J. Algebra Comb. Discrete Appl.
3(2) • 97–104

Received: 14 April 2015
Accepted: 16 Feburary 2016

Journal of Algebra Combinatorics Discrete Structures and Applications

New results on vertex equitable labeling

Research Article

Pon Jeyanthi, Anthony Maheswari, Mani Vijayalakshmi

Abstract: The concept of vertex equitable labeling was introduced in [9]. A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, \cdots, q$. A graph G is said to be a vertex equitable if it admits a vertex equitable labeling. In this paper, we prove that the graphs, subdivision of double triangular snake $S(D(T_n))$, subdivision of double quadrilateral snake $S(D(Q_n))$, subdivision of double alternate triangular snake $S(DA(T_n))$, subdivision of double alternate quadrilateral snake $S(DA(Q_n))$, $DA(Q_m) \odot nK_1$ and $DA(T_m) \odot nK_1$ admit vertex equitable labeling.

2010 MSC: 05C78, 05C38

Keywords: Vertex equitable labeling, Vertex equitable graph, Double triangular snake graph, Double alternate triangular snake graph, Double alternate quadrilateral snake graph

1. Introduction

All graphs considered here are simple, finite, connected and undirected. We follow the basic notation and terminology of graph theory as in [2]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey of graph labeling can be found in [1]. The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan [9]. Let G be a graph with p vertices and q edges and $A = \{0, 1, 2, \cdots, \lceil \frac{q}{2} \rceil\}$. A graph G is said to be vertex equitable if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, \cdots, q$, where $v_f(a)$ is the number of vertices v with $f(v) = a$ for $a \in A$. The vertex labeling f is known as vertex equitable labeling. A graph G is said to be a vertex equitable if it admits a vertex equitable labeling. In [9] they proved that the graphs like paths, bistars $B(n, n)$, combs,

Pon Jeyanthi (Corresponding Author); Research Centre, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur - 628 215, Tamilnadu, India (email: jeyajeyanthi@rediffmail.com).
Anthony Maheswari; Department of Mathematics, Kamaraj College of Engg. and Technology, Virudhunagar, Tamilnadu, India (email: bala_nithin@yahoo.co.in).
Mani Vijayalakshmi; Department of Mathematics, Department of Mathematics, Dr. G. U. Pope College of Engineering, Sawyerpuram, Thoothukudi District, Tamil Nadu, India (email: viji_mac@rediffmail.com).

cycles C_n if $n \equiv 0$ or 3 (mod 4), $K_{2,n}$, $C_3^{(t)}$ for $t \geq 2$, quadrilateral snakes, $K_2 + mK_1$, $K_{1,n} \cup K_{1,n+k}$ if and only if $1 \leq k \leq 3$, ladders, arbitrary super division of any path and cycle C_n with $n \equiv 0$ or 3 (mod 4) are vertex equitable. Also they proved that the graphs $K_{1,n}$ if $n \geq 4$, any Eulerian graph with n edges where $n \equiv 1$ or 2 (mod 4), the wheel W_n, the complete graph K_n if $n > 3$ and triangular cactus with $q \equiv 0$ or 6 or 9 (mod 12) are not vertex equitable. In addition, they proved that if G is a graph with p vertices and q edges, q is even and $p < \lceil \frac{q}{2} \rceil + 2$ then G is not vertex equitable. Motivated by these results, we [3]-[6] proved that T_p-trees, $T \odot \overline{K_n}$ where T is a T_p-trees with even number of vertices, $T\hat{\circ}P_n$, $T\hat{\circ}2P_n$, $T\hat{\circ}C_n (n \equiv 0 \ , 3 \pmod 4)$, $T\tilde{\circ}C_n (n \equiv 0 \ , 3 \pmod 4)$, bistar $B(n, n+1)$, square graph of $B_{n,n}$and splitting graph of $B_{n,n}$, the caterpillar $S(x_1, x_2, \cdots, x_n)$ and $C_n \odot K_1$, P_n^2, tadpoles, $C_m \oplus C_n$, armed crowns , $[P_m; C_n^2]$, $\langle P_m \hat{\circ} K_{1,n} \rangle$, kC_4-snakes for all $k \geq 1$, generalized kC_n-snakes if $n \equiv 0 \pmod 4$, $n \geq 4$ and the graphs obtained by duplicating an arbitrary vertex and an arbitrary edge of a cycle C_n, total graph of P_n, splitting graph of P_n and fusion of two edges of a cycle C_n are vertex equitable graphs.

In this paper, we prove that $S(D(T_n))$, $S(D(Q_n))$, $S(DA(T_n))$, $S(DA(Q_n))$, $DA(Q_m) \odot nK_1$ and $DA(T_m) \odot nK_1$ are vertex equitable graphs. We use the following definitions in the subsequent section.

Definition 1. *The double triangular snake $D(T_n)$ is a graph obtained from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} to the new vertices w_i and u_i for $i = 1, 2, \cdots, n-1$.*

Definition 2. *The double quadrilateral snake $D(Q_n)$ is a graph obtained from a path P_n with vertices $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} to the new vertices v_i, x_i and w_i, y_i respectively and then joining v_i, w_i and x_i, y_i for $i = 1, 2, \cdots, n-1$.*

Definition 3. *A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1}(alternatively) to the two new vertices v_i and w_i for $i = 1, 2, \cdots, n-1$.*

Definition 4. *A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1}(alternatively) to the two new vertices v_i, x_i and w_i, y_i respectively and adding the edges $v_i w_i$and $x_i y_i$ for $i = 1, 2, \cdots, n-1$.*

Definition 5. *Let G be a graph. The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.*

Definition 6. *The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.*

2. Main results

Theorem 2.1. *Let $G_1(p_1, q_1), G_2(p_2, q_2), \cdots, G_m(p_m, q_m)$ be vertex equitable graphs with q_i even ($i = 1, 2, \cdots, m$) and u_i, v_i be the vertices of G_i $(1 \leq i \leq m)$ labeled by 0 and $\frac{q_i}{2}$. Then the graph G obtained by identifying v_1 with u_2 and v_2 with u_3 and v_3 with u_4 and so on until we identify v_{m-1} with u_m is also a vertex equitable graph.*

Proof. First we assign the label $\frac{\sum_{j=1}^{i} q_j}{2}$, $1 \leq i \leq m-1$ to the common vertices between the two graphs G_i, G_{i+1}. Then we add the number $\frac{\sum_{j=1}^{i} q_j}{2}$ to all the remaining vertex labels of the graph G_{i+1}, $1 \leq i \leq m-1$. Hence the edge labels are $1, 2, \cdots, q_1$; $q_1+1, q_1+2, \cdots, q_1+q_2, q_1+q_2+1, q_1+q_2+2, \cdots, q_1+q_2+q_3; \cdots; \sum_{j=1}^{m-1} q_j + 1, \sum_{j=1}^{m-1} q_j + 2, \cdots, \sum_{j=1}^{m} q_j$. □

Theorem 2.2. *The graph $S(D(T_n))$ is a vertex equitable graph.*

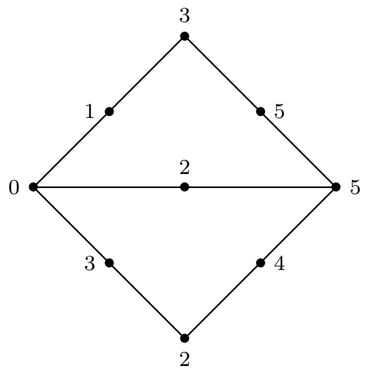

Figure 1.

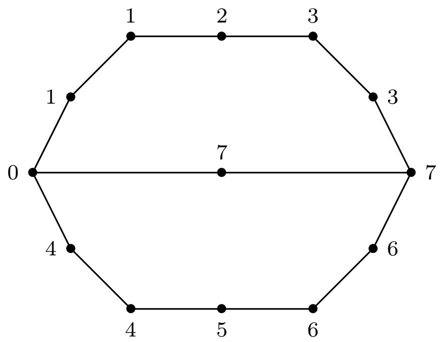

Figure 2.

Proof. The vertex equitable labeling shown in Figure 1 together with Theorem 2.1 proves the result. □

Theorem 2.3. *The graph $S(D(Q_n))$ is a vertex equitable graph.*

Proof. The vertex equitable labeling shown in Figure 2 together with Theorem 2.1 proves the result. □

Theorem 2.4. *The graph $S(DA(T_n))$ is a vertex equitable graph.*

Proof. Let $G = S(DA(T_n))$. Let $u_1, u_2, \cdots, u_n$ be the vertices of path P_n.

Case i. The triangle starts from u_1.

We construct $DA(T_n)$ by joining u_{2i-1} and u_{2i} to the new vertices v_i, w_i for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$. Let $V(G) = V(DA(T_n)) \cup \{u'_i | 1 \leq i \leq n-1\} \cup \left\{x_i, y_i, x'_i, y'_i | 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right\}$ and $E(G) = E(DA(T_n)) \cup \{u_i u'_i | 1 \leq i \leq n\} \cup \{u'_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{x_i v_i, x'_i w_i, v_i y_i, w_i y'_i | 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\} \cup \{u_{2i-1} x_i, u_{2i-1} x'_i, y_i u_{2i}, y'_i u_{2i} | 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\}$. We consider the following two sub cases:

Subcase i. n is even.

Here $|V(G)| = 5n - 1$ and $|E(G)| = 6n - 2$. Let $A = \{0, 1, 2, \cdots, 3n - 1\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \leq i \leq \frac{n}{2}$, $f(u_{2i-1}) = 6(i-1)$, $f(u'_{2i-1}) = 6i - 5$, $f(u_{2i}) = f(y_i) = 6i - 1$, $f(x_i) = f(y'_i) = 6i - 3$, $f(w_i) = f(x'_i) = 6i - 4$, $f(v_i) = 6i - 2$ and $f(u'_{2i}) = 6i$ if $1 \leq i \leq \frac{n-2}{2}$. It can be verified that the induced edge labels of $S(DA(T_n))$ are $1, 2, \cdots, 6n - 2$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(T_n))$.

Subcase ii. n is odd.

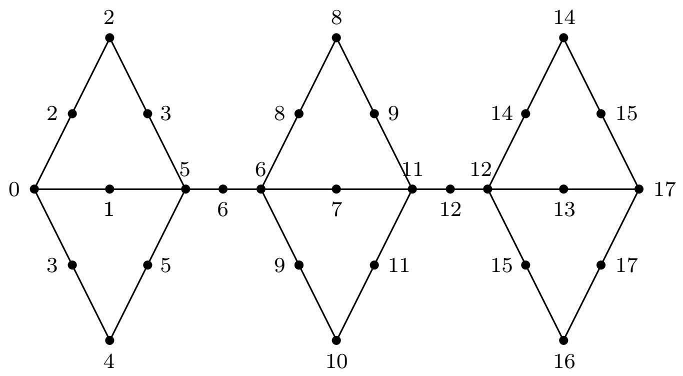

Figure 3.

Here $|V(G)| = 5n - 4$ and $|E(G)| = 6n - 6$. Let $A = \{0, 1, 2, \cdots, 3n - 3\}$. Define a vertex labeling $f : V(G) \to A$ as follows: We label the vertices u_{2i-1} $\left(1 \le i \le \left\lceil \frac{n}{2} \right\rceil\right)$ and $u_{2i}, u'_{2i-1}, u'_{2i}, v_i, v'_i, w_i, w'_i$ $\left(1 \le i \le \frac{n-1}{2}\right)$ as in sub case (i). It can be verified that the induced edge labels of $S(DA(T_n))$ are $1, 2, \cdots, 6n - 6$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(T_n))$.

Case ii. The triangle starts from u_2.

We construct $DA(T_n)$ by joining u_{2i} and u_{2i+1} to the new vertices v_i, w_i for $1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil$. Let $V(G) = V(DA(T_n)) \cup \{u'_i | 1 \le i \le n-1\} \cup \left\{x_i, y_i, x'_i, y'_i | 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil\right\}$ and $E(G) = E(DA(T_n)) \cup \{u_i u'_i | 1 \le i \le n\} \cup \{u'_i u_{i+1} | 1 \le i \le n-1\} \cup \{x_i v_i, x'_i w_i, v_i y_i, w_i y'_i | 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil\} \cup \{u_{2i} x_i, u_{2i} x'_i, u_{2i+1} y_i, u_{2i+1} y'_i | 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil\}$. We consider the following two sub cases:

Subcase i. n is odd.

Here $|V(G)| = 5n - 4$ and $|E(G)| = 6n - 6$. Let $A = \{0, 1, 2, \cdots, 3n - 3\}$. Define a vertex labeling $f : V(G) \to A$ as follows: $f(u_{2i-1}) = 6(i - 1)$ if $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$, for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, $f(u_{2i}) = f(u'_{2i-1}) = 6i - 5$, $f(u'_{2i}) = 6i - 4$, $f(w_i) = f(x'_i) = 6i - 3$, $f(x_i) = f(y'_i) = 6i - 2$, $f(y_i) = 6i$, $f(v_i) = 6i - 1$. It can be verified that the induced edge labels of $S(DA(T_n))$ are $1, 2, \cdots, 6n - 6$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(T_n))$.

Subcase ii. n is even.

Here $|V(G)| = 5n - 7$ and $|E(G)| = 6n - 10$. Let $A = \{0, 1, 2, \cdots, 3n - 5\}$. Define a vertex labeling $f : V(G) \to A$ as follows: We label the vertices u_{2i-1}, u'_{2i-1}, u_{2i} $\left(1 \le i \le \left\lceil \frac{n}{2} \right\rceil\right)$ and $v_i, v'_i, w_i, w'_i, x_i, x'_i, y_i, y'_i, u'_{2i}$ $\left(1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil\right)$ as in sub case (i). It can be verified that the induced edge labels of $S(DA(T_n))$ are $1, 2, \cdots, 6n - 10$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(T_n))$. □

An example for the vertex equitable labeling of $S(DA(T_6))$ where the two triangles start from u_1 is shown in Figure 3.

Theorem 2.5. *The graph $S(DA(Q_n))$ is a vertex equitable graph.*

Proof. Let $G = S(DA(Q_n))$. Let $u_1, u_2, \cdots, u_n$ be the vertices of path P_n.

Case i. The quadrilateral starts from u_1.

We construct $DA(Q_n)$ by joining u_{2i-1} and u_{2i} to the new vertices v_i, w_i and x_i, y_i respectively and then joining v_i, x_i and w_i, y_i for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$. Let $V(G) = V(DA(Q_n)) \cup \{u'_i | 1 \le i \le n-1\} \cup \left\{v'_i, w'_i, x'_i, y'_i, z_i, z'_i | 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\right\}$ and $E(G) = E(DA(Q_n)) \cup \{u_i u'_i | 1 \le i \le n\} \cup \{u'_i u_{i+1} | 1 \le i \le n -$

$1\} \cup \{v_iv_i', v_ix_i', x_iz_i, w_i'w_i, w_iy_i', y_i'y_i, y_iz_i' | 1 \le i \le \lfloor \frac{n}{2} \rfloor\} \cup \{u_{2i-1}v_i', u_{2i-1}w_i', u_{2i}z_i, u_{2i}z_i' | 1 \le i \le \lfloor \frac{n}{2} \rfloor\}$. We consider the following two sub cases:

Subcase i. n is even.

Here $|V(G)| = 7n - 1$ and $|E(G)| = 8n - 2$. Let $A = \{0, 1, 2, \cdots, 4n - 1\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \le i \le \frac{n}{2}$, $f(u_{2i-1}) = 8(i-1)$, $f(u_{2i}) = f(u_{2i-1}') = 8i - 1$, $f(x_i) = f(z_i) = 8i - 2$, $f(x_i') = 8i - 3$, $f(w_i) = f(w_i') = 8i - 7$, $f(y_i) = f(z_i') = 8i - 5$, $f(y_i') = 8i - 6$ and $f(u_{2i}') = 8i$ if $1 \le i \le \frac{n-2}{2}$. It can be verified that the induced edge labels of $S(DA(Q_n))$ are $1, 2, \cdots, 8n-2$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(Q_n))$.

Subcase ii. n is odd.

Here $|V(G)| = 7n - 6$ and $|E(G)| = 8n - 8$. Let $A = \{0, 1, 2, \cdots, 4n - 4\}$. Define a vertex labeling $f : V(G) \to A$ as follows: We label the vertices u_{2i-1} $\left(1 \le i \le \lceil \frac{n}{2} \rceil\right)$ and $u_{2i}, u_{2i-1}', u_{2i}', v_i, v_i', w_i, w_i', x_i, x_i', y_i, y_i', z_i, z_i'$ $\left(1 \le i \le \frac{n-1}{2}\right)$ as in sub case (i). It can be verified that the induced edge labels of $S(DA(Q_n))$ are $1, 2, \cdots, 8n - 8$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(Q_n))$.

Case ii. The quadrilateral starts from u_2.

We construct $DA(Q_n)$ by joining u_{2i} and u_{2i+1} to the new vertices v_i, w_i and x_i, y_i respectively and then joining v_i, x_i and w_i, y_i for $1 \le i \le \lceil \frac{n-2}{2} \rceil$. Let $V(G) = V(DA(Q_n)) \cup \{u_i' | 1 \le i \le n - 1\} \cup \left\{v_i', w_i', x_i', y_i', z_i, z_i' | 1 \le i \le \lceil \frac{n-2}{2} \rceil\right\}$ and $E(G) = E(DA(Q_n)) \cup \{u_iu_i' | 1 \le i \le n - 1\} \cup \{u_i'u_{i+1} | 1 \le i \le n - 1\} \cup \{v_iv_i', v_ix_i', x_iz_i, w_i'w_i, w_iy_i', x_ix_i', y_i'y_i, y_iz_i' | 1 \le i \le \lceil \frac{n-2}{2} \rceil\} \cup \{u_{2i}v_i', u_{2i}w_i', u_{2i+1}z_i, u_{2i+1}z_i' | 1 \le i \le \lceil \frac{n-2}{2} \rceil\}$. We consider the following two sub cases:

Subcase i. n is odd.

Here $|V(G)| = 7n - 6$ and $|E(G)| = 8n - 8$. Let $A = \{0, 1, 2, \cdots, 4n - 4\}$. Define a vertex labeling $f : V(G) \to A$ as follows:
For $1 \le i \le \lceil \frac{n}{2} \rceil$, $f(u_{2i-1}) = 8(i - 1)$, $f(u_{2i-1}') = 8i - 7$.
For $1 \le i \le \lfloor \frac{n}{2} \rfloor$, $f(u_{2i}) = 8i - 7$, $f(u_{2i}') = 8i$, $f(v_i) = f(v_i') = 8i - 3$, $f(x_i) = f(z_i) = 8i - 1$, $f(x_i') = 8i - 2$, $f(w_i) = f(w_i') = 8i - 6$, $f(y_i) = f(z_i') = 8i - 4$, $f(y_i') = 8i - 5$. It can be verified that the induced edge labels of $S(DA(Q_n))$ are $1, 2, \cdots, 8n - 8$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(Q_n))$.

Subcase ii. n is even.

Let $|V(G)| = 7n - 11$ and $|E(G)| = 8n - 14$. Let $A = \{0, 1, 2, \cdots, 4n - 7\}$. Define a vertex labeling $f : V(G) \to A$ as follows: We label the vertices u_{2i-1}, u_{2i}, u_{2i-1}' $\left(1 \le i \le \lceil \frac{n}{2} \rceil\right)$ and $u_{2i}', v_i, v_i', w_i, w_i', x_i, x_i', y_i, y_i'$, $\left(1 \le i \le \lceil \frac{n-2}{2} \rceil\right)$ as in sub case (i). It can be verified that the induced edge labels of $S(DA(Q_n))$ are $1, 2, \cdots, 8n-14$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $S(DA(Q_n))$. □

An example for the vertex equitable labeling of $S(DA(Q_7))$ where the two quadrilaterals start from u_1 is shown in Figure 4.

Theorem 2.6. *Let $G_1(p_1, q), G_2(p_2, q), \cdots, G_m(p_m, q)$) be vertex equitable graphs with q odd u_i, v_i be vertices of G_i $(1 \le i \le m)$ labeled by 0 and $\lceil \frac{q}{2} \rceil$. Then the graph G obtained by joining v_1 with u_2 and v_2 with u_3 and v_3 with u_4 and so on until joining v_{m-1} with u_m by an edge is also a vertex equitable graph.*

Proof. The graph G has $p_1 + p_2 + \cdots + p_m$ vertices and $mq + (m - 1)$ edges. Let f_i be the vertex equitable labeling of G_i $(1 \le i \le m)$ and let $A = \left\{0, 1, 2, \cdots, \lceil \frac{mq+m-1}{2} \rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as $f(x) = f_i(x) + \frac{(i-1)(q+1)}{2}$ if $x \in G_i$ for $1 \le i \le m$. The edge labels of G_i are increased by $(i - 1)(q + 1)$ for $i = 1, 2, \cdots, m$ under the new labeling f. The bridge between the two graphs G_i , G_{i+1} will get the label $i(q + 1)$, $1 \le i \le m - 1$. Hence the edge labels of G are distinct and is $\{1, 2, \cdots, mq + m - 1\}$. Also $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$. Then the graph G is a vertex equitable graph. □

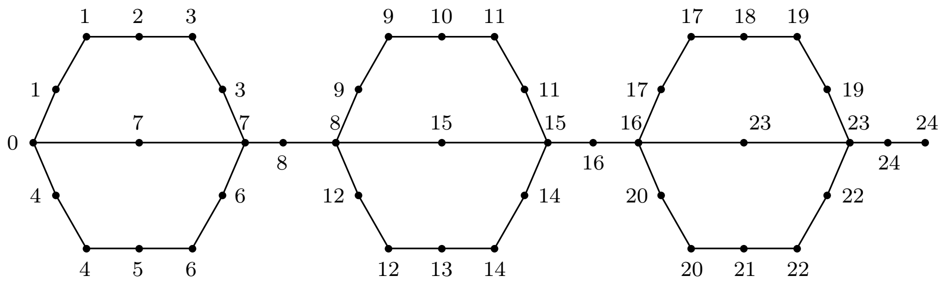

Figure 4.

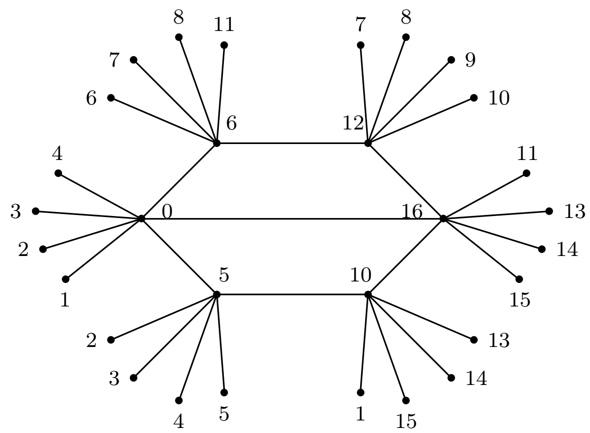

Figure 5.

Remark 2.7. *[7] The graph $DA(Q_m) \odot nK_1$ and $DA(T_m) \odot nK_1$ are vertex equitable graphs if $m, n = 1, 2$.*

Theorem 2.8. *The graph $DA(Q_2) \odot nK_1$ is a vertex equitable graph for $n \geq 3$*

Proof. Let $G = DA(Q_2) \odot nK_1$. Let $V(G) = \{u_1, u_2, v, w, x, y\} \cup \{u_{ij} | 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{v_i, w_i, x_i, y_i | 1 \leq i \leq n\}$ and $E(G) = \{u_1u_2, u_1v, vw, wu_2, u_1x, xy, yu_2\} \cup \{u_iu_{ij} | 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{vv_i, ww_i, xx_i, yy_i | 1 \leq i \leq n\}$. Here $|V(G)| = 6(n+1)$ and $|E(G)| = 6n+7$. Let $A = \left\{0, 1, 2, \cdots, \left\lceil \frac{6n+7}{2} \right\rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \leq i \leq n$, $f(u_{1i}) = i$, $f(v_i) = i+1$, $f(y_i) = n+2+i$, $f(u_i) = 0$, $f(u_2) = 3n+4$, $f(v) = n+1$, $f(w) = 2(n+1)$, $f(x) = n+2$, $f(y) = 2(n+2)$, $f(u_{2i}) = 3n+4-i$ if $1 \leq i \leq n-1$, $f(u_{2n}) = 2n+3$, $f(w_1) = 1$, $f(w_i) = 3n+5-i$ if $2 \leq i \leq n$, $f(x_i) = n+i+1$ if $1 \leq i \leq n-1$, $f(x_n) = 2n+3$. It can be verified that the induced edge labels of $DA(Q_2) \odot nK_1$ are $1, 2, \cdots, 6n+7$ and $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$. Hence f is a vertex equitable labeling of $DA(Q_2) \odot nK_1$. □

An example for the vertex equitable labeling of $DA(Q_2) \odot 4K_1$ is shown in Figure 5.

Theorem 2.9. *The graph $DA(Q_m) \odot nK_1$ is a vertex equitable graph for $m, n \geq 3$.*

Proof. By Theorem 2.8, $DA(Q_2) \odot nK_1$ is a vertex equitable graph. Let $G_i = DA(Q_2) \odot nK_1$ for $1 \leq i \leq m-1$. Since each G_i has $6n+7$ edges, by Theorem 2.6, $DA(Q_m) \odot nK_1$ admits vertex equitable labeling. □

An example for the vertex equitable labeling of $DA(Q_6) \odot 4K_1$ is shown in Figure 6.

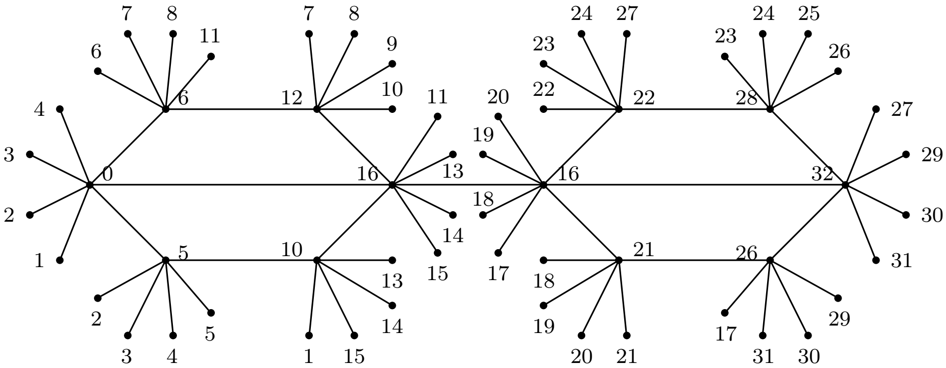

Figure 6.

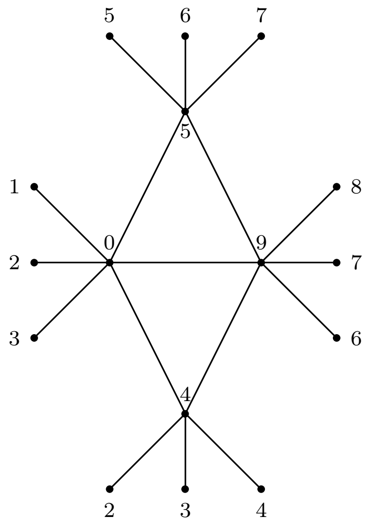

Figure 7.

Theorem 2.10. *The graph $DA(T_2) \odot nK_1$ is a vertex equitable graph for $n \geq 3$.*

Proof. Let $G = DA(T_2) \odot nK_1$. Let $V(G) = \{u_1, u_2, u, w\} \cup \{u_{ij} | 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{v_i, w_i | 1 \leq i \leq n\}$ and $E(G) = \{u_1u_2, u_1v, vu_2, u_1w, wu_2\} \cup \{u_iu_{ij} | 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{vv_i, ww_i | 1 \leq i \leq n\}$. Here $|V(G)| = 4(n+1)$ and $|E(G)| = 4n+5$. Let $A = \{0, 1, 2, \cdots, \lceil \frac{4n+5}{2} \rceil\}$. Define a vertex labeling $f : V(G) \to A$ as follows. For $1 \leq i \leq n$, $f(u_{1i}) = i$, $f(u_{2i}) = 2n+3-i$, $f(v_i) = i+1$, $f(w_i) = n+1+i$, $f(u_1) = 0$, $f(u_2) = 2n+3$, $f(v) = n+1$, $f(w) = n+2$. It can be verified that the induced edge labels of $DA(T_2) \odot nK_1$ are $1, 2, \cdots, 4n+5$ and $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$. Hence f is a vertex equitable labeling of $DA(T_2) \odot nK_1$. □

An example for the vertex equitable labeling of $DA(T_2) \odot 3K_1$ is shown in Figure 7.

Theorem 2.11. *The graph $DA(T_m) \odot nK_1$ is a vertex equitable graph for $m, n \geq 3$.*

Proof. By Theorem 2.10, $DA(T_2) \odot nK_1$ is a vertex equitable graph. Let $G_i = DA(T_2) \odot nK_1$,

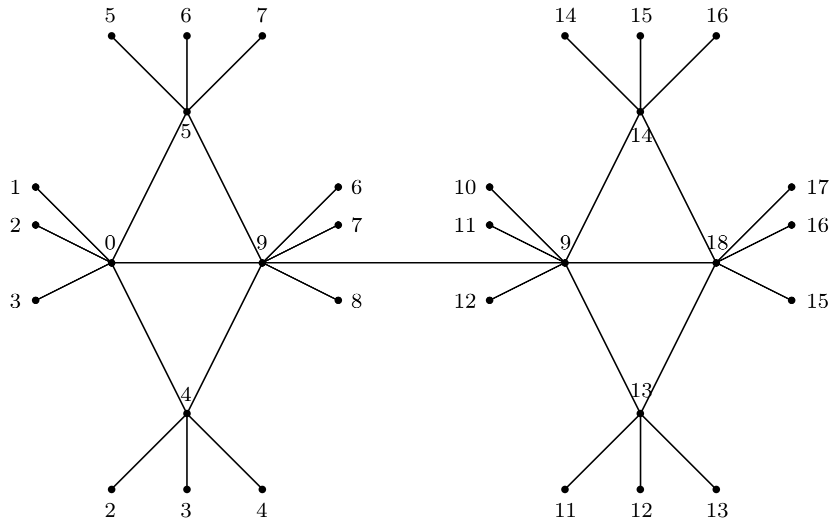

Figure 8.

$1 \leq i \leq m-1$. Since each G_i has $4n+5$ edges, by Theorem 2.6, $DA(T_m) \odot nK_1$ admits a vertex equitable labeling. □

An example for the vertex equitable labeling of $DA(T_m) \odot 4K_1$ is shown in Figure 8.

Acknowledgment: The authors would like to thank the referees for their valuable suggestions to improve the paper.

References

[1] J. A. Gallian, Graph labeling, Electron. J. Combin. (2015) (Dynamic Survey #DS6).
[2] F. Harary, Graph theory, Addison-Wesley, Reading Mass, 1972.
[3] P. Jeyanthi, A. Maheswari, Some results on vertex equitable labeling, Open J. Discrete Math. 2(2) (2012) 51–57.
[4] P. Jeyanthi, A. Maheswari, Vertex equitable labeling of transformed trees, J. Algorithms Comput. 44(1) (2013) 9–20.
[5] P. Jeyanthi, A. Maheswari, Vertex equitable labeling of cyclic snakes and bistar graphs, J. Sci. Res. 6(1) (2014) 79–85.
[6] P. Jeyanthi, A. Maheswari, M. Vijayalaksmi, Vertex equitable labeling of cycle and star related graphs, J. Sci. Res. 7(3) (2015) 33–42.
[7] P. Jeyanthi, A. Maheswari, Vertex equitable labeling of cycle and path related graphs, Util. Math. 98 (2015) 215–226.
[8] P. Jeyanthi, A. Maheswari, M. Vijayalakshmi, Vertex equitable labeling of double alternate snake graphs, J. Algorithms Comput. 46 (2015) 27–34.
[9] M. Seenivasan, A. Lourdusamy, Vertex equitable labeling of graphs, J. Discrete Math. Sci. Cryptogr. 11(6) (2008) 727–735.