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Dynamics of a Prey-Predator System with Harvesting Effect on Prey

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ABSTRACT This article is about the dynamic nature of a prey-predator model exposed to the harvesting effect on prey. Firstly, the model's fixed points' existence and stability are determined, and then, the presence and direction of a Neimark-Sacker bifurcation are examined. By using the bifurcation theory, we show that the system experiences Neimark-Sacker bifurcation. The hybrid control strategy is handled to control the chaos caused by the Neimark-Sacker bifurcation. Additionally, some numerical simulations are given to validate the theoretical outcomes obtained.

KEYWORDS
Prey-Predator model
Fixed point
Stability
Harvesting
Bifurcation
Chaos control

INTRODUCTION

Basic models based on biological assumptions are used to better understand the behavior of populations. Lotka and Volterra (Lotka 1925; Volterra 1978) are pioneers in proposing a mathematical prey-predator model. In the interaction of the prey and the predator, both the prey and the predator develop various strategies to survive and adapt to their environment. The balance of nature is maintained by the continuity of the life of the species. Models are used as tools to learn about future numbers of species. Here, the analysis conducted by including the factors affecting the population in the model allows us to reach more realistic results. It is possible to have information about the dynamics of the population with model analysis depending on these factors.

The analysis of prey-predator models with the harvesting effect has an important place in dynamic systems. Since there is great interest in the use of bioeconomic models (Clark 1985; Clark and Clark 1990), the dynamic behaviors of harvesting populations are examined in many studies. At the same time, this factor is effective in controlling populations (Liu *et al.* 2008; Paul *et al.* 2021). In most cases, the goal is not only population control; but also getting a significant harvest gain from the population. If the harvest pushes the population to extinction, this process should be stopped for a certain period. It is possible to reach qualified conclusions about the dynamics of these models with stability, bifurcation analysis, presence of chaotic behaviors and chaos control (Elaydi 1996; Gümüş and Feckan 2021; Kuznetsov *et al.* 1998; Liu *et al.* 2008; Madhusudanan *et al.* 2014; Murray 2002; Peng *et al.* 2009; Robinson 1998).

In the literature, many continuous-time prey-predator models have been introduced to explain the complex relationships between species. However, in ecology, populations evolve in discrete time steps, as there is no overlap between successive generations. It is therefore benefical to use difference equations (Ak Gümüş 2014; Gümüş and Kose 2012; Gümüş *et al.* 2022b,c; Merdan and Gümüş 2012; Merdan *et al.* 2018) or discrete-time systems involving prey-predator models (Danca *et al.* 2019; Elsadany *et al.* 2012; Gümüş 2020; Liu and Xiao 2007; Rana 2015), host-parasitoid models (Gümüş 2015; Gümüş and Kangalgil 2015; Gümüs *et al.* 2020), epidemic models (Gümüş *et al.* 2022a, 2019), and also fractional models (Selvam *et al.* 2020; Singh *et al.* 2019).

In prey-predator population models, one can say that the prey population has a limiting influence on the population dynamics since the size of the predator population depends on the size of the prey population. The size of predator populations that do not catch sufficient numbers of prey decreases. Therefore, small numerical changes in the prey population can cause large changes in the dynamics of such models. To maintain a balanced life in prey-predator populations, the prey population must have an appropriate growth rate. The harvesting factor on the prey will affect the growth rate of the prey population. In this study, our aim is to investigate prey-predator dynamics by examining the effect of the harvest factor on the internal growth rate of the prey population. For this purpose, the prey population's growth rate is taken as the bifurcation parameter, and results are obtained about the long-term behavior of the population.

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This paper examines a discrete-time prey-predator system that depicts interactions between two populations of non-overlapping generations that are affected by the harvesting effect.

$$
\begin{aligned}
x_{n+1} &= ax_n(1-x_n) - bx_ny_n - hx_n \\
y_{n+1} &= dx_ny_n
\end{aligned}
\qquad (1)
$$

where x_n and y_n denote the numbers of prey and predator in year (generation) n, respectively, and the parameters a, b, h, and d are all positive parameters. In this model, bx_n indicates the number of prey individuals ingested per unit area and per unit time by an individual predator, and dx_ny_n is the predator reaction. Here, a is the growth rate of the prey population which has a logistic growth rate, d is the growth rate of the predator population limited by the number of prey, and h is the harvesting rate, where $0 < h < 1$.

In a previous study (Danca *et al.* 2019), the dynamics of discrete-time prey-predator model (1) are presented without the harvesting effect. We explore the stability and bifurcation of the system (1) by incorporating the harvesting effect and observe the dynamics of the system. We refer to studies (Elaydi 1996; Liu and Xiao 2007; Din 2013) for some basic concepts that we have used throughout our paper.

The paper is arranged as follows: In Section 2, we present the existence and local asymptotic stability of fixed points of the system (1) in $\mathbb{R}^2_+$ with plots showing system behavior. In Section 3, the dynamics of system (1) which undergoes a Neimark-Sacker bifurcation are investigated by choosing a as a bifurcation parameter. The chaos emerging with the Neimark-Sacker bifurcation is controlled by a hybrid method. The dynamical characteristics of the system (1) are displayed via numerical simulations in the form of trajectories, bifurcation diagrams, and phase portraits. The last section provides a summary of the results.

THE EXISTENCE AND STABILITY OF FIXED POINTS OF SYSTEM (1)

The analyses of the system (1)'s fixed points' existence and local stability are presented in this section. First, let us examine the existence of all available fixed points of system (1). System (1) has a trivial (extinction) fixed point $E_0 = (0,0)$ for all positive parameters. If $a > h+1$, then, system (1) has an exclusion fixed point $E_1 = (\frac{a-h-1}{a}, 0)$.

If $a > \frac{d(h+1)}{d-1}$ such that $d > 1$, then $E^* = (\frac{1}{d}, \frac{ad-a-d-dh}{bd})$ is a unique coexistence fixed point of system (1).

Remark 1 *When $a < h+1$, the fixed point E_0 is locally asymptotically stable, and when $1 < d$ and $h+1 < a < \min(3-h, \frac{d(h+1)}{d-1})$, the fixed point E_1 is locally asymptotically stable. The magnitude of the eigenvalues of the Jacobian matrix determines the local stability conditions of the fixed points of discrete-time systems.*

- **The Local Stability of the Coexistence Fixed Point**

Let us investigate the locally asymptotic stability of the coexistence fixed point as follows:

$$E^* = (x^*, y^*) = (\frac{1}{d}, \frac{ad-a-d-dh}{bd}). \qquad (2)$$

where $a > \frac{d(h+1)}{d-1}$, $d > 1$. The Jacobian matrix of system (1) assessed at E^* is

$$J_{E^*} = \begin{pmatrix} 1-\frac{a}{d} & \frac{-b}{d} \\ \frac{-a-d+ad-dh}{b} & 1 \end{pmatrix}$$

and the characteristic polynomial of the Jacobian matrix is

$$F(\lambda) = \lambda^2 + [-2 + \frac{a}{d}]\lambda + a - \frac{2a}{d} - h.$$

So, we have the following Lemma.

Lemma 2 *Suppose that $a > \frac{d+dh+d^2h}{d-1}$, $d > 3$. Then the coexistence fixed point E^* is respectively locally asymptotically stable and unstable, if the following cases are provided:*

(i) *If $a < \frac{d(h+1)}{d-2}$, $h < \frac{1}{d^2-2d-1}$, then E^* is a sink point.*

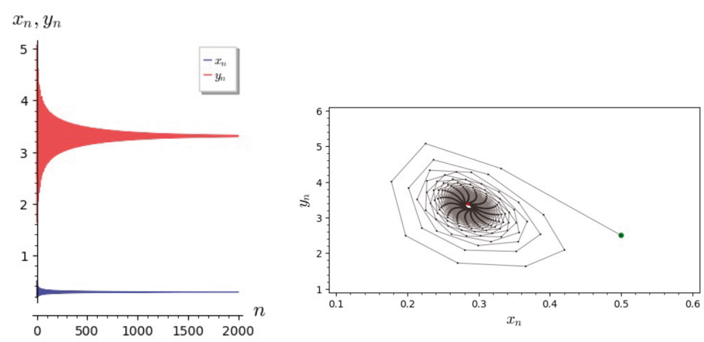
(ii) *If $a > \frac{d(h+1)}{d-2}$, $h < \frac{1}{d^2-2d-1}$, then E^* is a source point.*

We can give examples to confirm the results obtained in Lemma 2. The trajectories and phase portrait of the prey-predator densities are exhibited in Figure 1 and Figure 2 with the parameter values $b = 0.2$, and $d = 3.5$ which are taken from a previous study (Danca *et al.* 2019).

Example 3 *Let us take into account the following population model to expose the appearance of the trajectories and phase portrait of system (1) for the parameter values $a = 2.33$, $b = 0.2$, $d = 3.5$, and $h = 0.002$:*

$$
\begin{aligned}
x_{n+1} &= 2.33x_n(1-x_n) - 0.2x_ny_n - 0.002x_n, \\
y_{n+1} &= 3.5x_ny_n
\end{aligned}
\qquad (3)
$$

where the initial conditions are $x_0 = 0.5$ and $y_0 = 2.5$.



(a) (b)

Figure 1 **(a)** *The trajectories of the prey and predator densities in system (3) when $a = 2.33, b = 0.2, h = 0.002$, and $d = 3.5$.* **(b)** *The phase portrait of system (3) when $a = 2.33$, $b = 0.2$, $h = 0.002$ and $d = 3.5$.*

In this example, it is seen that the fixed point $(0.285714, 3.31143)$ is locally asymptotically stable (see Lemma 2-(i)).

Example 4 *Let us take into account the following population model to expose the appearance of the trajectories and phase portrait of system (1) for $a = 2.34$, $b = 0.2$, $h = 0.002$ and $d = 3.5$:*

$$
\begin{aligned}
x_{n+1} &= 2.34x_n(1-x_n) - 0.2x_ny_n - 0.002x_n, \\
y_{n+1} &= 3.5x_ny_n
\end{aligned}
\qquad (4)
$$

where the initial conditions are $x_0 = 0.5$ and $y_0 = 2.5$.

The fixed point $(0.285714, 3.34714)$ of system (4) with the selected values is unstable (see the Lemma 2-(ii)).

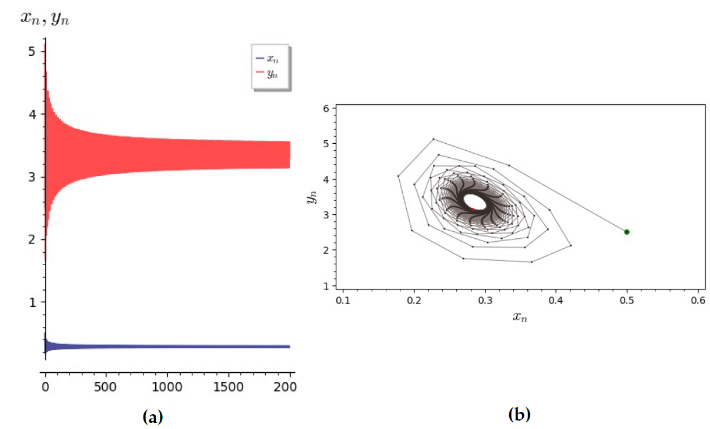


Figure 2 (a) *The trajectories of the prey and predator densities in system (4) when* $a = 2.34$, $b = 0.2$, $h = 0.002$, *and* $d = 3.5$. **(b)** *The phase portrait of system (4) when* $a = 2.34$, $b = 0.2$, $h = 0.002$ *and* $d = 3.5$.

NEIMARK-SACKER BIFURCATION ANALYSIS AND CHAOS CONTROL

- **Neimark-Sacker Bifurcation**

In this section, we discuss whether system (1) experiences a Neimark-Sacker bifurcation by using the bifurcation theory (Kuznetsov *et al.* 1998; Wiggins 2003).As the prey population's growth rate changes, we see that system (1) has a Neimark-Sacker bifurcation. In other words, a is taken as a bifurcation parameter to get the conditions of Neimark–Sacker bifurcations. The direction of the Neimark–Sacker bifurcation is also obtained for system (1). If system (1) ensures the eigenvalue assignment, transversality and nonresonance conditions, then the Neimark–Sacker bifurcation emerges at a bifurcation point a_{NS}. The conditions that cause the bifurcation to occur at the coexistence fixed point E^* are determined as

$$NSB_{E^*} = \{a, b, h, d \in \mathbb{R}_+ : d > 3,\ a_1 < a < a_2 \text{ and } a = a_{NS}\}$$

where

$$a_1 = -2\sqrt{d^4 - 2d^3 - d^2h} + 2(-d + d^2),$$

$$a_2 = +2\sqrt{d^4 - 2d^3 - d^2h} + 2(-d + d^2),$$

$$h < -2\sqrt{\frac{(-1+d)^2 d[-1 + (-2+d)d^2]}{(1+d)^4} + \frac{-1 + d[1 + 2(-2+d)d]}{(1+d)^2}},$$

and

$$a_{NS} = \frac{d(1+h)}{d-2}.$$

By using the transformation $u = x - \frac{1}{d}$, $v = y - \frac{ad-a-d-dh}{bd}$, the fixed point E^* is shifted to the origin. So, we obtain

$$\begin{pmatrix} u \\ v \end{pmatrix} \to J_{E^*} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} F_1(u,v) \\ F_2(u,v) \end{pmatrix}. \quad (5)$$

where

$$F_1(u,v) = -au^2 - buv + O(\|U\|^3) \quad (6)$$
$$F_2(u,v) = duv + O(\|U\|^3) \quad (7)$$

such that $U = (u,v)^T$. From here, system (1) becomes

$$(U_{n+1}) \to J_{E^*}(U_n) + \frac{1}{2}B(u_n, u_n) + \frac{1}{6}C(u_n, u_n, u_n) + O(\|U_n\|^4), \quad (8)$$

with the multilinear vector functions of $u, v, w \in \mathbb{R}^2$:

$$B(u,v) = \begin{pmatrix} B_1(u,v) \\ B_2(u,v) \end{pmatrix}$$

and

$$C(u,v,w) = \begin{pmatrix} C_1(u,v,w) \\ C_2(u,v,w) \end{pmatrix}.$$

These vectors are stated by

$$B_1(u,v) = \sum_{j,k=1}^{2} \frac{\partial^2 F_1}{\partial \xi_j \partial \xi_k}\Big|_{\xi=0} u_j v_k = -2au_1v_1 - b(u_2v_1 + u_1v_2)$$

$$B_2(u,v) = \sum_{j,k=1}^{2} \frac{\partial^2 F_2}{\partial \xi_j \partial \xi_k}\Big|_{\xi=0} u_j v_k = d(u_2v_1 + u_1v_2)$$

$$C_1(u,v,w) = \sum_{j,k=1}^{2} \frac{\partial^3 F_1}{\partial \xi_j \partial \xi_k \xi_l}\Big|_{\xi=0} u_j v_k w_l = 0$$

$$C_2(u,v,w) = \sum_{j,k=1}^{2} \frac{\partial^3 F_2}{\partial \xi_j \partial \xi_k \xi_l}\Big|_{\xi=0} u_j v_k w_l = 0.$$

For $a = a_{NS}$, the eigenvalues of the matrix J_{E^*} associated with the linearization in map (5) are conjugate complex numbers. These eigenvaues are

$$\lambda, \overline{\lambda}\,\Big|_{a=a_{NS}} = \frac{-5 + 2d - h \pm i\sqrt{(1+h)(-9+4d-h)}}{2(d-2)}$$

such that

$$|\lambda(a_{NS})| = 1.$$

For $a \in NSB_{E^*}$, we get

$$\frac{\partial\,|\lambda_i(a)|}{\partial a}\Big|_{a=a_{NS}} \neq 0\ ,\ i = 1,2. \quad (9)$$

Moreover, if

$$trJ(a)\,\Big|_{a=a_{NS}} \neq 0, -1, \quad (10)$$

then, we reach

$$\lambda^k(a_{NS}) \neq 1\ ,\ k = 1,2,3,4. \quad (11)$$

Let $q, p \in \mathbb{C}^2$ be the eigenvectors which correspond to the eigenvalues λ of $J(NSB_{E^*})$ and the eigenvalues $\overline{\lambda}$ of $J(NSB_{E^*})^T$, respectively. If these eigenvectors are computed with the Mathematica program, then we get

$$q \sim \left(\frac{-b(h+1) + ib\sqrt{(-9+4d-h)(h+1)}}{2d(h+1)}, 1\right)^T$$

and

$$p \sim \left(\frac{d(h+1) - id\sqrt{(-9+4d-h)(h+1)}}{2b(d-2)}, 1 \right)^T.$$

By using the inner product in $\mathbb{C}^2 : < p, q >= \overline{p_1}q_1 + \overline{p_2}q_2$, we get the following vector to normalize p in accordance with q

$$p \sim \left(\frac{d^2(1+h)}{bdi\sqrt{(-9+4d-h)(h+1)}}, \frac{2(d-2)}{-9+4d-h+i\sqrt{(-9+4d-h)(h+1)}} \right)^T$$

where $< p, q >= 1$. $\forall\, U \in \mathbb{R}^2$ can be uniquely represented for some z as

$$U = zq + \overline{zq} \tag{12}$$

Here, $\overline{z}$ denotes the conjugate of the complex number z, and $z =< p, U >$. For all sufficiently small $|a|$ about a_{NS}, we can convert system (1) as follows:

$$z \to \lambda(a)z + g(z, \overline{z}, a), \tag{13}$$

where $\lambda(a) = (1 + \omega(a))e^{i\theta(a)}$ for $\omega(a_{NS}) = 0$, and $g(z, \overline{z}, a)$ is a smooth function of z and $\overline{z}$. The Taylor expression of g with respect to $g(z, \overline{z})$ is

$$g(z, \overline{z}, a) = \sum_{k+l \geq 2} \frac{1}{k!l!} g_{kl}(a) z^k \overline{z}^l, \tag{14}$$

and the Taylor coefficients g_{kl} calculated through multilinear vector functions are expressed by the formulae

$$\begin{aligned} g_{20}(a_{NS}) &=< p, B(q, q) > \\ g_{11}(a_{NS}) &=< p, B(q, \overline{q}) > \\ g_{02}(a_{NS}) &=< p, B(\overline{q}, \overline{q}) > \\ g_{21}(a_{NS}) &=< p, C(q, q, \overline{q}) > . \end{aligned}$$

For system (5) which exhibits the Neimark-Sacker bifurcation, the coefficient $\varphi(a_{NS})$ determining the direction of the appearance of the invariant curve can be calculated as:

$$\varphi(a_{NS}) = Re\left(\frac{e^{-i\theta(a_{NS})} g_{21}}{2}\right) - Re\left(\frac{(1 - 2e^{i\theta(a_{NS})})e^{-2i\theta(a_{NS})}}{2(1 - e^{i\theta(a_{NS})})} g_{20} g_{11} \right) - \frac{1}{2}|g_{11}|^2 - \frac{1}{4}|g_{02}|^2 \tag{15}$$

where $e^{i\theta(a_{NS})} = \lambda(a_{NS})$. As a result, we get the following theorem regarding the Neimark-Sacker bifurcation:

Theorem 5 *If (10) holds, $\varphi(a_{NS}) \neq 0$ and the parameter a changes in the small vicinity of NSB_{E^*}, then system (1) experiences a Neimark-Sacker bifurcation at the only fixed point E^*. Moreover there is a unique attracting ($\varphi(a_{NS}) < 0$) or repelling (($\varphi(a_{NS}) > 0$)) invariant closed curve that bifurcates from E^*.*

Example 6 *Let us take into account the following system for the parameter values $b = 0.2$, $d = 3.5$, and $h = 0.002$,*

$$\begin{aligned} x_{n+1} &= 2.338x_n(1 - x_n) - 0.2x_n y_n - 0.002x_n, \\ y_{t+1} &= 3.5x_n y_n \end{aligned} \tag{16}$$

where $a_{NS} = 2.338$ is the Neimark-Sacker bifurcation point. The computation yields $(x^*, y^*) = (0.285714, 3.34)$, and the Jacobian matrix assessed at (x^*, y^*) is

$$J_{(x^*,y^*)} = \begin{pmatrix} 0.332 & -0.0571429 \\ 11.69 & 1 \end{pmatrix}.$$

The eigenvalues are $\lambda_{1,2} = 0.666 \pm 0.745952i$ such that $|\lambda_{1,2}| = 1$. Let $q, p \in \mathbb{C}^2$ be the complex eigenvectors corresponding to $\lambda_{1,2}$, respectively, $q \sim (-0.0285714 + 0.0638111i, 1)^T$ and $p \sim (5.845 - 13.0542i, 1)^T$. We get the vector $p \sim (-7.83563i, 0.5 - 0.223875i)^T$ by normalizing p according to q, such that $< p, q >= 1$. So, we obtain

$$\begin{aligned} g_{20}(a_{NS}) &= 2.338 + 1.31495i \\ g_{11}(a_{NS}) &= 2.238 + 1.09161i \\ g_{02}(a_{NS}) &= -2.538 + 0.868276i \\ g_{21}(a_{NS}) &= 0 \end{aligned}$$

where

$$\begin{aligned} F_1(u, v) &= -au^2 - buv + O(\|U\|^3) \\ F_2(u, v) &= duv + O(\|U\|^3) \end{aligned}$$

$$B(q, q) = \begin{pmatrix} 0.145029 - 0.323905i \\ -0.2 + 0.446678i \end{pmatrix}$$

$$C(q, q, q) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$C(q, q, \overline{q}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} B_1(u, v) &= -4.676u_1v_1 - 0.2(u_2v_1 + u_1v_2) \\ B_2(u, v) &= 3.5(u_2v_1 + u_1v_2) \end{aligned}$$

$$\begin{aligned} C_1(u, v, w) &= 0 \\ C_2(u, v, w) &= 0. \end{aligned}$$

From (15), we get $\varphi(a_{NS}) = -3.57837 < 0$. Consequently, the Neimark-Sacker bifurcation emerges at $a_{NS} = 2.338$. The Figure 3 gives the bifurcation and phase portraits of system (16) with the initial conditions $x_0 = 0.5$ and $y_0 = 2.5$. Figure 3.(a) shows Neimark-Sacker bifurcation diagram of the system (16). The phase portraits of system (16) are presented in Figure 3.(b)-(d).

- **Chaos control**

For many researchers, the focus point is the control of chaos in dynamic systems. It is possible to avoid chaos with some chaos strategies applied to systems (Danca *et al.* 2019; Din *et al.* 2017; Gümüş and Feckan 2021; Gümüş *et al.* 2022b; Liu *et al.* 2008; Yuan and Yang 2015). We apply a controlling strategy based on the hybrid control feedback methodology to control the chaos in system (1).

As system (1) undergoes a Neimark-Sacker bifurcation at the fixed point (x^*, y^*), the corresponding controlled system can be handled as follows:

$$\begin{aligned} x_{n+1} &= \beta[ax_n(1 - x_n) - bx_n y_n - hx_n] + (1 - \beta)x_n \\ y_{n+1} &= \beta dx_n y_n + (1 - \beta)y_n \end{aligned} \tag{17}$$

where β is the control parameter for $0 < \beta < 1$. The Jacobian matrix of the controlled system (17) is provided by

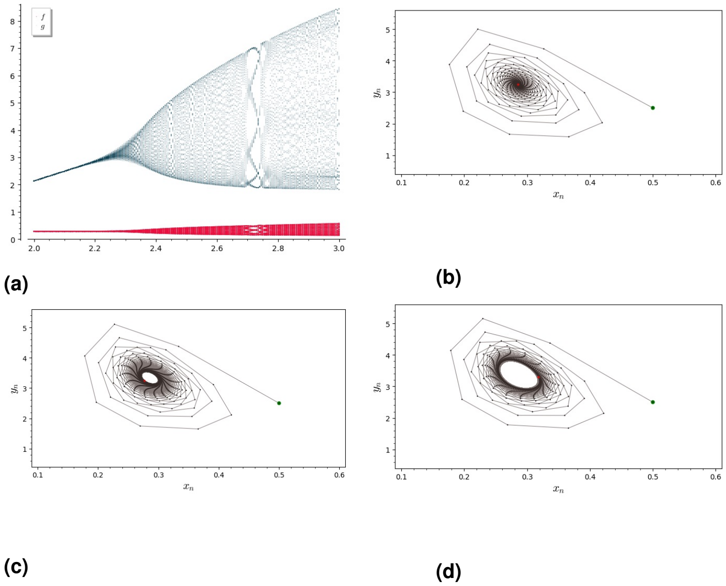


Figure 3
(a) *Bifurcation diagram of the prey-predator system (16) with the parameter values* $a \in (2,3)$, $b = 0.2$, $d = 3.5$, and $h = 0,002$. **(b)** *The phase portrait of system (16) when* $a = 2.31, b = 0.2, d = 3.5$, and $h = 0,002$. **(c)** *The phase portrait of system (16) when* $a = 2.338, b = 0.2$, $d = 3.5$, and $h = 0,002$. ***(d)*** *The phase portrait of system (16) when* $a = 2.35, b = 0.2, d = 3.5$, and $h = 0,002$.

$$\begin{bmatrix} 1-\beta+(-h+a(1-x^*)-ax^*-by^*)\beta & -bx^*\beta \\ dy^*\beta & 1-\beta+dx^*\beta \end{bmatrix}.$$

If

$$\left|\frac{a\beta}{d}-2\right| < 1+\frac{-a\beta(1+\beta)+d(1+(-1+a-h)\beta^2)}{d} < 2$$

is provided, then the positive fixed point (x^*, y^*) of the controlled system (17) is locally asymptotically stable.

Example 7 *We consider the parameters* $b = 0.2$, $d = 3.5$, $h = 0.002$, *and* $a = 2.35$ *for the initial conditions* $x_0 = 0.5$ *and* $y_0 = 2.5$. *For these parametric values, the controlled system is*

$$\begin{aligned} x_{n+1} &= \beta[2.35x_n(1-x_n)-0.2x_ny_n-hx_n]+(1-\beta)x_n \quad (18)\\ y_{n+1} &= 3.5\beta x_ny_n+(1-\beta)y_n \end{aligned}$$

and system (18) has a unique coexistence fixed point $(x^*, y^*) = (0.285714, 3.38286)$. *Additionally, the Jacobian matrix evaluated at* $(0.285714, 3.38286)$ *is*

$$\begin{bmatrix} 1-0.671429\beta & -0.0571429\beta \\ 11.84\beta & 1 \end{bmatrix} \quad (19)$$

and the characteristic equation (19) is obtained as

$$\lambda^2+(-2+0.671429\beta)\lambda+1-0.671429\beta+0.676571\beta^2 = 0. \quad (20)$$

From the Jury condition, we conclude that if $0 < \beta < 0.9923991$, *then the roots of (20) lie in a unit open disk. Therefore, the Neimark-Sacker bifurcation is fully controlled for values* β *in the obtained range.*

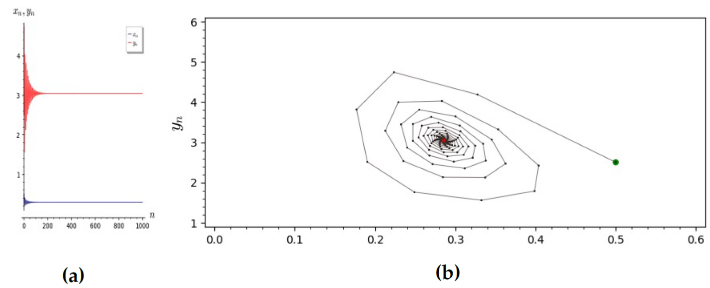


Figure 4 **(a)** *The trajectories of the controlled system (18) for* $b = 0.2$, $d = 3.5$, *h=0.002*, $a = 2.35$, *and* $\beta = 0.9$. **(b)** *The phase portrait of the controlled system (18) for* $b = 0.2$, $d = 3.5$, *h=0.002*, $a = 2.35$, *and* $\beta = 0.9$.

CONCLUSION

Harvesting in a natural population is one of the most important concerns in population ecology. In this study, the dynamics of system (1) are investigated depending on the harvest effect applied to the prey population. We determine that system (1) has a trivial (extinction) fixed point E_0, an exclusion fixed point E_1, and a coexistence fixed point E^*. The stability conditions of extinction and exclusion fixed points are investigated. The stability and bifurcation conditions of the coexistence fixed point of system (1) are also obtained. To examine the Neimark-Sacker bifurcation, the growth rate of the prey population a is taken as a bifurcation parameter. The stabilization of the unstable fixed point of system (1) is provided by the hybrid control method. The hybrid control strategy allows us to successfully control the chaotic behavior by suppressing the unstable fixed point. The dynamic properties of system (1) are presented by the trajectories, phase portraits, and bifurcation diagram belonging to system (1) by means of SageMath (see Kapçak (2018)). Furthermore, diagrams presenting the dynamic behaviour of system (1) with and without the harvesting effect are included in Figure 5 and Figure 6. A comparison is provided by giving the bifurcation value obtained without the harvesting effect. These dynamic behaviours are applied to understand the difference caused by the harvesting effect. In the examples given, the system behaviour is examined by choosing the initial point close to the fixed point.

Without the harvesting effect while system (1) undergoes a Neimark-Sacker bifurcation for $a = \frac{d}{d-2}$, with the harvesting effect, it undergoes a Neimark-Sacker bifurcation for $a = \frac{d(1+h)}{d-2}$. For $h = 0.025$, the bifurcation values are $a = 2.88864$ and $a = 2.81818$ with and without the harvesting effect, respectively. With this effect, the system will continue to remain stable for a certain period. If h is taken as 0.002, the bifurcation point is obtained as $a = 2.82382$. The smaller the effect value, the shorter the equilibrium time of the system. In other word, as the harvesting effect value increases, the bifurcation of the system will be delayed. We conclude that the harvesting effect on the prey population delays the Neimark-Sacker bifurcation (see (Danca *et al.* 2019)). Thus, the population will remain in equilibrium for a while.

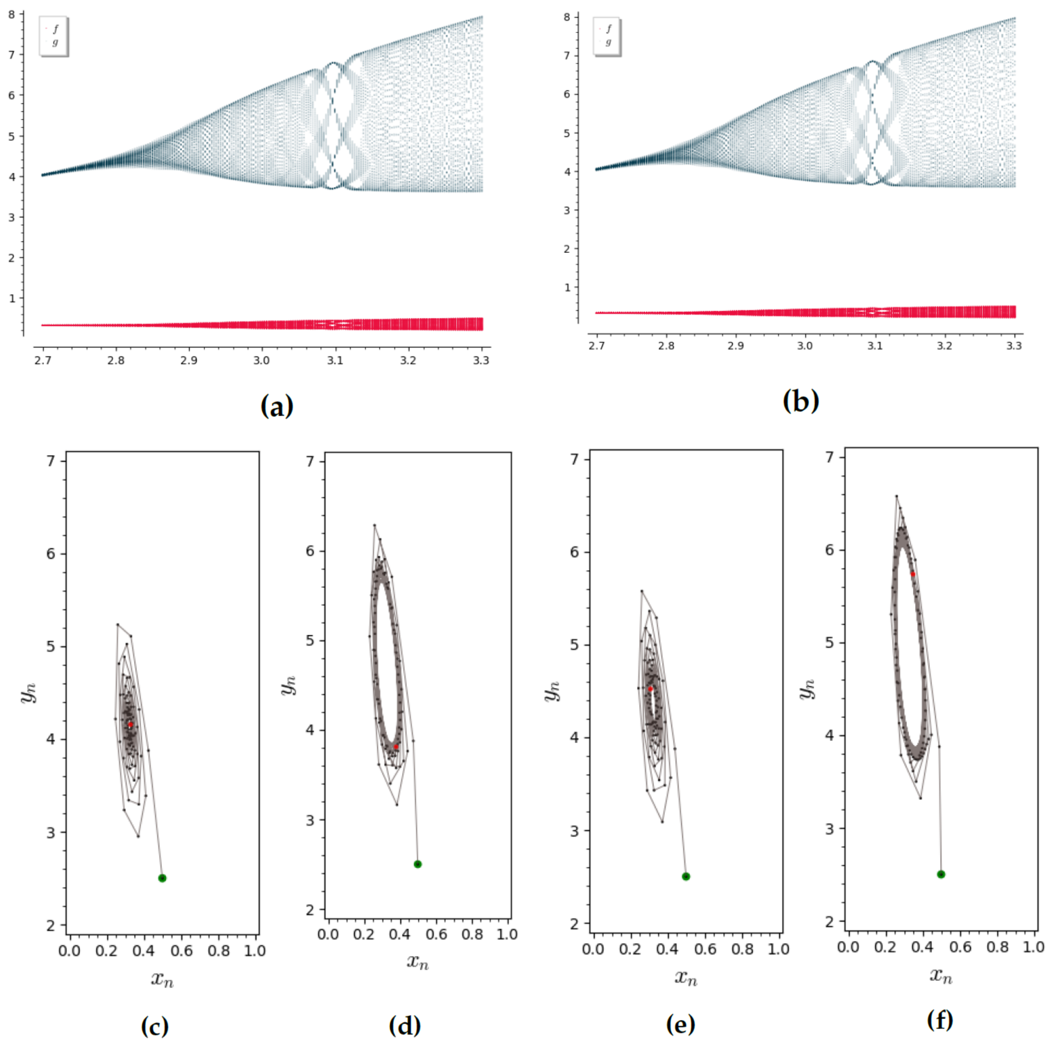


Figure 5

(a) *Bifurcation diagram of the prey-predator system (1) without the harvesting effect for the parameter values* $a \in (2.7, 3.3)$, $b = 0.2$, and $d = 3.1$. **(b)** *Bifurcation diagram of the prey-predator system (1) with the harvesting effect for* $a \in (2.7, 3.3)$, $b = 0.2$, $d = 3.1$ and $h = 0.020$. **(c)** *The phase portrait of system (1) without the harvesting effect for* $a = 2.7, b = 0.2$, and $d = 3.1$ **(d)** The phase portrait of system (1) without the harvesting effect for $a = 2.9, b = 0.2, d = 3.1$. **(e)** *The phase portrait of system (1) with the harvesting effect for* $a = 2.8, b = 0.2, d = 3.1$, and $h = 0.020$. ***(f)*** *The phase portrait of system (1) with the harvesting effect for* $a = 3, b = 0.2, d = 3.1$, and $h = 0.020$.

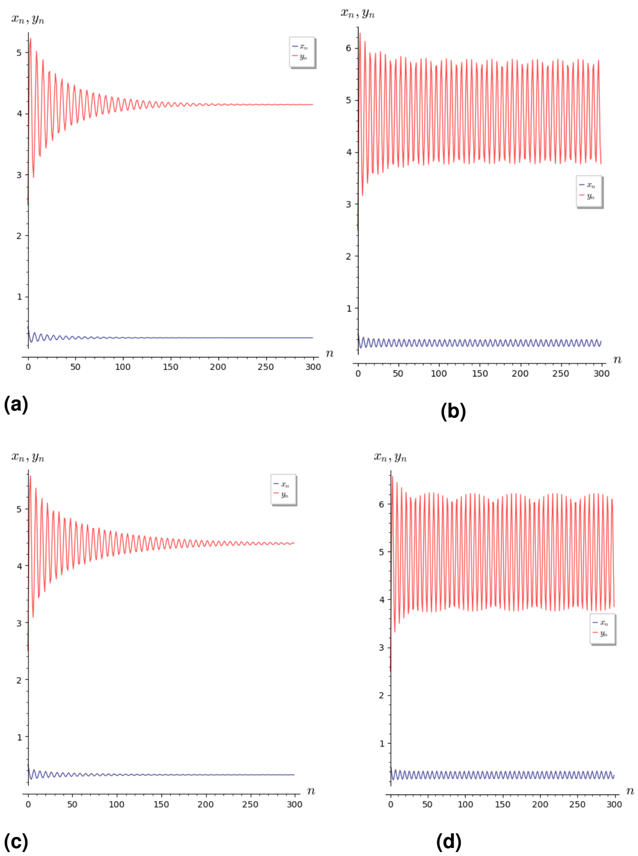


Figure 6
(a) *Time series diagram of system (1) without the harvesting effect for the parameter values* $a = 2.7$, $b = 0.2$, and $d = 3.1$. **(b)** Time series diagram of system (1) without the harvesting effect for the parameter values $a = 2.9$, $b = 0.2$, and $d = 3.1$. **(c)** Time series diagram of system (1) with the harvesting effect for the parameter values $a = 2.8$, $b = 0.2$, $d = 3.1$ and $h = 0.020$. **(d)** Time series diagram of system (1) with the harvesting effect for the parameter values $a = 3$, $b = 0.2$, $d = 3.1$and $h = 0.020$.

Conflicts of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Availability of data and material

Not applicable.

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