Dark Energy Classification in Higher Dimensional FRW Model According to State

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Abstract- In this study, we obtain scalar potential in higher dimensional FRW model via considering exponential acceleration. Also, we classify dark energy models for higher dimensional FRW model in accordance with state parameters (r, s) by considering exponential acceleration. In this respect, we obtain cosmological constant for r = 1and s = 0 as dark energy candidate. We obtain Phantom energy for r <1 and s > 0 dark energy candidate.

Keywords Dark energy, higher dimensional, FRW model, state parameter.

1. Introduction

Many studies showed that the dark energy using the positive densities and negative pressure occur acceleration of the universe in recent years [1]. The negative pressure can be obtain by using conservation equations, which is $(T^{ik}_{;k}=0)$ for (n + 2) dimensions [2].

$$\dot{\rho}+(n+1)H(\rho+p)=0 \tag{1}$$

Einstein's field equations are as follows [3]:

$$G_{ab}\equiv R_{ab}-\frac{1}{2}g_{ab}R=\chi T_{ab} \tag{2}$$

FRW metric (n + 2) dimension is as follows:

$$dS^2=dt^2-a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(dX_n)^2\right] \tag{3}$$

in which a(t) expresses scale parameter, k = 0, ±1 is used as curvature parameter and $dX_n^2=d\theta_1^2+Sin^2\theta_1 d\theta_2^2+\ldots+Sin^2\theta_1 Sin^2\theta_2 \ldots Sin^2\theta_{n-1}d\theta_n^2$ [4]. We will decide energy-momentum tensor in accordance with perfect fluid.

$$T_{ab}=(\rho+p)U_aU_b-pg_{ab} \tag{4}$$

in which ρ shows density and P is utilized as pressure, U_a, U_b are determined as the components of the speed [3]. Einstein's field equations for eqs. (3) and (4) is:

$$\frac{n(n+1)}{2}\left[\frac{\dot{a}^2+k}{a^2}\right]=\frac{8\pi G}{c^4}\rho \tag{5}$$

$$-n\frac{\ddot{a}}{a}-\frac{n(n-1)}{2}\left[\frac{\dot{a}^2+k}{a^2}\right]=\frac{8\pi G}{c^4}p \tag{6}$$

in which $8\pi G = c^4$, k = 0 and α scale factor [5]. The condition of [6]-[8]

$$a(t)=e^{\lambda t^{\beta}} \quad \lambda > 0 \text{ and } 0 < \beta < 1\ \lambda \tag{7}$$

From eqs. (5), (6) and (7), we have

$$\rho=\frac{n(n+1)\lambda^2\beta^2t^{2(\beta-1)}}{2} \tag{8}$$

and

$$p=-n\lambda\beta(\beta-1)t^{\beta-2}-n\lambda^2\beta^2t^{2(\beta-1)}\left(\frac{n+1}{2}\right) \tag{9}$$

We get parameters such as density and pressure for the scalar field:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \frac{n(n+1)\lambda^2\beta^2 t^{2(\beta-1)}}{2} \tag{10}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -n\lambda\beta(\beta-1)t^{\beta-2} - n\lambda^2\beta^2 t^{2(\beta-1)}\left(\frac{n+1}{2}\right) \tag{11}$$

Based on Equations (10) and (11), we calculate scalar field

$$\phi = 2\sqrt{-n\lambda\beta(\beta-1)}\,t^{\frac{\beta}{2}} + C \tag{12}$$

in which, C defines integration constant. According to Equations (10) and (11), we obtain scalar potential

$$V(\phi) = \frac{1}{2}\left[n(n+1)\lambda^2\beta^2 t^{2(\beta-1)} + n\lambda\beta(\beta-1)t^{\beta-2}\right] \tag{13}$$

Deceleration parameter is shown below [9]:

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{14}$$

From eqs. (7) and (14), we obtain

$$q = -\frac{(\beta-1)}{\lambda\beta t^\beta} - 1 \tag{15}$$

The dark energy can be derived according to the w state parameter. In addition, the dark energy can be classified by using the geometric part of the field equations, with the parameters received r and s values. Such a classification was obtained for four-dimensional space-time [10]. For (n + 2) dimensional FRW space-time is obtained as follows.

$$r = \frac{\dddot{a}}{aH^3} \tag{16}$$

$$s = \frac{r-1}{(n+1)(q-\frac{1}{2})} \tag{17}$$

State parameters for the cosmological constant model are r = 1 and s = 0; state parameters for Chaplyin gas cosmological model are r > 1, s < 0 and state parameters for the phantom energy model are r < 1, s > 0 [10]-[19]. When the expression of the scale factor a is written instead of the equation (16).

$$r = 1 + \frac{(\beta-1)(\beta-2)}{\lambda^2\beta^2 t^{2\beta}} + \frac{3(\beta-1)}{\lambda\beta t^\beta} \tag{18}$$

From eqs. (15), (17) and (18), we have

$$s = \frac{\frac{(\beta-1)(\beta-2)}{\lambda^2\beta^2 t^{2\beta}} + \frac{3(\beta-1)}{\lambda\beta t^\beta}}{\frac{(n+1)(\beta-1)}{\lambda\beta t^\beta} + \frac{(n+1)}{2}} \tag{19}$$

2. Conclusion

In this study, for λ > 0, 0 < β < 1 λ = 1/2, β = 1/2, and t = 1 parameters and the state n = 2, r = 1and s = 0 are obtained. We can say that dark energy candidate of universe is a cosmological constant model. For t = 2 and t = 3, r < 1, s > 0 are obtained. In this case, phantom energy is determined the candidate for dark energy of universe. For λ = 1/2, β = 1/2, and t = 4 parameters and the state n = 2, r = 1 and s = 0 are obtained and we can say that the cosmological constant model is the candidate in dark energy for universe. r < 1, s > 0 are obtained for t ≥ 5 and in this case phantom energy is the candidate in dark energy for universe.

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