, the members tendency to make groups and reduce the distribution property. As the number of objectives are increase from 5 to 15, the performance of the proposals are also increases. Especially for higher number of objectives best member helps to improve the accuracy. For the three methods compared with each other method 2 and three gives almost same performance since they are based on distances. However best members could not improve the performance of the DE-based MOEA/D algorithm, although they gives better performances for 15 objectives.

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