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Research Article **Araştırma Makalesi**

Importance of Stability Analysis for Sustainable Fisheries in the Absence of Important Data

Önemli Verilerin Eksikliğinde Kararlılık Analizinin Sürdürülebilir Balıkçılıktaki Önemi

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Abstract: Assessment of fish stocks is especially important to avoid overfishing and obtain sustainable fishing policies, and there are many stock assessment methods such as XSA, VPA, BMS, CMSY, and MSVPA to analyze fish stocks. However, these assessment methods require an important amount of data for fish stocks such as diet data, natural mortality, fishing mortality, abundance index of species, predator ratio estimates, and so on. Unfortunately, we do not have such data for most of the fish stocks, and obtaining such data requires an important amount of money and time, but we still can predict important information about fish stocks such as biomass of fish stocks, the maximum sustainable yield, the biomass of fish lost or gained due to predator-prey relations, and even can track the effect of harvesting on predator-prey relations by building a mathematical model for fish populations and implementing a stability analysis. To obtain these outputs, we only need landing data and implement a parameter estimation constrained on stability conditions derived from the stability analysis of the mathematical model. Shortly, this study shows us how important stability analysis is to obtain important information about fish populations in the absence of important data.

Keywords
- Food chain
- Predator-prey relation
- Equilibrium
- Stability
- Sustainable fishery

Özet: Aşırı avcılığın önlenmesi ve sürdürülebilir balıkçılık politikalarının elde edilmesi için balık stoklarının yönetimi çok önemlidir ve balık stoklarını analiz etmek için XSA, VPA, BMS, CMSY ve MSVPA gibi birçok stok yönetim metotları vardır. Fakat bu stok yönetim metotlarını kullanmak için balık stoklarının beslenme verisi, doğal ölüm oranı, avlanan balık miktarı, balık stoku miktarı indeksi, avcı balık oranı gibi önemli dataların olması gerekir. Ne yazık ki, birçok balık stoku için bu tarz verilere sahip değiliz ve bu verileri elde etmek ekonomik olarak çok maliyetli ve zaman alıcı. Fakat bu veriler elimizde olmasa da matematiksel modeller ve kararlılık analizi yardımıyla balık stoku miktarı, maksimum sürdürülebilir avlanma miktarı, av-avcı ilişkisinin balık miktarına etkisi, avcılığın türler arası av-avcı ilişkisi üzerine etkisi gibi balık stokları ile alakalı birçok önemli bilgiye ulaşabiliriz. Bu önemli model çıktılarını elde edebilmemiz için, sadece avlanan balık miktarı verisi ve oluşturulan matematiksel modelin kararlılık analizine bağlı parametre tahmini yapmak yeterlidir. Kısacası bu çalışma, balık popülasyonları ile alakalı önemli veriler elimizde olmadığında, kararlılık analizinin balık popülasyonları ile alakalı önemli bilgileri elde etmedeki öneminden bahsetmektedir.

Anahtar kelimeler
- Besin zinciri
- Av-avcı ilişkisi
- Denge noktası
- Kararlılık analizi
- Sürdürülebilir balıkçılık

1. INTRODUCTION

Fish populations are very important members of aquatic systems since they play an important role in food webs from bottom-up or top-down in ecosystems. Thus, to sustain resilient and healthy ecosystems, the dynamics of fish populations are crucial in aquatic systems. Besides their importance in aquatic ecosystems, they are also important for humans worldwide as a source of food. However, fish populations have been facing overfishing worldwide due to wrong harvesting strategies or high exploitation/harvest rates applied to fish populations (Hilborn, 2012; Bardey, 2019). Thus, before applying any harvesting strategies, it is important to analyze the aquatic ecosystem in terms of the abundance of important species in the targeted fish population's food web.

There have been many conventional stock assessment methods for investigation on the abundance of fish populations such as XSA, VPA, BMS, CMSY, and MSVPA but most of such methods consider a single species model and require additional data in addition to the landing data. The most complete one among these assessment methods is the MSVPA method since this stock assessment method considers predator-prey relations by using multi-species models rather than a single-species model by including important predator and prey effects on a target fish stock. However, to perform MSVPA, detailed food-habit information is required. For example, the MSVPA assessment method requires data such as natural mortality, an estimate of fishing mortality from the previous year, an abundance index of species, suitability estimates, weight-at-age (or average weights), predator ratio estimates, and diet data. Unfortunately, we don't have these data for most of the fish populations that have been harvested. Almost similar data is needed for the other conventional stock assessment methods as well (Magnusson, 1995; Daskalov et al., 2020; Demirel et al., 2020). Thus, we cannot apply any of these stock assessment methods in the absence of the data mentioned above.

However, we still can investigate fish stocks and their current status to derive a sustainable fishery when we have only the landing data of fish stocks and the knowledge of important predators and prey that are significantly affecting the size of fish stocks. For example, the Atlantic bonito (*Sarda sarda*) and *Mnemiopsis leidyi* are the most significant predators has been affecting the size of the Black Sea anchovy in the Black Sea and zooplanktons are the main source of food for the Black Sea anchovy population. Thus, we need to include these predator-prey effects in the assessment of the Black Sea anchovy. One of the well-known tools helping to investigate the abundance or status of important fish stocks in food webs is the building of mathematical models for targeted fisheries. Using mathematical models for fisheries is a common method in fishery management when having limited data or just having landing data (Kot, 2001; Neubert, 2003; Demir and Lenhart, 2019). But solely using mathematical models is not enough to understand the dynamics of species and avoid overfishing. Therefore, it is also important to implement stability analyses to avoid overfishing and understand the dynamics of key species that are most influential as main predators or prey for fish stocks in food webs (Panja & Mondal 2015; Bentounsi et al., 2017; Agmour et al., 2018; Demir & Lenhart, 2019; Demir & Lenhart, 2021). Since fish populations depend on primary producers that are at the bottom of the food chain and consist of phytoplankton and zooplankton, it is essential to include them as important species in the investigation of fish stocks.

Therefore, considering food webs and implementing stability analysis for fishery models is essential in the management of fisheries (Kot, 2001; Bergland et al., 2018; Harun et al., 2019; Didiharyono et al., 2021). That is why, in this study, I used a food chain model and applied stability analysis to understand the dynamics and status of fish stocks before applying any harvesting strategies. The food chain model used in this study consisted of three species: two of them are fish populations and one of them is a zooplankton population.

2. MATERIAL AND METHODS

Even if this is not a data-driven study, I assume that we have only landing data and all the parameters are estimated depending on this landing data. A similar parameter estimation is proposed in the study of Demir and Lenhart, 2019 in the case study of the Black Sea anchovy assessment. Thus, in this study, parameter values were obtained depending on the stability conditions driven in this study (details are given at the end of subsection 2.4). Since the general goal of this study is to show how important stability analysis is when we only have landing data, I keep this study more theoretical rather than a specific case study. The fishery model was first introduced in the model formulation part (Eq. 1). Then, ensuring the existence of a solution for the fishery model, all the possible equilibrium points were found and investigated. After that, the stability analysis of important equilibrium points was numerically made. Finally, after capturing important values of harvest rates for a sustainable fishery, the predator-prey system was investigated under different levels of harvesting in the result section.

2.1 Model Formulation

I used a food chain model with three trophic levels to represent the behavior of the food web system, consisting of fish populations P_1 and P_2 as well as a zooplankton population, Z as prey of P_1 and P_2. In this food web, P_2 is also considered as a predator of P_1 in the food chain model given in Eq. (1). In this study, I consider the harvesting of P_1 with the harvest term, $h_1(t)P_1(t)$, which is proportional to the fish population $P_1(t)$and the harvest rate, $h_1(t)$, which represents the amount of fish taken from the system at time t. Similarly, I consider the harvesting of P_2 with the harvesting term, $h_2(t)P_2(t)$. Figure 1 shows the consumption of each compartment and Table 1 shows the description of the parameters given in the following food chain model

$$
\begin{aligned}
\frac{dP_1}{dt} &= r_1 P_1\left(1-\frac{P_1}{K_1}\right)+ m_0 Z P_1 - m_1 P_1 P_2 - h_1 P_1,\\
\frac{dP_2}{dt} &= r_2 P_2\left(1-\frac{P_2}{K_2}\right)+ m_2 P_1 P_2 + m_3 Z P_2 - h_2 P_2,\\
\frac{dZ}{dt} &= r_3 Z\left(1-\frac{Z}{K_3}\right)- m_4 Z P_1 - m_5 Z P_2
\end{aligned}
\tag{1}
$$

with the initial conditions: $P_1(0)=P_{1,0}$, $P_2(0)=P_{2,0}$, and $Z(0)=Z_0$. The terms m_0ZP_1, $m_1P_1P_2$, $m_2P_1P_2$, m_3ZP_2, m_4ZP_1, and m_5ZP_2 represent interaction terms among species. For example, $m_1P_1P_2$, is a decay term for the fish population, P_1 due to the predation of P_1 by P_2 and $m_2P_1P_2$ is a growth term for the fish population, P_2 gained from the consumption of the fish population P_1. I also consider logistic growth rates for each population with intrinsic growth rates r_i and carrying capacities K_i for *i = 1, 2, 3*.

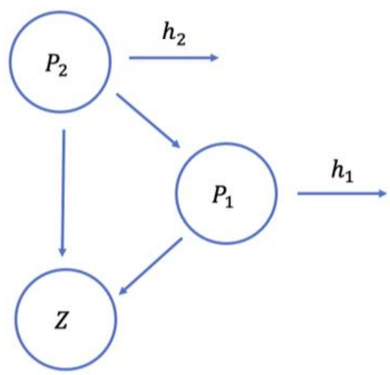


Figure 1. Flow diagram of the model illustrating the consumption among the compartments.

2.2. Positivity and Boundedness of the Model Outputs

In this part, the positivity and boundedness of the state variables in Eq. (1) were shown. For P_1, P_2, and Z with their initial conditions, there exists constants M_1, M_2, $M_3 > 0$ such that $0 < P_1(t) \le M_1$, $0 < P_2(t) \le M_2$, $0 < Z(t) \le M_3$ for all $t \in [0, T]$. Here T denotes the final time. Firstly, I will show that $0 < Z(t) \le M3$ for all $t \in [0,T]$. Then, the same technique will work for $0 < P_1(t) \le M_1$ and $0 < P_2(t) \le M_2$ for all $t \in [0, T]$.

$$\frac{dZ}{dt} = r_3 Z\left(1-\frac{Z}{K_3}\right)- m_4 Z P_1 - m_5 Z P_2$$

The integration factor technique will be used to show $Z(t) > 0$ for all $t \in [0,T]$, but firstly I substitute $Z = \frac{1}{\hat{Z}}$ into the above equation to obtain the following linear equation:

$$\frac{d\hat{Z}}{dt} = -(r_3 - m_4 P_1 - m_5 P_2)\hat{Z} + \frac{r_3}{K_3}$$

letting $\varphi(P_1, P_2) = -(r_3 - m_4P_1 - m_5P_2)$, we can write the above equation in the following linear form as:

$$\frac{d\hat{Z}}{dt} = \varphi(P_1, P_2)\hat{Z} + \frac{r_3}{K_3}$$

multiplying both sides of the equation by the integral factor $\mu = e^{\int_0^t \varphi(P_1,P_2)ds}$ and taking the integral over the interval $t \in [0, T]$, the following will be obtained as

$$\hat{Z}(t)e^{\int_0^t \varphi(P_1,P_2)ds} = \hat{Z}_0 e^{\int_0^t \varphi(P_1,P_2)ds} + \int_0^t e^{\int_0^t \varphi(P_1,P_2)ds} \frac{r_3}{K_3} > 0.$$

Since $\hat{Z}_0$, r_3, K_3, and the exponential function given in the above equation are positive, we can obtain $\hat{Z}(t)>0$. Thus, it follows that $Z(t)>0$ for all $t \in [0,T]$. With a similar approach, we can get $P_2 > 0$ and $P_1 > 0$ for all $t \in [0,T]$. Now, let us first show that $Z(t)$ has an upper bound over the interval $[0,T]$. Since all the coefficients are defined as positive and the states are positive, we can get the following inequality

$$\frac{dZ}{dt} = r_3 Z\left(1 - \frac{Z}{K_3}\right) - m_4ZP_1 - m_5ZP_2 \leq r_3 Z\left(1 - \frac{Z}{K_3}\right) \leq r_3 Z$$

arranging the above inequality and taking the integral from 0 to t, where $t \in [0,T]$ and T in $\mathbb{R}$, we will obtain

$$\int_0^t \frac{dZ}{Z} \leq \int_0^t r_3\, ds\,.$$

When we solve the integral, we obtain

$$Z \leq Z_0 e^{r_3 t} \text{ for all } t \in [0,T]$$

and for $M_3 = Z_0 e^{r_3 t}$, we can reach out the following result

$$0 < Z \leq M_3 \text{ for all } t \in [0,T].$$

Similarly, we can bound $P_1(t)$ by using $0 < Z \leq M_3$, and then bound $P_2(t)$ over the interval $t \in [0,T]$ as $0 < P_1(t) \leq M_1$ and $0 < P_2(t) \leq M_2$.

2.3. Existence of Equilibrium Points

Now I am going to examine the stability of the food chain model given in Eq. 1. First, let's set the time derivative parts equal to zero to obtain the equilibrium points of Eq.1 as

$$\begin{aligned}
0 &= r_1 P_1\left(1 - \frac{P_1}{K_1}\right) + m_0ZP_1 - m_1P_1P_2 - h_1P_1, \\
0 &= r_2 P_2\left(1 - \frac{P_2}{K_2}\right) + m_2P_1P_2 + m_3ZP_2 - h_2P_2, \\
0 &= r_3 Z\left(1 - \frac{Z}{K_3}\right) - m_4ZP_1 - m_5ZP_2
\end{aligned} \tag{2}$$

when we arrange Eq. 2, we will get the following

$$\begin{aligned}
0 &= P_1\left(r_1\left(1 - \frac{P_1}{K_1}\right) + m_0Z - m_1P_2 - h_1\right), \\
0 &= P_2\left(r_2\left(1 - \frac{P_2}{K_2}\right) + m_2P_1 + m_3Z - h_2\right), \\
0 &= Z\left(r_3\left(1 - \frac{Z}{K_3}\right) - m_4P_1 - m_5P_2\right).
\end{aligned} \tag{3}$$

Now, let's get the nullclines of the above equations as

Nullclines for P_1 $P_1 = 0$ or $P_1 = \frac{K_1}{r_1}(r_1 + m_0Z - m_1P_2 - h_1)$

Nullclines for P_2 : $P_2 = 0$ or $P_2 = \frac{K_2}{r_2}(r_2 + m_2P_1 + m_3Z - h_2)$

Nullclines for Z : $Z = 0$ or $Z = \frac{K_3}{r_3}(r_3 - m_4P_1 - m_5P_2)$.

We then can get the equilibrium points of the food chain model in the order $E = (P_1^*, P_2^*, Z^*)$:

Case 1: Assume $P_1 = 0$, then we will obtain $P_2 = 0$ or $P_2 = \frac{K_2}{r_2}(r_2 + m_3 Z - h_2)$.
If $P_2 = 0$, we will then get $Z = 0$ or $Z = K_3$ which implies that $E_1 = (0,0,0)$ or $E_2 = (0,0,K_3)$.
If $P_2 = \frac{K_2}{r_2}(r_2 + m_3 Z - h_2)$, we have $Z = 0$ or $Z = \frac{K_3}{r_3}(r_3 - m_5 P_2)$ which follows that

$$E_3 = (0, K_2, 0) \text{ or } E_4 = \left(0, \frac{K_2}{r_2}(r_2 + m_3 Z - h_2), Z^*\right)$$

where $Z^* = \frac{1+\frac{K_2}{r_3} m_5 \left(\frac{h_2}{r_2} - 1\right)}{\frac{1}{K_3} + \frac{K_2}{r_2 r_3} m_3 m_5}$. The equilibrium point E_4 is biologically feasible if $r_2 + m_3 Z > h_2$ and $\frac{r_3}{m_5 K_2} + \frac{h_2}{r_2} > 1$.

Case 2: Assume $P_1 = \frac{K_1}{r_1}(r_1 + m_0 Z - m_1 P_2 - h_1)$, then we have $P_2 = 0$ or $P_2 = \frac{K_2}{r_2}(r_2 + m_2 P_1 + m_3 Z - h_2)$. If $P_2 = 0$, then we have Z = 0 or $Z = \frac{K_3}{r_3}(r_3 - m_4 P_1)$, which implies that

$$E_5 = \left(\frac{K_1}{r_1}(r_1 - h_1), 0, 0\right) \text{ or } E_6 = \left(\frac{K_1}{r_1}(r_1 - h_1 + m_0 Z^*), 0, Z^*\right)$$

where $Z^* = \frac{1+\frac{K_1}{r_3} m_4 \left(\frac{h_1}{r_1} - 1\right)}{\frac{1}{K_3} + \frac{K_1}{r_1 r_3} m_0 m_4}$.The equilibrium point E_5 is positive for $r_1 > h_1$, and so it is biologically feasible if $r_1 > h_1$. The equilibrium point E_6 is biologically feasible if $r_1 + m_0 Z^* > h_1$ and $\frac{r_3}{m_4 K_1} + \frac{h_1}{r_1} > 1$.

If $P_2 = \frac{K_2}{r_2}(r_2 + m_2 P_1 + m_3 Z - h_2)$, then we have Z = 0 or $Z = \frac{K_3}{r_3}(r_3 - m_4 P_1 - m_5 P_2)$. For Z = 0, the following equilibrium point is obtained

$$E_7 = \left(\frac{K_1}{r_1}(r_1 - h_1 - m_1 P_2^*), P_2^*, 0\right)$$

where $P_2^* = \frac{r_2 - h_2 + \frac{K_1}{r_1} m_2 (r_1 - h_1)}{\frac{r_2}{K_2} + \frac{K_1}{r_1} m_1 m_2}$. The equilibrium point E_7 is biologically feasible if $r_1 > h_1 + m_1 P_2^*$ and $r_2 + K_1 m_2 > h_2 + \frac{K_1}{r_1} m_2 h_1$.

For $Z = \frac{K_3}{r_3}(r_3 - m_4 P_1 - m_5 P_2)$, the coexisting equilibrium point is obtained as $E_8 = (P_1^*, P_2^*, Z^*)$, where

$$P_1^* = \frac{K_1}{r_1}(r_1 + m_0 Z^* - m_1 P_2^* - h_1)$$
$$P_2^* = \frac{K_2}{r_2}(r_2 + m_2 P_1^* + m_3 Z^* - h_2)$$
$$Z^* = \frac{K_3}{r_3}(r_3 - m_4 P_1^* - m_5 P_2^*)$$

By solving the above equations for the equilibrium point E_8, the following is obtained as

$$P_1^* = \frac{r_1 - h_1 + K_3 m_0 - \left(\frac{K_3}{r_3} m_0 m_5 + m_1\right)\left(\frac{r_2 - h_2 + K_3 m_3}{\frac{r_2}{K_2} + \frac{K_3}{r_3} m_3 m_5}\right)}{\frac{r_1}{K_1} + \frac{K_3}{r_3} m_0 m_4 + \left(\frac{K_3}{r_3} m_0 m_5 + m_1\right)\left(\frac{m_2 - \frac{K_3}{r_3} m_3 m_4}{\frac{r_2}{K_2} + \frac{K_3}{r_3} m_3 m_5}\right)}$$

$$P_2^* = \frac{r_2 - h_2 + K_3 m_3 - (m_2 - \frac{K_3}{r_3} m_3\, m_4) P_1^*}{\frac{r_2}{K_2} + \frac{K_3}{r_3} m_3 m_5}$$

$$Z^* = \frac{K_3}{r_3}\,(r_3 - m_4 P_1^* - m_5 P_2^*)$$

and so, the equilibrium point E_8 is biologically feasible if $P_1^* > 0$, $P_2^* > 0$ and $r_3 > m_4 P_1^* + m_5 P_2^*$.

2.4. Stability Analysis of Coexisting Equilibrium Points

Now, I discuss the stability of equilibrium points in which at least two of the species coexist. Thus, the equilibrium points E_4, E_6, E_7, and E_8 will be investigated in this section. To investigate the stability of the food chain model at these equilibrium points, we first need to obtain the Jacobian (community) matrix of the food chain model. After that, the stability of these equilibrium points will be checked by using the Jacobian matrix at these equilibrium points. Note that when all the eigenvalues of this matrix are negative at an equilibrium point, this equilibrium point will be called a stable equilibrium. When at least one of the eigenvalues of this matrix at an equilibrium point is non-negative, then this equilibrium point is unstable.

Let us get the Jacobian matrix to check the stability of the equilibrium points. Assume that the functions *F*, *G*, and *H* equal the RHS of the food chain system (1) as follows:

$$\begin{aligned}
\frac{dP_1}{dt} &= r_1 P_1 \left(1 - \frac{P_1}{K_1}\right) + m_0 Z P_1 - m_1 P_1 P_2 - h_1 P_1 = F(P_1, P_2, Z) \\
\frac{dP_2}{dt} &= r_2 P_2 \left(1 - \frac{P_2}{K_2}\right) + m_2 P_1 P_2 + m_3 Z P_2 - h_2 P_2 = G(P_1, P_2, Z) \\
\frac{dZ}{dt} &= r_3 Z \left(1 - \frac{Z}{K_3}\right) - m_4 Z P_1 - m_5 Z P_2 = H(P_1, P_2, Z)
\end{aligned} \tag{4}$$

The Jacobian matrix will be in the form:

$$J = \begin{pmatrix} \frac{\partial F}{\partial P_1} & \frac{\partial F}{\partial P_2} & \frac{\partial F}{\partial Z} \\ \frac{\partial G}{\partial P_1} & \frac{\partial G}{\partial P_2} & \frac{\partial G}{\partial Z} \\ \frac{\partial H}{\partial P_1} & \frac{\partial H}{\partial P_2} & \frac{\partial H}{\partial Z} \end{pmatrix}$$

When we get the partial derivatives of the system (4), we obtain the following

$$J = \begin{pmatrix} r_1(1 - \frac{2P_1^*}{K_1}) + m_0 Z^* - m_1 P_2^* - h_1 & -m_1 P_1^* & m_0 P_1^* \\ m_2 P_2^* & r_2(1 - \frac{2P_2^*}{K_2}) + m_2 P_1^* + m_3 Z^* - h_2 & m_3 P_2^* \\ -m_4 Z^* & -m_5 Z^* & r_3(1 - \frac{2Z^*}{K_3}) - m_4 P_1^* - m_5 P_2^* \end{pmatrix}$$

After obtaining the Jacobian matrix, the stability of equilibrium points is investigated by substituting the equilibrium points to the Jacobian matrix, J. Firstly, the stability of the coexisting equilibrium point,

E_8 was investigated by using parameter values given in Table 1. Note that this study is not data-driven. If so, we could estimate these parameter values given in Table 1 fitting the model with landing data under the constraints of the stability of the predator-prey system when all the species coexist. Note that one directly can apply stability requirements for parameters during the parameter estimation, or first estimates parameters and then check the stability requirements with estimated parameters and arrange the upper and lower bound of the parameters' initial guesses until satisfying the stability requirements for the estimated parameters. One could fit the model below for a specific case study as:

$$\min\left(\frac{\sum_{k=1}^{n}\left(L_k-\hat{L}_k\right)^2}{\sum_{k=1}^{n}(L_k)^2}+\frac{\sum_{k=1}^{n}(L_k^*-\hat{L}_k^*)^2}{\sum_{k=1}^{n}(L_k^*)^2}\right)$$

where the letter n denotes the number of data points in the above formula, L_k is the landing data of species P_1, and $\hat{L}_k$ is the predicted landing that is obtained from the term h_1P_1 of the model. Similarly, L_k^* is the landing data of species P_2, and $\hat{L}_k^*$ is the predicted landing that is obtained from the term h_2P_2 of the model.

Table 1: Parameter descriptions and values used in Stability analysis of equilibrium points. Here e is a scientific notation in MATLAB and it is a shorthand for 10.

Parameters	Descriptions	Unit	Value	Source
$P_{1,0}$	Initial biomass of fish population, P_1	Tonnes	$9e^3$	Assumed
$P_{2,0}$	Initial biomass of fish population, P_2	Tonnes	$6e^2$	Assumed
Z_0	Initial biomass of zooplankton, Z	Tonnes	$3e^7$	Assumed
r_1	Intrinsic growth rate of fish population, P_1	days^{-1}	0.4	Assumed
r_2	Intrinsic growth rate of fish population, P_2	days^{-1}	0.3	Assumed
r_3	Intrinsic growth rate of zooplankton, Z	days^{-1}	0.5	Assumed
K_1	Carrying capacity of fish population, P_1	Tonnes	$1e^{+5}$	Assumed
K_2	Carrying capacity of fish population, P_2	Tonnes	$2e^{+3}$	Assumed
K_3	Carrying capacity of zooplankton, Z	Tonnes	$1e^{+7}$	Assumed
m_0	Growth rate of P_1 due to predation of Z	$(\text{days x Tonnes})^{-1}$	$3e^{-7}$	Assumed
m_1	Consumption rate of P_1 due to its predator P_2	$(\text{days x Tonnes})^{-1}$	$5e^{-5}$	Assumed
m_2	Growth rate of P_2 due to predation of P_1	$(\text{days x Tonnes})^{-1}$	$8e^{-6}$	Assumed
m_3	Growth rate of P_2 due to predation of Z	$(\text{days x Tonnes})^{-1}$	$4e^{-7}$	Assumed
m_4	Consumption rate of Z due to its predator P_1	$(\text{days x Tonnes})^{-1}$	$5e^{-5}$	Assumed
m_5	Consumption rate of Z due to its predator P_2	$(\text{days x Tonnes})^{-1}$	$4e^{-5}$	Assumed
h_1	Harvest rate of fish population P_1	days^{-1}	0.4	Assumed
h_2	Harvest rate of fish population P_2	days^{-1}	0.4	Assumed

To keep the study general, I did not fit the model with data instead the parameter values given in Table 1 are obtained by varying each parameter to reach a stable coexisting state for the three species. After obtaining these parameter values, I applied stability analysis to the other equilibrium points E_4, E_6, and E_7 as well by using the same parameter values given in Table 1. Then, the harvesting rates h_1 and h_2 are varied to see the effect of harvesting on the stability of the predator-prey system for the equilibrium points, E_4, E_6, E_7, and E_8. Also, note that the initial values of species given in Table 1 are chosen to be around the equilibrium point, E_8. Details are given in the result section.

3. RESULTS

In this part, firstly the stability of the equilibrium points, E_4, E_6, E_7, and E_8 was numerically investigated. Then, the status of the predator-prey dynamic of species was analyzed when different levels of harvesting were applied.

3.1. Stability Analysis of the Equilibrium point, E_8

Now, let's check the stability of the equilibrium point, $E_8 = \left(\frac{K_1}{r_1}(r_1 + m_0 Z^* - m_1 P_2^* - h_1), \frac{K_2}{r_2}(r_2 + m_2 P_1^* + m_3 Z^* - h_2), \frac{K_3}{r_3}(r_3 - m_4 P_1^* - m_5 P_2^*)\right)$. Here, the equilibrium point E_8 is biologically feasible if $P_1^* > 0$, $P_2^* > 0$, and $r_3 > m_4 P_1^* + m_5 P_2^*$. The Jacobian Matrix of E_8 is

$$J\Big|_{E_8} = \begin{pmatrix} r_1(1 - \frac{2P_1^*}{K_1}) + m_0 Z^* - m_1 P_2^* - h_1 & -m_1 P_1^* & m_0 P_1^* \\ m_2 P_2^* & r_2(1 - \frac{2P_2^*}{K_2}) + m_2 P_1^* + m_3 Z^* - h_2 & m_3 P_2^* \\ -m_4 Z^* & -m_5 Z^* & r_3(1 - \frac{2Z^*}{K_3}) - m_4 P_1^* - m_5 P_2^* \end{pmatrix}$$

where

$$P_1^* = \frac{r_1 - h_1 + K_3 m_0 - (\frac{K_3}{r_3} m_0 m_5 + m_1)\left(\dfrac{r_2 - h_2 + K_3 m_3}{\frac{r_2}{K_2} + \frac{K_3}{r_3} m_3 m_5}\right)}{\frac{r_1}{K_1} + \frac{K_3}{r_3} m_0 m_4 + (\frac{K_3}{r_3} m_0 m_5 + m_1)\left(\dfrac{m_2 - \frac{K_3}{r_3} m_3 m_4}{\frac{r_2}{K_2} + \frac{K_3}{r_3} m_3 m_5}\right)}$$

$$P_2^* = \frac{r_2 - h_2 + K_3 m_3 - (m_2 - \frac{K_3}{r_3} m_3 m_4) P_1^*}{\frac{r_2}{K_2} + \frac{K_3}{r_3} m_3 m_5}$$

$$Z^* = \frac{K_3}{r_3}(r_3 - m_4 P_1^* - m_5 P_2^*)$$

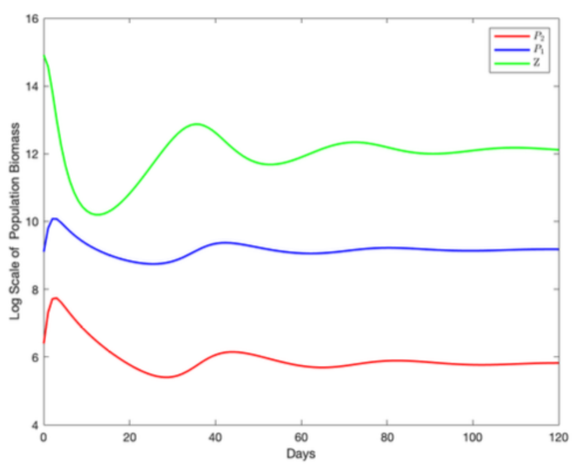
Now, I am going to obtain the characteristic polynomial of E_8 from the following matrix $J^* = |J_{|E_8} - \lambda I|$, where

$$J^* = \begin{vmatrix} r_1(1 - \frac{2P_1^*}{K_1}) + m_0 Z^* - m_1 P_2^* - h_1 - \lambda & -m_1 P_1^* & m_0 P_1^* \\ m_2 P_2^* & r_2(1 - \frac{2P_2^*}{K_2}) + m_2 P_1^* + m_3 Z^* - h_2 - \lambda & m_3 P_2^* \\ -m_4 Z^* & -m_5 Z^* & r_3(1 - \frac{2Z^*}{K_3}) - m_4 P_1^* - m_5 P_2^* - \lambda \end{vmatrix}$$

such that

$$
\begin{aligned}
&P(\lambda) = \\
&\left(r_1(1-\frac{2P_1^*}{K_1})+m_0Z^*-m_1P_2^*-h_1-\lambda\right)\left[\left(r_2(1-\frac{2P_2^*}{K_2})+m_2P_1^*+m_3Z^*-h_2-\lambda\right)\right. \\
&\left.\left(r_3(1-\frac{2Z^*}{K_3})-m_4P_1^*-m_5P_2^*-\lambda\right)+m_3m_5P_2^*Z^*\right] \\
&+m_1P_1^*\left[m_2P_2^*\left(r_3(1-\frac{2Z^*}{K_3})-m_4P_1^*-m_5P_2^*-\lambda\right)+m_3m_4P_2^*Z^*\right] \\
&+m_0P_1^*\left[m_4Z^*\left(r_2(1-\frac{2P_2^*}{K_2})+m_2P_1^*+m_3Z^*-h_2-\lambda\right)-m_2m_5P_2^*Z^*\right]=0
\end{aligned}
$$

As can be seen, it is not easy to examine the stability analysis analytically. Therefore, the stability analysis was examined numerically by using parameter values given in Table 1. This stability analysis showed that the equilibrium point, E_8 is stable since the eigenvalues are obtained as $\lambda_1 = -0.08$, $\lambda_2 = -0.08$, and $\lambda_3 = -0.05$ by using parameter values from Table 1 (see the left-hand side plot in Figure 2). The equilibrium point, E_8 is obtained as (P_1^*, P_2^* , Z^*)= (9e^3, 4.7e^2,6e^6). After obtaining this stability result, I obtained the right-hand side plot in Figure 2 by changing h_2 from 0.4 to 0.58. This investigation shows that the fish population P_2 will collapse if the harvest rate h_2 equals 0.58 or above it. Besides this investigation, the harvest rate h_1 is varied to see the effect of harvesting on the dynamic of the fish population, P_1. When h_1 is taken as 0.5, then we obtain the solid blue curve, which is a different equilibrium state, given in Figure 3. If we set h_1 = 0.6, we will get the dashed blue curve that decays to zero. This analysis indicates that the coexistence equilibrium state of the system can change depending on the harvest rates h_1 and h_2.



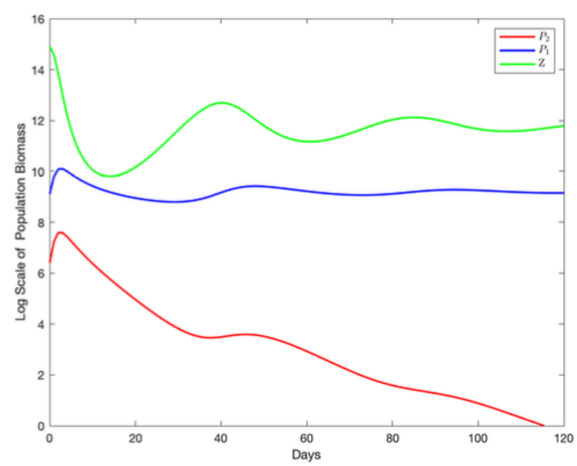


Figure 2. Dynamics of the species at the equilibrium point, E_8. The left plot is obtained by using parameter values given in Table 1. The right plots were obtained by using parameter values given in Table 1 but h_2 is taken as 0.58 instead of 0.4

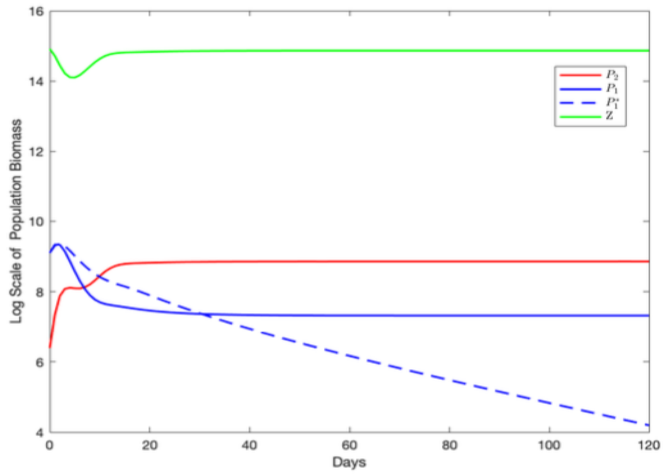


Figure 3. Dynamics of the species at the equilibrium point, E_8 by varying the harvest rate h1 and fixing the rest of the parameter values in Table 1. The solid blue curve is obtained for h_1 = 0.5, and the dashed blue curve is obtained for h_1 = 0.6

3.2. Stability Analysis of the Equilibrium point, E7

Let's investigate the stability of the equilibrium point $E_7 = \left(\frac{K_1}{r_1}(r_1 - h_1 - m_1 P_2^*), P_2^*, 0\right)$ which is the zooplankton-free equilibrium point and $P_2^* = \frac{r_2 - h_2 + \frac{K_1}{r_1} m_2 (r_1 - h_1)}{\frac{r_2}{K_2} + \frac{K_1}{r_1} m_1 m_2}$. The equilibrium point E_7 is biologically feasible if $r_1 > h_1 + m_1 P_2^*$ and $r_2 + K_1 m_2 > h_2 + \frac{K_1}{r_1} m_2 h_1$. The Jacobian matrix of E_7 is

$$J\Big|_{E_7} = \begin{pmatrix} r_1(1 - \frac{2P_1^*}{K_1}) - m_1 P_2^* - h_1 & -m_1 P_1^* & m_0 P_1^* \\ m_2 P_2^* & r_2(1 - \frac{2P_2^*}{K_2}) + m_2 P_1^* - h_2 & m_3 P_2^* \\ 0 & 0 & r_3 - m_4 P_1^* - m_5 P_2^* \end{pmatrix}$$

and the characteristic polynomial of E_7 from the following matrix

$$\left|J|_{E_7} - \lambda I\right| = \begin{vmatrix} r_1(1 - \frac{2P_1^*}{K_1}) - m_1 P_2^* - h_1 - \lambda & -m_1 P_1^* & m_0 P_1^* \\ m_2 P_2^* & r_2(1 - \frac{2P_2^*}{K_2}) + m_2 P_1^* - h_2 - \lambda & m_3 P_2^* \\ 0 & 0 & r_3 - m_4 P_1^* - m_5 P_2^* - \lambda \end{vmatrix}$$

obtained as

$$P(\lambda) = \left(r_3 - m_4P_1^* - m_5P_2^* - \lambda\right)\Big[(r_1(1 - \frac{2P_1^*}{K_1}) - m_1P_2^* - h_1 - \lambda)$$
$$(r_2(1 - \frac{2P_2^*}{K_2}) + m_2P_1^* - h_2 - \lambda) + m_1m_2P_1^*P_2^*\Big] = 0$$

Since the equilibrium point E_7 cannot be biologically feasible if the condition, $r_1 > h_1 + m_1P_2^*$ does not hold, h_1 has to be less than $r_1 = 0.4$. Thus, when we investigate the system with $h_1 = 0.4$, we see that both P_1 and P_2 species will be collapsing, as shown in the left plot of Figure 4.

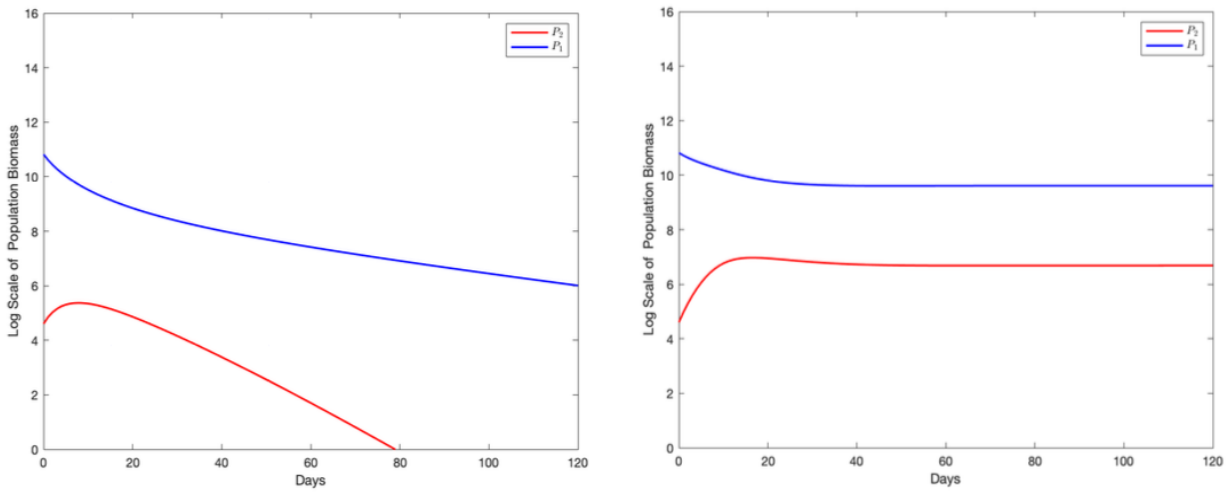


Figure 4. Dynamics of the species at Equilibrium points, E_7. The left plot was obtained by using parameter values from Table 1. The right plot was obtained using parameter values from Table 1 but $h_1 = h_2 = 0.35$ instead of $h_1 = h_2 = 0.4$

However, when we change both harvest rates h_1 and h_2 from 0.4 to 0.35, then we obtain a feasible and stable state for the equilibrium point, E_7 (see the right plot given in Figure 4). This stable equilibrium point is (P_1^*, P_2^* , Z^*)= ($1e^4$, $2e^2$,0) with the eigenvalues $\lambda_1 = -0.04$, $\lambda_2 = -0.04$, and $\lambda_3 = -0.01$. As we decrease the harvest rates, the new stable state will be more abundant and resilient for the system in the absence of zooplankton. For instance, for the harvest rates $h_1 = h_2 = 0.3$, this equilibrium point will be (P_1^*, P_2^* , Z^*)= (1.5e^4, 8e^2,0). This simple investigation shows how important stability analysis is for fishery management.

3.3. Stability Analysis of the Equilibrium point, E_6

Let's examine the stability of the equilibria point $E_6 = (P_1^*, 0, Z^*)$ which is the top-predator-free equilibrium, and biologically feasible when $r_1 + m_0Z^* > h_1$ and $\frac{r_3}{m_4 K_1} + \frac{h_1}{r_1} > 1$ since $P_1^* = \frac{K_1}{r_1}(r_1 - h_1 + m_0Z^*)$ and $Z^* = \frac{1+\frac{K_1}{r_3}m_4\,(\frac{h_1}{r_1}-1)}{\frac{1}{K_3}+\frac{K_1}{r_1\,r_3}m_0\,m_4}$.

The Jacobian matrix with the equilibrium point $E_6 = (P_1^*, 0, Z^*)$ is

$$J\Big|_{E_6} = \begin{pmatrix} r_1(1-\frac{2P_1^*}{K_1})+m_0Z^*-h_1 & -P_1^*m_1 & m_0P_1^* \\ 0 & r_2+m_2P_1^*+m_3Z^*-h_2 & 0 \\ -m_4Z^* & -m_5Z^* & r_3(1-\frac{2Z^*}{K_3})-m_4P_1^* \end{pmatrix}.$$

Now, let us first get the Characteristic Polynomial, $P(\lambda) = det(|J-\lambda I|)$ to obtain eigenvalues of the J, where I is a 3x3 unit matrix.

$$\left|J|_{E_6}-\lambda I\right| = \begin{vmatrix} r_1(1-\frac{2P_1^*}{K_1})+m_0Z^*-h_1-\lambda & -P_1^*m_1 & m_0P_1^* \\ 0 & r_2+m_2P_1^*+m_3Z^*-h_2-\lambda & 0 \\ -m_4Z^* & -m_5Z^* & r_3(1-\frac{2Z^*}{K_3})-m_4P_1^*-\lambda \end{vmatrix}$$

We now have the following characteristic polynomial as

$$P(\lambda) = \left(r_2+m_2P_1^*+m_3Z^*-h_2-\lambda\right)\Bigg[(r_1(1-\frac{2P_1^*}{K_1})+m_0Z^*-h_1-\lambda)(r_3(1-\frac{2Z^*}{K_3}) - m_4P_1^*-\lambda)+m_0m_4Z^*P_1^*\Bigg]=0$$

where $P_1^* = \frac{K_1}{r_1}(r_1 - h_1 + m_0Z^*)$ and $Z^* = \frac{1+\frac{K_1}{r_3}m_4\,(\frac{h_1}{r_1}-1)}{\frac{1}{K_3}+\frac{K_1}{r_1\,r_3}m_0\,m_4}$. Now, we need to find the eigenvalues of the characteristic polynomial given above.

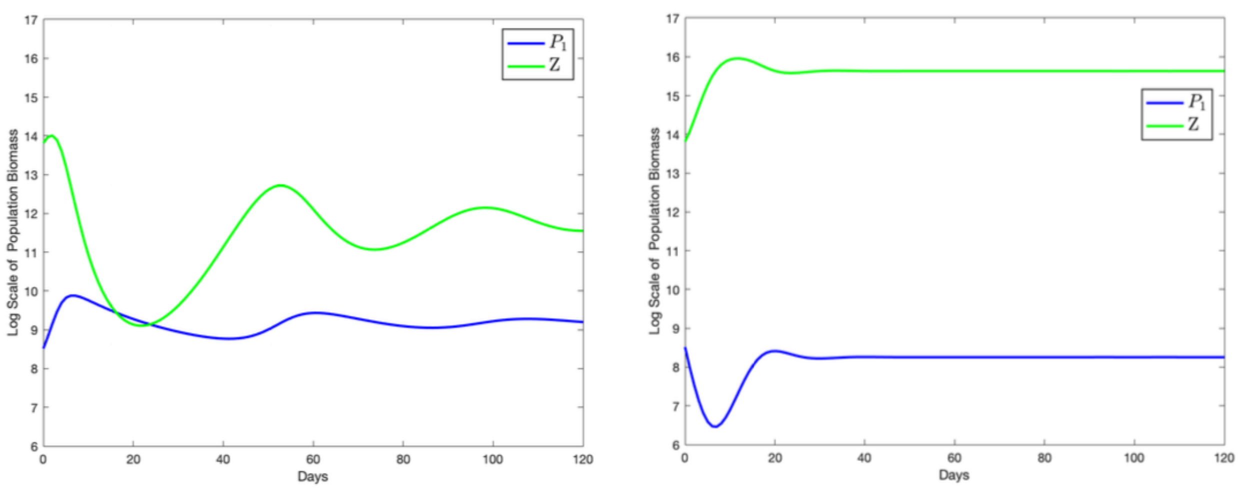


Figure 5. Dynamics of the species at Equilibrium points, E_6. The left plot was obtained by using parameter values from Table 1. The right plot was obtained using parameter values from Table 1 but $h_1 = 0.6$ instead of $h_1 = 0.6$

Since it is not easy to find the eigenvalues analytically, I will find them numerically for the given values of each parameter in Table 1. Since the eigenvalues are negative at the Equilibrium point E_6 as $\lambda_1 = -1.16$, $\lambda_2 = -0.43$, and $\lambda_3 = -0.04$, this equilibrium point is stable (see Figure 5 for different values of the harvest rate, h_1). This analysis indicates that increasing the harvest rate, h_1 causes a reduction in this fish population's abundance and causes increases in zooplankton's abundance in the food web.

3.4. Stability Analysis of the Equilibrium point, E4

Now, let's investigate the last coexisting equilibrium point

$$E_4 = \left(0, \frac{K_2}{r_2}\,(r_2 + m_3 Z - h_2), Z^*\right),$$

where $Z^* = \dfrac{1+\frac{K_2}{r_3} m_5\,(\frac{h_2}{r_2}-1)}{\frac{1}{K_3}+\frac{K_2}{r_2\,r_3} m_3\, m_5}$. The equilibrium point E_4 is biologically feasible if $r_2 + m_3 Z > h_2$ and $\frac{r_3}{m_5\,K_2} + \frac{h_2}{r_2} > 1$.

$$J\Big|_{E_4} = \begin{pmatrix} r_1 + m_0 Z^* - m_1 P_2^* - h_1 & 0 & 0 \\ m_2 P_2^* & r_2(1-\frac{2P_2^*}{K_2}) + m_3 Z^* - h_2 & m_3 P_2^* \\ -m_4 Z^* & -m_5 Z^* & r_3(1-\frac{2Z^*}{K_3}) - m_5 P_2^* \end{pmatrix}$$

$$\Big| J|_{E_4} - \lambda I \Big| = \begin{vmatrix} r_1 + m_0 Z^* - m_1 P_2^* - h_1 - \lambda & 0 & 0 \\ m_2 P_2^* & r_2(1-\frac{2P_2^*}{K_2}) + m_3 Z^* - h_2 - \lambda & m_3 P_2^* \\ -m_4 Z^* & -m_5 Z^* & r_3(1-\frac{2Z^*}{K_3}) - m_5 P_2^* - \lambda \end{vmatrix}$$

$$P(\lambda) = \left(r_1 + m_0 Z^* - m_1 P_2^* - h_1 - \lambda\right)\Big[(r_2(1-\frac{2P_2^*}{K_2}) + m_3 Z^* - h_2 - \lambda) \\ (r_3(1-\frac{2Z^*}{K_3}) - m_5 P_2^* - \lambda) + m_3 m_5 Z^* P_2^*\Big] = 0$$

When we investigate the food chain system in the absence of the fish population, P_1, we see that both P_1 and Z species are increasing and reaching a stable state, as shown in the left plot of Figure 6. When we change the harvest rates h_1 from 0.4 to 0.6, we obtain a slightly different stable state (see the right plot given in Figure 6).

In this section, it is not discussed but one also can predict the amount of fish consumed by its predator by calculating the term $m_1 P_1 P_2$ in Eq.1 and predict the gain due to the consumption of its prey by calculating the term $m_0 Z P_1$ for the fish population P_1 and can do the same calculation for the other fish population P_2.

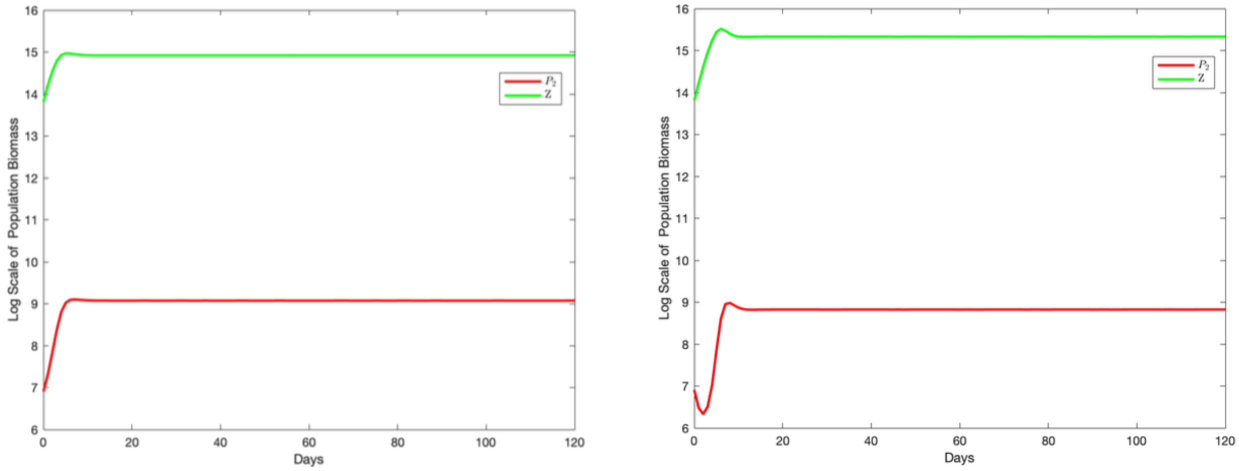


Figure 6. Dynamics of the species at Equilibrium points, E_4. The left plot was obtained by using parameter values from Table 1. The right plot was obtained using parameter values from Table 1 but $h_2 = 0.6$ instead of $h_2 = 0.4$

4. DISCUSSIONS

The fishery model used in the study was not fitted with any specific data to keep the study more general, but one could fit this fishery model or any fishery model with landing data to estimate species-specific parameter values conditional on the stability results of his/her fishery model. It is not hard to see that the results obtained in the study can hold in any case study. For example, in the absence of main food sources such as zooplankton, fish populations will be affected negatively and even can collapse in the absence of main food resources as shown in the study. Similarly, when we increase fishing efforts and pressure on fish populations, their size will be negatively affected. Thus, the results obtained in this study can hold in any case study.

This modeling method requires a few data as compared with the other fishery assessment methods such as XSA, VPA, BMS, CMSY, and MSVPA. In this method, having landing data is enough to capture important features of fish stocks such as biomass of fish stocks, the maximum sustainable yield, the biomass of fish lost or gained due to predator-prey relations, and the effect of harvesting on predator-prey relations as discussed in the result section. However, the other assessment methods require an important amount of data and estimates for fish stocks such as diet data, natural mortality, fishing mortality (landing data), abundance index of species, suitability estimates, weight-at-age (or average weights), predator ratio estimates, and so on. Obtaining such rich data requires an important amount of money and time.

Furthermore, most of such assessment methods consider single-species models instead of multi-species models in the investigation of fish stocks without including any predator-prey effects on fish stock dynamics. However, using single-species models often has overestimated sustainable harvest levels since the actual population levels are lowered due to food chain interactions, and too often traditional fishery management that uses single-species models has failed to take a precautionary approach to maintain and protect sustainable fisheries, biodiversity, and marine ecosystem function (Lauck et al.,1998; Foley, 2013; Demir & Lenhart, 2019). Therefore, this study was considered a multi-species fishery model in the investigation of fish stocks to eliminate any risk of overestimation of the maximum sustainable yield and collapses in fish stocks due to overfishing. This is another advantage of this modeling method besides the requirement of less data for the assessment of fish stocks.

This investigation also showed that stability analysis of fishery models is crucial in fishery management since it allows us to identify the upper or lower bounds of harvesting rates to reach a resilient and healthy food web where targeted fish populations are. For example, in this study, the stability analysis for coexistence equilibrium, E_8 shows that the harvest rates h_1 and h_2 are very important for the dynamic of fish populations and their fade in the food web since increases in these harvest rates may cause a reduction or even a collapse in fish populations (Figure 3) and analysis of this equilibrium point lets us figure out the critical upper bound of harvest rates for these fish populations that correspond to the maximum sustainable yield in literature. Thus, it is recommended to investigate the critical upper bounds of harvest rates for targeted fisheries by implementing stability analysis before applying any harvesting strategies.

The investigation of equilibrium points, especially the equilibrium point E_7 shows that the zooplankton population is very crucial for the fade of fish populations in the food web (Figure 4). In the absence of zooplankton, both fish populations are collapsing. Thus, this result indicates that any violation in the lower level of food webs directly affects upper levels.

Also, note that the estimated parameters in the study are conditional on the stability of the multi-species model used. It means that not only the targeted fish stocks but also the other species included in the study will be sustained in the long term if we apply the outputs of the study, especially the maximum sustainable (landing) yield. Note that even if this technique is applied to sustain mainly fish stocks, it also can be used to sustain any population and even an important proportion of a food web in an aquatic system if we have time series biomass or density data of the most important species in a food web. Therefore, to control and sustain our ecosystems we need to control and help our environment with a little touch as this study recommends.

In addition to the outputs covered in this study, one could estimate the optimal sustainable yield besides the maximum sustainable yield with no extra data but coupling the fishery model with optimal control tools (Neubert, 2003; Kelly et al., 2016) and could obtain optimal predator-prey dynamics among species (Demir & Lenhart, 2019). When catch per unit effort (CPUE) data is available for a fish population besides the landing data, one can also predict the optimal number of fishing fleets that need to be used in harvesting the optimal sustainable yield as proposed in the study Demir and Lenhart, 2019.

5. CONCLUSION

This study indicates that one can predict the status of fish stocks and obtain important outputs of fisheries such as the maximum sustainable yield, current biomass dynamics of fish stocks, and dynamics of their predators and prey thanks to the landing data and including the most influential predators and preys on fish stocks by using fishery models supported with stability analysis. This method requires only landing data as compared with other conventional assessment methods that require very rich data. Obtaining such data requires an important amount of money and time. Furthermore, most of these assessment methods consider single-species models when important outputs of fisheries are driven and this approach ignores and misses predator-prey effects on fishery management. Therefore, the method used in this study is more complete and requires less data as compared to other assessment methods to derive important outputs for fishery management.

This study also shows that it is essential to investigate the equilibrium points of species and their stability for a fishery model used in the investigation of fish populations to avoid overfishing and eliminate the risk of any collapse in fish populations due to overfishing (Figures 2, 3, and 4). It is also important to investigate the effect of fishing on the food web by including other important key species in fishery models besides the targeted fish populations as this study does. For instance, the zooplankton population is included in this study to see and track the effect of the fisheries on zooplankton abundance, and this also lets us examine the dynamics of the fish population in the absence of zooplankton.

One of the main recommendations of the study is that if the policymakers of fisheries consider the maximum sustainable (landing) yield as the maximum amount that can be harvested from the system, then they will be able to not only sustain fish stocks but also sustain the other species included in such an analysis. This modeling technique can also be used to investigate a food web in an aquatic system when time series of density (or abundance) data is available for the most influential species in the food web. Thus, this technique used in the study is also ecosystem friendly.

6. FUTURE WORK

The conventional stock assessment methods and the method used in this study lead us to investigate and obtain important outputs for fishery management, but the outputs come from deterministic models that provide rough predictions and do not consider variations affecting the birth rates of fish populations due to environmental changes. Also, these assessment methods do not consider measurement errors of data. Thus, to make the predicted outputs much better, one can use stochastic differential equations coupled with a measurement model as proposed by Marino et al., 2019. In this way, one can consider the birth rate variation and measurement errors in data.

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CONFLICT OF INTEREST

The author declares that no financial interests or personal relationships may affect this work.

AUTHOR CONTRIBUTIONS

Single author.

ETHICAL STATEMENTS

Local Ethics Committee Approval was not obtained because experimental animals were not used in this study

DATA AVAILABILITY STATEMENT

Research data is not shared.

REFERENCES

Agmour, I., Achtaich, N., & Foutayeni, Y. E. (2018). Stability Analysis of a Competing Fish Populations Model with the Presence of a Predator, *International Journal of Nonlinear Science*, *26*(2), 108-121.

Bardey, D. J. (2019). Overfishing: Pressure on our Oceans. *Research in Agriculture, Livestock and Fisheries*, *6*(3), 397-404.

Bentounsi, M., Agmour, I., Achtaich, N., & El Foutayeni, Y. (2017). Stability Analysis of a Biological Model of a Marine Resources Allowing Density Dependent Migration. *International Frontier Science Letters*, *12*, 22-34.

Bergland, H., Pedersen, P. A., & Wyller, J. (2018). Stable and unstable equilibrium states in a fishery–aquaculture model. *Natural Resource Modeling*, *32*(2), e12200.

Daskalov, G. M., Demirel, N., Ulman, A., Georgieva, Y., & Zengin, M. (2020). Stock dynamics and predator–prey effects of Atlantic bonito and bluefish as top predators in the Black Sea. *ICES Journal of Marine Science*, *77*(7-8), 2995–3005, https://doi.org/10.1093/icesjms/fsaa182

Demir, M. & Lenhart, S. (2019). Optimal sustainable fishery management of the Black Sea anchovy with food chain modeling framework. *Natural Resource Modeling*, *33*(2), 1–29.

Demir, M. & Lenhart, S. (2021). A Spatial Food Chain Model for the Black Sea Anchovy and its Optimal Fishery. *Discrete and Continuous Dynamical Systems Series B*, *26*(1), 155-171.

Demirel, N., Zengin, M., & Ulman, A. (2020). First Large-Scale Eastern Mediterranean and Black Sea Stock Assessment Reveals a Dramatic Decline. *Frontiers in Marine Science*, *7*(103). https://doi.org/10.3389/fmars.2020.00103

Didiharyono, D., Toaha, S., Kusuma, J., & Kasbawati. (2021). Stability analysis of two predators and one prey population model with harvesting in fisheries management. *Earth and Environmental Science*, *921*, 012005. https://doi.org/10.1088/1755-1315/921/1/012005

Foley, C. M. (2013). A Difference in Perspective: A Comparison of Ecosystem and Single-Species Focused Fisheries Management. Open Access Master's Theses. Paper 45. https://doi.org/10.23860/thesis-foley-catherine-2013

Harun, M., George, L., Colleta, A., & Wasike, A. (2019). Stability and Bifurcation Analysis of a Fishery Model with Allee Effects. *Mathematical Modelling and Applications*, *4*(1), 1-9. https://doi.org/10.11648/j.mma.20190401.11

Hilborn, R. (2012). *Overfishing: What everyone needs to know*. Oxford University Press.

Lauck, T., Clark, C. W., Mangel, M., & Munro, G. R. (1998). Implementing the precautionary principle in fisheries management through marine reserves. *Ecological Applications*, *8*, 72–78.

Kelly, M. R., Xing, Y., & Lenhart, S. (2016). Optimal fish harvesting for a population modeled by a nonlinear parabolic partial differential equation. *Natural Resource Modeling, 29*(1), 36-70.

Kelly, M. R., Neubert, M., & Lenhart, S. (2019). Marine reserves and optimal dynamic harvesting when fishing damages habitat. *Theoretical Ecology*, *12*(2), 131–144.

Kot, M. (2001). *Elements of mathematical ecology*. Cambridge University Press.

Marino, J. A. Jr., Peacor, S. D., Bunnell, D. B., Vanderploeg, H. A., Pothoven, S. A., Elgin, A. K., Bence, J. R., Jiao, J., & Ionides, E. L. (2019). Evaluating consumptive and nonconsumptive predator effects on prey density using field time-series data. *Ecology 100*(3), e02583. https://doi.org/10.1002/ecy.2583

Magnusson, K. G. (1995). An overview of the multispecies VPA: theory and applications. *Reviews in Fish Biology and Fisheries, 5*, 195-202.

Panja, P. & Mondal, S. K. (2015). Stability analysis of coexistence of three species prey-predator model. *Nonlinear Dynamics, 81*(1-2), 373-382.