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Research Article

Bulk arrival queue with unreliable server, balking and modified Bernoulli vacation

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Abstract

In this paper, we concentrate on the analysis of the breakdown of an unreliable server with batch arrival retrial queue and non-mandatory re-service with modified Bernoulli vacation. The behavior of impatient customers is considered for this analysis. The basic presumption of this paper is that there is a delay following a breakdown before the repair begins. After receiving the service, a customer gets two different possibilities: those who can depart the system or, in case of some customers, they can retry the service. Using the Supplementary Variable Technique, the steady state has been derived and its results are compared with previous findings.

Mathematics Subject Classification (2020). 60K25, 60K20, 90B22

Keywords. Re-attempt queue, non-mandatory re-service, modified Bernoulli vacation, impatient customers

1. Introduction

For a long period of time, queueing researchers are interested in analyzing queuing models with vacation. Performance modelling is one of the key issues that affects the design, development, configuration, and change of any real-time system in an era of technological advancement. Call centres, Supermarket, Web services, Railway stations, Communication networks, Government offices, long lineups at Airports and Telecommunication networks are just a few scenarios where queuing models are implemented.

In [10], the queueing model along with retrial and reneging customers has been investigated. Various ideas about queues with server vacations are discussed in [9]. A unique server (retrial queue) with general retrial times, two phases (I and II) of service, and compulsory participation in the second service, Bernoulli vacation and balking are used in [2]. A Non-Markovian retrial queue with Bernoulli schedule and active breakdowns are examined by [3]. Batch arrivals vacation lineup with two forms of heterogeneous (service) is described in [28] along with balking and re-service in that queue. In [15] an analysis of the $M^{(X)}/(G1, G2)/1$ retrial queue with general retrial times, in which the server provides two phases of varying service to each customer under Bernoulli vacation schedules is considered. An Improved Round-Robin (IRR) queue management algorithm for elastic and inelastic traffic flows is proposed in [26]. The BRR scheduler handles inelastic

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and DRR-SFF handles the elastic flows. Since the data flows are categorized and different treatments are given as per their nature this algorithm shows better results when compared with primitive models.

Steady-state results for queue sizes, average waiting time of the customer in line-up and mean queue size, Bernoulli process scheduling system, and vacations with generic distribution, in addition to some special cases and deriving some known results, are obtained in [25]. As per [12], repairs can be done in phase (single) or in series of phases (optional), depending on breakdown. More focus has been placed on the numerous retrial lineups with non-persistent (impatient) customers including accidental and active breakdown of the server, such retrial models are taken into consideration by [4]. In the study [16], $M^{[X]}/G/1$ queueing system with Bernoulli schedule server vacations and random system breakdowns with general delay and general repair periods is investigated. Stable state solutions have been found by applying the supplementary variables method. In [13] N-policy, bulk arrival with Bernoulli feedback, the system accumulates N customers, who are then repaired by the repairman in k phases are discussed.

In [22], the majority of research on retrial queueing systems makes use of the classical retrial policy, in which each customer who is retrying, conducts service attempts independently of other customers at exponentially distributed inter-retrial periods. The majority of unreliable repetition queues make the assumption that a malfunctioning server can be rectified instantly. Zhang [29] investigated an $M/M/1$ retrial queue with passive breakdowns and active breakdowns from an economic viewpoint, in which whenever any type of breakdowns occurs, the server immediately enters a repair stage, and indeed the repair times for these two types of breakdowns are identical and have an exponential distribution. Krishna Kumar and Raja [17] examined the analysis of an $M/M/c$ retrial queue with feedback, balking and also gave the control retrial rate some consideration. In a Bernoulli feedback retrial queue with balking, where the server performs a different vacation policy, Ke and Chang [14] looked at the facts. The behaviours of the $M/M/c$ retrial queue with feedback and geometric loss were discussed by [23]. In [27] studied a single server retrial queue with two service phases, the second of which is elective, with the server operated under a Bernoulli vacation schedule. It's fascinating to note that different vacation schedules, such as exhaustive service, 1-limited service, Bernoulli vacation, and modified Bernoulli vacation schedule,are compared in terms of performance metrics like orbit size, server utilization, and the probability that the system is empty. To demonstrate how system parameters affect performance measures, they provided a detailed numerical study.

A batch arrival Bernoulli vacation queue with two stages of service has been discussed by [8]. In [21] the BBPC (Batch Bernoulli Process and Catastrophes) that occur by the DTRP (DiscreteTime Renewal Process) was discussed. This concept finds its application in DCS (Digital Communication Systems) and CN (computer networks). In this queue, an unreliable server uses a randomized vacation policy with a maximum of M consecutive vacations. They have presumpted that the server will become inactive and wait for a customer to enter the queue if system is empty by the end of the M^{th} vacation. Additionally, the server may break down in accordance with the Poison procedure and the repair time is distributed generally. To establish the ideal randomized policy, the author developed a cost model. Especially these journals [6,7,19,30], are focused on balking while also discussing other topics like repair, vacation Interruption, working vacation policy, and Bernoulli feedback. Negative consumers, BMAP (Batch Markovian Arrival Process), and arrival that happens as a renewal process are all explored by [20], while disasters and negative arrivals are scrutinized by [18]. In [1] investigated monotonicity and comparability of an batch arrival and service retrial queues with two way communication. Along with demonstrating the monotonicity of the embedded Markov chain's transition operator in terms of convex ordering [5], also investigated the $M/G/1$ queue using repeated attempts and two-phase service. In circumstances in which consumers may repeat the same service and rate of arrival changed with server status, an MMPI (Markov Modulated Poisson Inputs) and RS (Repeated Service) as well as queue with generalized server time utilizing policies(N-Policy) is investigated by [24].

2. Essence of the propounded model:

The Retrial-Queueing Model (RQM), which involves two phases of service, has been the subject of a number of preceding literatures. In our proposed model, the major focus is on retrial queuing model with two phases of services with a modified Bernoulli vacation. The distinct characteristic of our model is considering the case of balking, bulk arrival, repair along with modified Bernoulli vacation. The approach of our model is highly suitable for use in real-world contexts including Mobile Processing Systems (MPS), Computer Networking Procedures, Manufacturing Industries, Communication Networks, Transportation Departments, Banks and Post Offices. The rest of the paper is organized as: section 3 provides the Proposed Model description, section 4 presents Performance Measures, section 5 and 6 follows the special cases and numerical findings and finally, conclusions are made.

3. Proposed model description

1. We assume a queueing system in which customers arrive in batches according to Poisson process with parameter $\lambda(>0)$. Let I_i represent the number of customers belonging to the i^{th} bulk arrival where I_i, $i = 1, 2, 3, ..$, $P[I_i = n] = L_n$, $n = 1, 2, 3, ..$ and $I(\imath_w)$ denotes the probability generating function of X. An incoming customer joins an orbit with probability 1-d while the server is available or balks (does not enter) with probability d $(0 \le d \le 1)$ when the server is unavailable.

2. All of customers depart our service place (area) then joined group of waiting (blocked) customers known as main pool (orbit) if there is no waiting area and a customer arrives and findout the server is active to serve its requests instantly. Later, the orbiting customers attempt to obtain their service.

$$\text{Retrialing Process} = \begin{cases} \varrho(\hbar_t) & \mapsto \text{PDF (Probability Distribution Function)} \\ \varrho^*(\imath_u) & \mapsto \text{LST (Laplace-Stieltjes Transform)} \\ \Upsilon(\imath_u)d\imath_u = \frac{d\varrho(\imath_u)}{1-\varrho(\imath_u)} & \mapsto \text{CCR (Conditional Completion Rates)} \end{cases}$$

3. A server delivers RS (regular service) and NMR to every work. The RS is completed ,then its opt for NMR to the same service is taken without joining main pool with probability r $(0 \le r \le 1)$ or may departing the system with probability $\bar{r} = 1 - r$.

$$\text{Servicing Process} = \begin{cases} B_a(\hbar_t) & \mapsto \text{PDF for RS} \\ B_b(\hbar_t) & \mapsto \text{PDF for NMR} \\ B_a^*(\imath_u) & \mapsto \text{LST for RS} \\ B_b^*(\imath_u) & \mapsto \text{LST for NMR} \\ \Theta_1(\imath_u)d\imath_u = \frac{dB_a(\imath_u)}{1-B_a(\imath_u)} & \mapsto \text{CCR for RS} \\ \Theta_2(\imath_u)d\imath_u = \frac{dB_b(\imath_u)}{1-B_b(\imath_u)} & \mapsto \text{CCR for NMR} \end{cases}$$

Additionally, we present the random variable $C(\hbar_t)$ which represents the servers states and is given by

$$C(\hbar_t) = \begin{cases} 0 & \mapsto \text{idle at } \hbar_t \\ 1 & \mapsto \text{busy at } \hbar_t \\ 2 & \mapsto \text{on NMR at } \hbar_t \\ 3 & \mapsto \text{delaying repair on normal service at } \hbar_t \\ 4 & \mapsto \text{delaying repair on NMR at } \hbar_t \\ 5 & \mapsto \text{is on vacation at } \hbar_t \end{cases}$$

4. After the RS is finished, vacation is initiated with probability a $(0 \leq a \leq 1)$, or the service facility may waiting until another customer arrives with probability $1 - a$.

$$\text{Vacation Process} = \begin{cases} V(\hbar_t) & \mapsto \text{PDF} \\ V^*(\imath_u) & \mapsto \text{LST (Laplace-Stieltjes transform)} \\ v(\imath_u) = \frac{dV(\imath_u)}{1-V(\imath_u)} & \mapsto \text{CCR for vacation} \end{cases}$$

5. The service channels may fail for a short period due to a random system failure. The exogenous Poisson process with rates α_1 and α_2 generates the server's life times. The server is sent for repair as soon as a breakdown occurs; during a certain time, it stop provides service, any new customers and awaits the beginning of the repair work, we might refer to its server's duration,

$$\text{Repairing Process} = \begin{cases} W_a(\hbar_t) & \mapsto \text{PDF for RS} \\ W_b(\hbar_t) & \mapsto \text{PDF for NMR} \\ W_a^*(\imath_u) & \mapsto \text{LST for RS} \\ W_b^*(\imath_u) & \mapsto \text{LST for NMR} \\ \Phi_1(\imath_v)d\imath_v = \frac{dW_a(\imath_v)}{1-W_a(\imath_v)} & \mapsto \text{CCR for RS} \\ \Phi_2(\imath_v)d\imath_v = \frac{dW_b(\imath_v)}{1-W_b(\imath_v)} & \mapsto \text{CCR for NMR} \end{cases}$$

3.1. Notations:

In addition let $\varrho^0(\hbar_t), B_a^0(\hbar_t), B_b^0(\hbar_t)$, $W_a^0(\hbar_t)$, $W_b^0(\hbar_t)$, $V^0(\hbar_t)$ be the passed (elapsed) reattempt time, RS, NMR, delaying repair (time) on RS, delaying repair (time) on NMR and vacation time at time $\hbar_t$.

For the process $\{X(\hbar_t), \hbar_t \geq 0\}$, the probabilities are defined as

$$K_0(\hbar_t) = P\{C(\hbar_t) = 0, X(\hbar_t) = 0\}$$

$$K_n(\imath_u, \hbar_t)d\imath_u = P\{C(\hbar_t) = 0, X(\hbar_t) = n, \imath_u \leq A^0(t) < \imath_u + d\imath_u\}, n \geq 1$$

$$S_{a,n}(\imath_u, \hbar_t)d\imath_u = P\{C(\hbar_t) = 1, X(\hbar_t) = n, \imath_u \leq B_a^0(\hbar_t) < \imath_u + d\imath_u\}, \quad \imath_u, n \geq 0$$

$$S_{b,n}(\imath_u, \hbar_t)d\imath_u = P\{C(\hbar_t) = 2, X(\hbar_t) = n, \imath_u \leq B_b^0(t) < \imath_u + d\imath_u\}$$

$$T_{a,n}(\imath_u, \hbar_t)d\imath_u = P\{C(\hbar_t) = 3, X(\hbar_t) = n, \imath_v \leq B_a^0(\hbar_t) < \imath_v + d\imath_v / \ B_a^0(\hbar_t) = \imath_u\}$$

$$T_{b,n}(\imath_u, \hbar_t)d\imath_u = P\{C(\hbar_t) = 4, X(\hbar_t) = n, \imath_v \leq B_b^0(\hbar_t) < \imath_v + d\imath_v / \ B_b^0(\hbar_t) = \imath_u\}$$

$$R_n(\imath_u, \hbar_t)d\imath_u = P\{C(\hbar_t) = 5, X(\hbar_t) = n, \imath_u \leq V^0(\hbar_t) < \imath_u + d\imath_u\}, n \geq 0$$

In following sections, the relevant probabilities are used

$K_0(\hbar_t) \rightarrow$ probability that the system is inactive at $\hbar_t$.

$K_n(\imath_u, \hbar_t) \rightarrow$ probability at $\hbar_t$ there are precisely n customers in the main pool with retrial (elapsed time) of the customer undergoing retrial is $\imath_u$.

$S_{a,n}(\imath_u, \hbar_t) \rightarrow$ probability that at $\hbar_t$ there are exactly n customers in main pool with the elapsed time of service of customer undergoing service is $\imath_u$.

$S_{b,n}(\imath_u, \hbar_t) \rightarrow$ probability that at $\hbar_t$ there are precisely n customers in the orbit with elapsed re-service time of customer undergoing service is $\imath_u$.

$T_{a,n}(\imath_u, \imath_v, \hbar_t) \rightarrow$ probability that at $\hbar_t$ there are precisely n customers in the orbit with elapsed service time of customer undergoing service is $\imath_u$ and the elapsed delaying repair of server is $\imath_v$.

$T_{b,n}(\imath_u, \imath_v, \hbar_t) \rightarrow$ probability that at $\hbar_t$ there are precisely n customers in the orbit with elapsed re - service time of the test customer undergoing service is $\imath_u$ and the elapsed delaying repair of server is $\imath_v$.

$R_n(\imath_u, \hbar_t) \rightarrow$ probability that at $\hbar_t$ there are precisely n customers in the orbit with vacation of elapsed time is $\imath_u$.

Let $[\hbar_{tn};$ n = 1, 2,...$]$ denote a sequence of epochs during which regular services, vacation (modified Bernoulli), or repairs are completed. Embedded in the retrial queueing system is a Markov chain composed of random vectors $Z_n = X(\hbar_{tn}+), N(\hbar_{tn}+)$.

Assuming that the sequence satisfies the stability criteria, we define the limiting probabilities are

$$K_0 = \lim_{\hbar_t \to \infty} K_0(\hbar_t)$$

$$K_n(\imath_u) = \lim_{\hbar_t \to \infty} K_n(\imath_u, \hbar_t), \imath_u \geq 0, n \geq 1$$

$$S_{a,n}(\imath_u) = \lim_{\hbar_t \to \infty} S_{a,n}(\imath_u, \hbar_t) \quad \imath_u \geq 0, n \geq 0$$

$$S_{b,n}(\imath_u) = \lim_{\hbar_t \to \infty} S_{b,n}(\imath_u, \hbar_t) \quad \imath_u \geq 0, n \geq 0$$

$$T_{a,n}(\imath_u, \imath_v) = \lim_{\hbar_t \to \infty} T_{a,n}(\imath_u, \hbar_t)$$

$$T_{b,n}(\imath_u, \imath_v) = \lim_{\hbar_t \to \infty} T_{b,n}(\imath_u, \hbar_t)$$

$$R_n(\imath_u) = \lim_{\hbar_t \to \infty} R_n(\imath_u, \hbar_t) \qquad \text{exist}$$

3.2. Steady state equation

This model is formulated by using the method of supplementary variable technique to formulate the system of governing equations

$$\lambda dK_0 = (1-a)\left[\bar{r}\int_0^\infty S_{(1,0)}(\imath_u)\Theta_1(\imath_u)d\imath_u + \int_0^\infty S_{(2,0)}(\imath_u)\Theta_2(\imath_u)d\imath_u\right] + \int_0^\infty R_0(\imath_u)v(\imath_u)d\imath_u \tag{3.1}$$

The above equation represent idle state

$$\frac{dK_n(\imath_u)}{d\imath_u} + (\lambda + \Upsilon(\imath_u))K_n(\imath_u) = 0,\ n \geq 1 \tag{3.2}$$

The equations are (3.2)-(3.12) represents retrial, services (normal and Non-Mandatory Re-Service), repair and modified Bernoulli vacation states.

$$\frac{dS_{a,0}(\imath_u)}{d\imath_u} + [d\lambda + \alpha_1 + \Theta_1(\imath_u)]S_{1,0}(\imath_u) = \int_0^\infty \mathit{\Phi}_1(\imath_v)\, T_{a,0}(\imath_u,\imath_v)d\imath_v, \quad n = 0 \tag{3.3}$$

$$\frac{dS_{a,n}(\imath_u)}{d\imath_u} + [d\lambda + \alpha_1 + \Theta_1(\imath_u)]S_{a,n}(\imath_u) = d\lambda\sum_{k=1}^{\infty} L_k S_{a,n-k}(\imath_u) + \int_0^\infty \mathit{\Phi}_1(\imath_v)\, T_{a,n}(\imath_u,\imath_v)d\imath_v \tag{3.4}$$

$$\frac{dS_{b,0}(\imath_u)}{d\imath_u} + [d\lambda + \alpha_2 + \Theta_2(\imath_u)]S_{b,0}(\imath_u) = \int_0^\infty \mathit{\Phi}_2(\imath_v)\, T_{b,0}(\imath_u,\imath_v)d\imath_v, n = 0 \tag{3.5}$$

$$\frac{dS_{b,n}(\imath_u)}{d\imath_u} + [d\lambda + \alpha_2 + \Theta_2(\imath_u)]S_{b,n}(\imath_u) = d\lambda\sum_{k=1}^{\infty} L_k S_{b,n-k}(\imath_u) + \int_0^\infty \mathit{\Phi}_2(\imath_v) T_{b,n}(\imath_u,\imath_v)d\imath_v \tag{3.6}$$

$$\frac{dT_{1,0}(\imath_u,\imath_v)}{d\imath_v} + (d\lambda + \mathit{\Phi}_1(\imath_v))\, T_{a,0}(\imath_u,\imath_v) = 0 \tag{3.7}$$

$$\frac{dT_{a,n}(\imath_u,\imath_v)}{d\imath_v} + (d\lambda + \mathit{\Phi}_1(\imath_v))\, T_{a,n}(\imath_u,\imath_v) = d\lambda\sum_{k=1}^{\infty} L_k\, T_{a,n-k}(\imath_u,\imath_v) \tag{3.8}$$

$$\frac{dT_{b,0}(\imath_u,\imath_v)}{d\imath_v} + (d\lambda + \mathit{\Phi}_2(\imath_v))\, T_{b,0}(\imath_u,\imath_v) = 0 \tag{3.9}$$

$$\frac{dT_{b,n}(\imath_u,\imath_v)}{d\imath_v} + (d\lambda + \mathit{\Phi}_2(\imath_v))T_{2,n}(\imath_u,\imath_v) = d\lambda\sum_{k=1}^{\infty} L_k\, T_{b,n-k}(\imath_u,\imath_v) \tag{3.10}$$

$$\frac{dR_0(\imath_u)}{d\imath_u} + (d\lambda + v(\imath_u))R_0(\imath_u) = 0 \tag{3.11}$$

$$\frac{dR_n(\imath_u)}{d\imath_u} + (d\lambda + v(\imath_u))R_n(\imath_u) = d\lambda\sum_{k=1}^{\infty} L_k R_{n-k}(\imath_u) \tag{3.12}$$

The steady-state boundary conditions are as follows at $\imath_u = 0$.

$$K_n(0) = (1-a)\left[\bar{r}\int_0^\infty S_{(a,n)}(\imath_u)\Theta_1(\imath_u)d\imath_u + \int_0^\infty S_{(b,n)}(\imath_u)\Theta_2(\imath_u)d\imath_u\right] + \int_0^\infty R_n(\imath_u)v(\imath_u)d\imath_u - d\lambda K_0 \tag{3.13}$$

$$S_{a,n}(0) = \int_0^\infty K_{n+1}(\imath_u)\Upsilon(\imath_u)d\imath_u + \lambda\sum_{k=1}^{\infty} L_k K_{n-(k-1)}(\imath_u,\imath_v) + d\lambda K_0, n \geq 1 \tag{3.14}$$

$$S_{b,n}(0) = r\int_0^\infty S_{a,n}(\imath_u)\Theta_1(\imath_u)d\imath_u, n \geq 1 \tag{3.15}$$

$$T_{a,n}(\imath_u, 0) = \alpha_1 S_{a,n}(\imath_u), n \geq 0 \tag{3.16}$$

$$T_{b,n}(\imath_u, 0) = \alpha_2 S_{b,n}(\imath_u), n \geq 0 \tag{3.17}$$

$$R_n(0) = a\bar{r}\int_0^\infty S_{a,n}(\imath_u)\Theta_1(\imath_u)d\imath_u + a\int_0^\infty S_{b,n}(\imath_u)\Theta_2(\imath_u)d\imath_u \tag{3.18}$$

The normalizing condition is given by

$$K_0 + \textstyle\sum_{n=1}^\infty \int_0^\infty K_n(\imath_u)d\imath_u + \sum_{n=0}^\infty \Big[\int_0^\infty S_{a,n}(\imath_u)d\imath_u + \int_0^\infty S_{b,n}(\imath_u)d\imath_u + \int_0^\infty R_n(\imath_u)d\imath_u + \int_0^\infty\int_0^\infty T_{a,n}(\imath_u,\imath_v)d\imath_u d\imath_v + \int_0^\infty\int_0^\infty T_{b,n}(\imath_u,\imath_v)d\imath_u d\imath_v\Big] = 1 \tag{3.19}$$

Now, we define the generating functions as

$K(\imath_u,\imath_w) = \sum_{n=1}^\infty K_n(\imath_u){\imath_w}^n$; $\quad K(0,\imath_w) = \sum_{n=1}^\infty K_n(0){\imath_w}^n$;

$R(\imath_u,\imath_w) = \sum_{n=1}^\infty R_n(\imath_u){\imath_w}^n$; $\quad R(0,\imath_w) = \sum_{n=1}^\infty R_n(0){\imath_w}^n$

$S_a(\imath_u,\imath_w) = \sum_{n=1}^\infty S_{a,n(\imath_u)}{\imath_w}^n$; $\quad S_a(0,\imath_w) = \sum_{n=1}^\infty S_{a,n}(0){\imath_w}^n$;

$S_b(\imath_u,\imath_w) = \sum_{n=1}^\infty S_{b,n(\imath_u)}{\imath_w}^n$; $\quad \mathbf{S}_b(0,\imath_w) = \sum_{n=1}^\infty S_{b,n}(0){\imath_w}^n$;

$T_a(\imath_u,\imath_v,\imath_w) = \sum_{n=1}^\infty T_{a,n(\imath_u,\imath_v)}{\imath_w}^n$; $\quad T_a(\imath_u,0,\imath_w) = \sum_{n=1}^\infty T_{a,n(\imath_u,0)}{\imath_w}^n$;

$T_b(\imath_u,\imath_v,\imath_w) = \sum_{n=1}^\infty T_{b,n(\imath_u,\imath_v)}{\imath_w}^n$; $\quad T_b(\imath_u,0,\imath_w) = \sum_{n=1}^\infty T_{b,n(\imath_u,0)}{\imath_w}^n$;

Multiply Eq.(3.2) to (3.18) by $(\imath_w)^n$ we get

$$\frac{dK(\imath_u,\imath_w)}{d\imath_u} + (\lambda + \Upsilon(x))K(\imath_u,\imath_w) = 0 \quad n \geq 1 \tag{3.20}$$

$$\frac{dS_a(\imath_u,\imath_w)}{d\imath_u} + [d\lambda(1-L(\imath_w)) + \alpha_1 + \Theta_1(\imath_u)]S_a(\imath_u,\imath_w) = \int_0^\infty \Phi_1(\imath_v)T_a(\imath_u,\imath_v,\imath_w)d\imath_v, \quad n = 0 \tag{3.21}$$

$$\frac{dS_b(\imath_u,\imath_w)}{d\imath_u} + [d\lambda(1-L(\imath_w)) + \alpha_2 + \Theta_2(\imath_u)]S_b(\imath_u,\imath_w) = \int_0^\infty \Phi_2(\imath_v)T_b(\imath_u,\imath_v,\imath_w)d\imath_v, \quad n = 0 \tag{3.22}$$

$$\frac{dT_a(\imath_u,\imath_w)}{d\imath_v} + (d\lambda(1-L(\imath_w)) + \Phi_1(\imath_v))\,T_a(\imath_u,\imath_v,\imath_w) = 0, n \geq 1 \tag{3.23}$$

$$\frac{dT_b(\imath_u,\imath_w)}{d\imath_v} + (d\lambda(1-L(\imath_w)) + \Phi_2(\imath_v))\,T_b(\imath_u,\imath_v,\imath_w) = 0, n \geq 1 \tag{3.24}$$

$$\frac{dR(\imath_u,\imath_w)}{d\imath_u} + (d\lambda(1-L(\imath_w)) + v(\imath_u))R(\imath_u,\imath_w) = 0, n \geq 1 \tag{3.25}$$

The steady state B.C at $\imath_u = 0$ and $\imath_v = 0$ are

$$K(0,\imath_w) = (1-a)\left[\bar{r}\int_0^\infty S_a(\imath_u,\imath_w)\Theta_1(\imath_u)d\imath_u + \int_0^\infty S_2(\imath_u,\imath_w)\Theta_2(\imath_u)d\imath_u\right] + \int_0^\infty R(\imath_u,\imath_w)v(\imath_u)d\imath_u - d\lambda K_0 \tag{3.26}$$

$$S_a(0,\imath_w) = \frac{1}{\imath_w}\int_0^\infty K(\imath_u,\imath_w)\Upsilon(\imath_u)d\imath_u + \lambda\frac{L(\imath_w)}{\imath_w}\int_0^\infty K(\imath_u,\imath_w)d\imath_u + d\lambda\frac{L(\imath_w)}{\imath_w}K_0 \tag{3.27}$$

$$S_b(0,\imath_w) = r\int_0^\infty S_a(\imath_u,\imath_w)\Theta_1(\imath_u)d\imath_u \tag{3.28}$$

$$T_a(\imath_u,0,\imath_w) = \alpha_1 S_a(\imath_u,\imath_w), n \geq 0 \tag{3.29}$$

$$T_b(\imath_u,0,\imath_w) = \alpha_2 S_b(\imath_u,\imath_w), n \geq 0 \tag{3.30}$$

$$R(0,\imath_w) = a\bar{r}\int_0^\infty S_a(\imath_u,\imath_w)\Theta_1(\imath_u)d\imath_u + a\int_0^\infty S_b(\imath_u,z)\Theta_2(\imath_u)d\imath_u, n \geq 0 \tag{3.31}$$

Solving Eq.(3.20) to Eq.(3.31), it follows that

$$K(\imath_u,\imath_w) = K(0,\imath_w)[1-\varrho(\imath_u)]e^{-\lambda\imath_u} \tag{3.32}$$

$$S_a(\imath_u,\imath_w) = S_a(0,\imath_w)[1-B_a(\imath_u)]e^{-\tau_1(\imath_w)\imath_u} \tag{3.33}$$

$$S_b(\imath_u,\imath_w) = S_b(0,\imath_w)[1-B_b(\imath_u)]e^{-\tau_2(\imath_w)\imath_u} \tag{3.34}$$

$$T_a(\imath_u,\imath_v,\imath_w) = T_a(\imath_u,0,\imath_w)[1-W_a(\imath_v)]e^{-h(\imath_w)\imath_v} \tag{3.35}$$

$$T_b(\imath_u,\imath_v,\imath_w) = T_b(\imath_u,0,\imath_w)[1-W_b(\imath_v)]e^{-h(\imath_w)\imath_v} \tag{3.36}$$

$$R(\imath_u,\imath_w) = R(0,\imath_w)[1-V(\imath_u)]e^{-h(\imath_w)\imath_u} \tag{3.37}$$

where $\tau_1(\imath_w) = h(\imath_w) + \alpha_1[1-(W_a^*(h(\imath_w)))]$, $\tau_2(\imath_w) = h(\imath_w) + \alpha_2[1-(W_b^*(h(\imath_w)))]$ and $h(\imath_w) = d\lambda(1-L(\imath_w))$.

We define the partial probability generating functions as

$K(\imath_w) = \int_0^\infty K(\imath_u,\imath_w)d\imath_u$,

$S_a(\imath_w) = \int_0^\infty S_a(\imath_u,\imath_w)d\imath_u$, $S_b(\imath_u,\imath_w) = \int_0^\infty S_b(\imath_u,\imath_w)d\imath_u$,

$T_a(\imath_u,\imath_w) = \int_0^\infty T_a(\imath_u,\imath_v,\imath_w)d\imath_u$, $T_b(\imath_u,\imath_w) = \int_0^\infty T_b(\imath_u,\imath_v,\imath_w)d\imath_u$,

$T_a(\imath_w) = \int_0^\infty T_a(\imath_u,\imath_w)d\imath_u$, $T_b(\imath_w) = \int_0^\infty T_b(\imath_u,\imath_w)d\imath_u$,

$R(\imath_w) = \int_0^\infty R(\imath_u,\imath_w)d\imath_u$

Integrating the Eq.(3.32) to (3.37) from 0 to ∞ with respect to $\imath_u$, we get

$$K(\imath_w) = K_0\left\{\frac{d(1-\varrho^*(\lambda))\{L(\imath_w)B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]-\imath_w\}}{\{\imath_w-[\varrho^*(\lambda)+L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]\}}\right\} \tag{3.38}$$

$$S_a(\imath_w) = K_0\left\{\frac{K_0(1-B_a^*(\tau_1(\imath_w)))h(\imath_w)\varrho^*(\lambda)}{\tau_1(\imath_w)\{\imath_w-[\varrho^*(\lambda)+L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]\}}\right\} \tag{3.39}$$

$$S_b(\imath_w) = K_0\left\{\frac{r(B_a^*(\tau_2(\imath_w))-1)B_a^*(\tau_1(\imath_w))h(\imath_w)\varrho^*(\lambda)}{\tau_2(\imath_w)\{\imath_w-[\varrho^*(\lambda)+L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]\}}\right\} \tag{3.40}$$

$$T_a(\imath_w) = K_0\left\{\frac{\alpha_1(1-B_a^*(\tau_1(\imath_w)))(W_a^*h(\imath_w))-1)\varrho^*(\lambda)}{\tau_1(\imath_w)\{\imath_w-[\varrho^*(\lambda)+L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]\}}\right\} \tag{3.41}$$

$$T_b(\imath_w) = K_0\left\{\frac{\alpha_2 r(1-B_b^*(\tau_2(\imath_w)))(W_b^*(h(\imath_w))-1)\varrho^*(\lambda)}{\tau_2(\imath_w)\{\imath_w-[\varrho^*(\lambda)+L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]\}}\right\} \tag{3.42}$$

$$R(\imath_w) = K_0\left\{\frac{a(V^*(h(\imath_w))-1)\varrho^*(\lambda)[\bar{r}+rB_b^*(\tau_2(\imath_w))]}{\{\imath_w-[\varrho^*(\lambda)+L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r}+rB_b^*(\tau_2(\imath_w))][1-a+aV^*h(\imath_w)]\}}\right\} \tag{3.43}$$

Since $K_0 \mapsto$ Probability that the unreliable server is inactive, applying the condition(normalizing)

$$K_0 + K(1) + S_a(1) + S_b(1) + T_a(1) + T_b(1) + R(1) = 1$$

$$K_0 = \left\{\frac{1-[(1-\varrho^*(\lambda))E(X)^{(1)} - d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}]}{1-[(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}]}\right\} \tag{3.44}$$

Using Eq.(3.38) to (3.44), we define PGF of the $H(\imath_w)$ (Number of Customers in the queue) and $K(\imath_w)$ (Number of Customers in the system), we obtain

$$K(\imath_w) = K_0 + K(\imath_w) + R(\imath_w) + \imath_w(S_a(\imath_w) + S_b(\imath_w) + T_a(\imath_w) + T_b(\imath_w))$$

$$K(\imath_w) = K_0 \left\{ \frac{\begin{array}{r}\{\imath_w[1-d(1-\varrho^*(\lambda))] + [(\imath_w-1)\varrho^*(\lambda) + (d-1)\imath_w(1-\varrho^*(\lambda))] \\ B_a^*(\tau_1(\imath_w))[\bar{r} + rB_b^*(\tau_2(\imath_w))]\}\end{array}}{\begin{array}{r}\{\imath_w - [\varrho^*(\lambda) + L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r} + rB_b^*(\tau_2(\imath_w))] \\ [1-a+aV^*h(\imath_w)]\}\end{array}} \right\} \tag{3.45}$$

$$H(\imath_w) = K_0 + K(\imath_w) + R(\imath_w) + S_a(\imath_w) + S_b(\imath_w) + T_a(\imath_w) + T_b(\imath_w)$$

$$H(\imath_w) = K_0 \left\{ \frac{\begin{array}{r}\{\imath_w[1-d(1-\varrho^*(\lambda))] - \varrho^*(\lambda) + (d-1)X(\imath_w)(1-\varrho^*(\lambda)) \\ B_a^*(\tau_1(\imath_w))[\bar{r} + rB_b^*(\tau_2(\imath_w))][1-a+aV^*(h(\imath_w))]\}\end{array}}{\begin{array}{r}\{\imath_w - [\varrho^*(\lambda) + L(\imath_w)(1-\varrho^*(\lambda))]B_a^*(\tau_1(\imath_w))[\bar{r} + rB_b^*(\tau_2(\imath_w))] \\ [1-a+aV^*h(\imath_w)]\}\end{array}} \right\} \tag{3.46}$$

4. Performance measures

This section includes system performance metrics, as well as some relevant system probabilities when the system is in various states.

Now we analyze our model's system performance measures. The following result are obtained from Eq.(3.38) to (3.43), by setting $\imath_w = 1$ and using L - Hospital's rule we get

Let $K(1)$ be the probability that the server is inactive during the retry period.

$$K(1) = K_0 \left\{ \frac{d(1-\varrho^*(\lambda))[E(X)^{(1)} + d\lambda E(X)^{(1)}\{(b_1^{(1)} + rb_2^{(1)} + av^{(1)})\}]}{\begin{array}{r}1 - [(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} \\ +r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}]\end{array}} \right\} \tag{4.1}$$

Let $S_a(1)$ be the probability that the server is busy during regular service in the steady-state

$$S_a(1) = K_0 \left\{ \frac{\varrho^*(\lambda)\lambda E(X)^{(1)} b_1^{(1)}}{\begin{array}{r}1 - [(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + \\ r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}]\end{array}} \right\} \tag{4.2}$$

Let $S_b(1)$ be the probability that the server is busy during NMR in the steady-state

$$S_b(1) = K_0 \left\{ \frac{r\varrho^*(\lambda)\lambda E(X)^{(1)} b_2^{(1)}}{\begin{array}{r}1 - [(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + \\ r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}]\end{array}} \right\} \tag{4.3}$$

Let $T_a(1)$ be the probability that the server is under delaying repair in Regular service

$$T_a(1) = K_0 \left\{ \frac{\varrho^*(\lambda) b_1^{(1)} \lambda E(X)^{(1)} w_1^{(1)}}{\begin{array}{r} 1-[(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + \\ r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}] \end{array}} \right\} \tag{4.4}$$

Let $T_b(1)$ be the probability that the server is under delaying repair in NMR

$$T_b(1) = K_0 \left\{ \frac{r\varrho^*(\lambda) b_2^{(1)} \lambda E(X)^{(1)} w_2^{(1)}}{\begin{array}{r} 1-[(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + \\ r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}] \end{array}} \right\} \tag{4.5}$$

Let $R(1)$ be the probability that the server is on MBV

$$R(1) = K_0 \left\{ \frac{K_0 a\varrho^*(\lambda) \lambda E(X)^{(1)} v^{(1)}}{\begin{array}{r} 1-[(1-\varrho^*(\lambda))E(X)^{(1)} + d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + \\ r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + av^{(1)}\}] \end{array}} \right\} \tag{4.6}$$

Now, we derive some performance measures of the system

$L_{sy} \mapsto$ Mean number of customer in the system,

Differentiating (3.45) with respect to $\imath_w$ and evaluated at $\imath_w = 1$ to get L_{sy} under steady state conditions.

$L_{sy} = \lim_{\imath_w \to 1} K'(\imath_w)$

$$L_{sy} = K_0 \left[\frac{(Dr' Nr_1'' - Nr_1' Dr'')}{2(Dr')^2} \right]$$

$$Nr_1' = (\varrho^*(\lambda)) - (1-d)(1-\varrho^*(\lambda))\{d\lambda E(X)^{(1)}([1+\alpha_1 w_1^{(1)}]b_1^{(1)} + r[1+\alpha_2 w_2^{(1)}]b_2^{(1)})\}$$

$$Nr_1'' = \left[\begin{array}{l} (d\lambda E(X)^{(1)})^2\{([1+\alpha_1 w_1^{(1)}]^2 b_1^{(2)} + r[1+\alpha_2 w_2^{(1)}]^2 \ b_2^{(2)} \\ +\alpha_1 b_1^{(1)} w_1^{(2)} + r\alpha_2 b_2^{(1)} w_2^{(2)})\} + 2d\lambda E(X)^{(2)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} \\ +r[1+\alpha_2 w_2^{(1)}]b_2^{(1)}\}(d-1)(1-\varrho^*(\lambda)) + 2(d\lambda E(X)^{(1)})^2 \\ \{([1+\alpha_1 w_1^{(1)}]b_1^{(1)} r[1+\alpha_2 w_2^{(1)}]b_2^{(1)})\}[\varrho^*(\lambda) + (d-1)(1-\varrho^*(\lambda))] \end{array} \right]$$

$$Dr' = \left[\begin{array}{l} 1-[(1-\varrho^*(\lambda))E(X)^{(1)} + \ d\lambda E(X)^{(1)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} \\ +r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + aV^{(1)}\}] \end{array} \right]$$

$$Dr'' = - \left[\begin{array}{l} (d\lambda)^2([1+\alpha_1 w_1^{(1)}]^2 b_1^{(2)} + r[1+\alpha_2 w_2^{(1)}]^2 b_2^{(2)} + \alpha_1 b_1^{(1)} w_1^{(2)} \\ +r\alpha_2 b_2^{(1)} w_2^{(2)} + aV^{(2)}) + d\lambda E(X)^{(2)}\{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} \\ +r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} + aV^{(1)}\} + E(X)^{(2)} + 2(1-\varrho^*(\lambda))E(X)^{(1)} \\ \{[1+\alpha_1 w_1^{(1)}]b_1^{(1)} + r[1+\alpha_2 w_2^{(1)}] \ b_2^{(1)} + aV^{(1)}\} \\ +2\{(d\lambda E(X)^{(1)})^2[1+\alpha_1 w_1^{(1)}]b_1^{(1)} r[1+\alpha_2 w_2^{(1)}]b_2^{(1)} \\ +aV^{(1)}([1+\alpha_1 w_1^{(1)}]b_1^{(1)} + r[1+\alpha_2 w_2^{(1)}]b_2^{(1)})\} \end{array} \right]$$

$L_{qe} \mapsto$ Average number of customer in the main pool,

Differentiating Eq.(3.46) with respect to $\imath_w$ and evaluated at $\imath_w = 1$ to get L_{qe} under steady state conditions.

$L_{qe} = \lim_{\imath_w \to 1} H'(\imath_w)$

$$L_{qe} = K_0 \left[\frac{(Dr' Nr_2'' - Nr_2' Dr'')}{2(Dr')^2} \right]$$

$$Nr_2' = \left[\begin{array}{l} [1 - d(1 - \varrho^*(\lambda))] + (d-1)(1 - \varrho^*(\lambda)) E(X)^{(1)} (1 + d\lambda\{[1 + \alpha_1 w_1^{(1)}] b_1^{(1)} \\ + r[1 + \alpha_2 w_2^{(1)}] b_2^{(1)}\}) \end{array} \right]$$

$$Nr_2'' = (d-1)(1 - \varrho^*(\lambda)) \left[\begin{array}{l} E(X)^{(2)} + (d\lambda E(X)^{(1)})^2 \{([1 + \alpha_1 w_1^{(1)}]^2 b_1^{(2)} \\ + r[1 + \alpha_2 w_2^{(1)}]^2 \ b_2^{(2)} + \alpha_1 b_1^{(1)} w_1^{(2)} + r\alpha_2 b_2^{(1)} w_2^{(2)})\} \\ + d \ \lambda E(X)^{(2)} \{[1 + \alpha_1 w_1^{(1)}] b_1^{(1)} + r[1 + \alpha_2 w_2^{(1)}] b_2^{(1)}\} \\ + 2\{E(X)^{(1)} \lambda d E(X)^{(1)} \{[1 + \alpha_1 w_1^{(1)}] b_1^{(1)} + r[1 + \alpha_2 w_2^{(1)}] b_2^{(1)}\} \\ + (d\lambda E(X)^{(1)})^2 [1 + \alpha_1 w_1^{(1)}] b_1^{(1)} r[1 + \alpha_2 w_2^{(1)}] b_2^{(1)}\} \end{array} \right]$$

by applying *Little's formula* we get $W_{qe} = \lambda L_{qe}$ and $W_{sy} = \lambda L_{sy}$

5. Special cases

The following are some of the special cases of the proposed model.

Case(i): Without non-mandatory re-service, without modified Bernoulli vacation and without balking
Let $\alpha_1 = 0, \alpha_2 = 0, a = 1$ and $\varrho^*(\lambda) \to 1$

$$K(\imath_w) = K_0 \left[\frac{B_1^*(\tau_1(\imath_w))(\imath_w - 1)}{\{\imath_w - [\varrho^*(\lambda) + L(\imath_w)(1 - \varrho^*(\lambda))] B_1^*(\tau_1(\imath_w))\}} \right]$$

$$K_0 = \frac{1 - E(X)^{(1)}(1 - \varrho^*(\lambda)) - E(X)^{(1)} \{\lambda[(1 + \alpha_1) w_1^{(1)}) + v^{(1)}]\}}{\varrho^*(\lambda)}$$

Case(ii): Without modified Bernoulli vacation and without balking

Let $\alpha_1 = 0, \alpha_2 = 0, a = 1$ and $\varrho^*(\lambda) \to 1$

$$K(\imath_w) = K_0 \left[\frac{B_1^*(\tau_1(\imath_w))(\imath_w - 1)[\bar{r} + r B_2^*(\tau_2(\imath_w))]}{\{\imath_w - B_1^*(\tau_1(\imath_w))[\bar{r} + r B_2^*(\tau_2(\imath_w))] \, V^*(h(\imath_w))\}} \right]$$

$$K_0 = 1 - E(X)^{(1)}(\lambda[(1 + \alpha_1) w_1^{(1)}] + v^{(1)})$$

Case(iii): Without non-mandatory re-service, without balking and $r = 0$

$$K(\imath_w) = K_0 \left[\frac{B_1^*(\tau_1(\imath_w))(\imath_w - 1)}{\{\imath_w - [\varrho^*(\lambda) + L(\imath_w)(1 - \varrho^*(\lambda))] B_1^*(\tau_1(\imath_w))[1 - a + a V^*(h(\imath_w))]\}} \right]$$

$$K_0 = \frac{1 - E(X)^{(1)}(1 - \varrho^*(\lambda)) - E(X)^{(1)} \{\lambda[(1 + \alpha_1) w_1^{(1)} + v^{(1)}]\}}{\varrho^*(\lambda)}$$

These agree on the outcome of the [11]

6. Numerical illustration

Numerical analysis of the proposed model has been presented below for varying system parameters like the arrival rate, service rate, retrial rate, and breakdown rate. The coding was carried out using the MATLAB programming language to calculate the numerical findings. The following default parameter values $\lambda = 0.6$, $\alpha_1 = 5$, $\alpha_2 = 6$, $r = 0.3$ and $d = 0.9$ are used in computation.

Figure 1 and Figure 3 make it abundantly evident that as the retrial rate (Υ) and vacation rate(v) are grow, so does the system's idle state (K_0). Figure 2 depicts how the system's idle state (K_0) reduces when the FRS (First Regular Service) rate (Θ_1) rises.

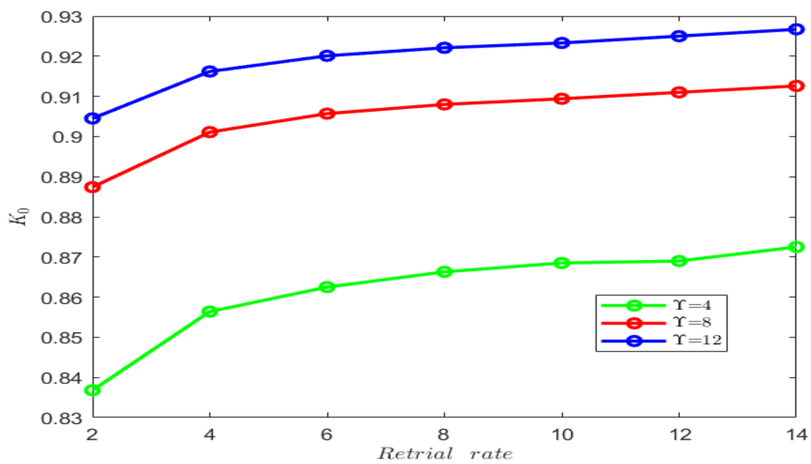

Figure 1. K_0 versus Retrial rate

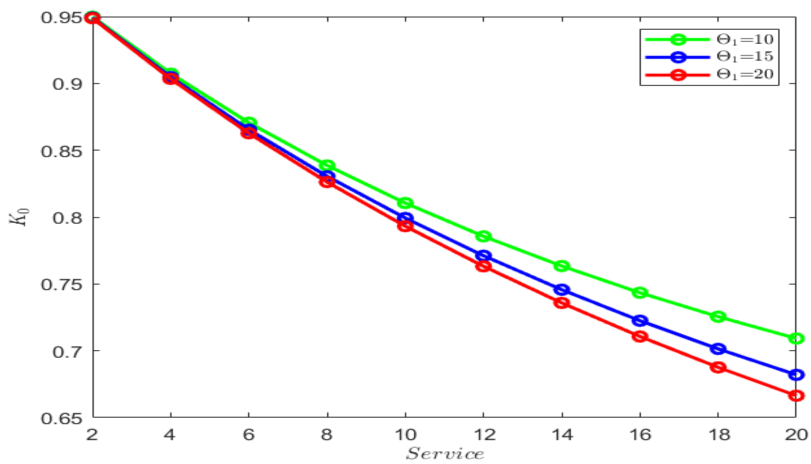

Figure 2. K_0 versus Normal Sevice

7. Conclusion

In the proposed model, an unreliable-server queue, where the customers arrive in batches of variable sizes is considered. We assumed generic distribution for service times, vacation times and delayed repair times. We presented our model with batches of reneging or balking customers in a system of variable size. Different performance measures and

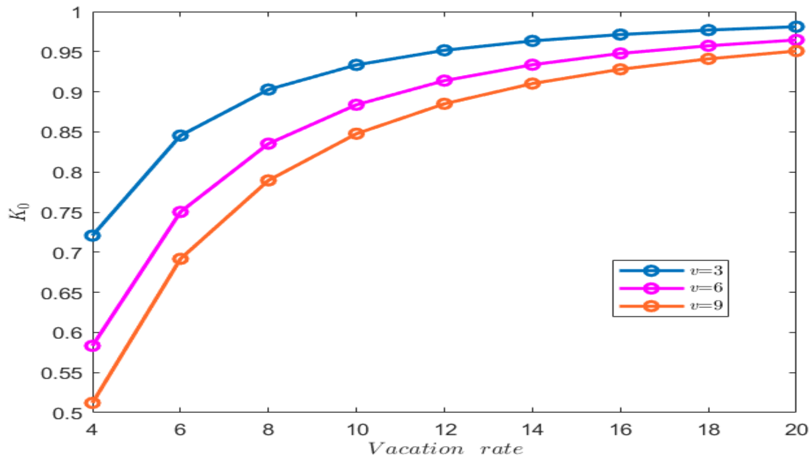

Figure 3. K_0 versus Vacation rate

special cases have been derived and the simulation of our model is executed using Matlab. A comparative analysis is presented which proves the significance of our proposed model. The outcome of this study has its applications in Mobile Processing Systems, Computer (communication) Systems, Satellite Communication, and Mailing Systems.

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