Ordu Üniversitesi Bilim ve Teknoloji Dergisi
Ordu Üniv. Bil. Tek. Derg.

Ordu University Journal of Science and Technology
Ordu Univ. J. Sci. Tech.

E-ISSN: 2146-6459 | 2022, 12(2), 159-166 | Araştırma Makalesi / Research Article

https://doi.org/10.54370/ordubtd.1183760

A Study over Transformations of the Parameters of Richards Growth Model into the Biologically Meaningful Parameters

Volkan Oda[1], Mehmet Korkmaz[2], Halil Şenol[3]

[1] Giresun University, Tirebolu Mehmet Bayrak Vocational High School, Finance Program, Giresun
[2] Ordu University, Faculty of Science and Arts, Department of Mathematics, Ordu
[3] Giresun University, Faculty of Engineering, Department of Energy Systems Engineering, Giresun

Geliş Tarihi / Received Date: 05.10.2022 Kabul Tarihi / Accepted Date: 17.11.2022

Abstract

In this study, it is explained how the Richards model, which is one of the sigmoid growth models, is transformed into a mechanical model with biologically significant parameters such as maximum growth value, lag time and maximum specific growth rate. Detail transformations of the Richards growth model containing 4 parameters with biologically significant parameters calculated with the help of the first and second derivatives were given. In order to test the accuracy of this model, the model prediction values were analyzed by using the height data of the E. Camaldulensis Dehn tree by years. The estimated values of the biologically significant parameters of Richards growth model containing 4 parameters were compared with the estimated values of the modified Gompertz, Logistic and Bertalanffy models containing the other 3 parameters. Error Sum of Squares, Coefficients of Determination and Akaike Information Criteria were used as model evaluation criteria for all proposed models, and it was concluded that the Richards growth model containing 4-parameters predicted relatively better than other models.

Keywords: richards growth model, sigmoidal growth model, mechanical model, akaike information criteria, coefficients of determination

Richards Büyüme Modelinin Parametrelerinin Biyolojik Olarak Anlamlı Parametrelere Dönüşümü Üzerine Bir Çalışma

Öz

Bu çalışmada, sigmoid büyüme modellerinden biri olan Richards modelinin maksimum büyüme değeri, gecikme süresi ve maksimum spesifik büyüme hızı gibi biyolojik olarak önemli parametrelerle mekanik bir modele nasıl dönüştürüldüğü anlatılmaktadır. Birinci ve ikinci türevler yardımıyla hesaplanan biyolojik olarak önemli 4 parametre içeren Richards büyüme modelinin detay dönüşümleri verildi. Bu modelin doğruluğunu test etmek için E. Camaldulensis Dehn ağacının yıllara göre yükseklik verileri kullanılarak model tahmin değerleri analiz edilmiştir. Biyolojik olarak önemli 4 parametreli Richards büyüme modelinin tahmini değerleri, diğer 3 parametreyi içeren modifiye Gompertz, Logistic ve Bertalanffy modellerinin tahmin değerleri ile karşılaştırıldı. Önerilen tüm modeller için model değerlendirme kriterleri olarak Hata Kareler Toplamı, Belirleme Katsayıları ve Akaike Bilgi Kriterleri kullanılmış ve 4 parametreli Richards büyüme modelinin diğer modellere göre nispeten daha iyi tahmin ettiği sonucuna varılmıştır.

Anahtar Kelimeler: richards büyüme modeli, sigmoidal büyüme modeli, mekanik model, akaike bilgi kriterleri, belirleme katsayıları

Sorumlu Yazar / Corresponding Author: Mehmet Korkmaz mkorkmaz52@yahoo.com

Introduction

Explaining growth scientifically is advantageous in many ways. The mathematical growth model should be determined according to the obtained data values. However, it is possible that the events related to growth can be interpreted and that decisions can be made (Narinc et al., 2010a).

The shape of growth curves depends on the species of the creature, its race, environmental conditions, sex, age and character of the measured character. The growth characteristics at different times are examined with the help of growth curves. Growth curve models; in the period up to the end of the growth event, have the biological meaning of parameters that explain the physiological mechanism of growth (Şenol, 2020). These parameters provide the understanding of the complex structure of the growth process and the factors affecting growth during this process. The most important benefit of growth curve models is that some information collected at different times and interpreted very difficultly can be interpreted biologically (Narinc et al., 2010b).

Detection and estimation of non-linear growth models is more difficult than linear growth models. The results are determined by means of iterations using different methods. Partial derivatives of model equations are calculated by using statistical package programs and numerical methods. These numerical approaches are used instead of analytical solutions and generally produce approximate results. In such cases, it is appropriate to directly use the partial derivatives from the numerical approaches to provide more efficient and more precise parameter estimates. The mathematical properties of growth models and the implications of growth parameters have been discussed by some researchers (Gregorczyk, 1998).

Growth models are used to estimate some unknown values of populations. There are two kinds of mathematical models. These are sigmoidal models and mechanical models. Sigmoidal models do not directly give an idea of the system and these models describe the general shape of a data set and also these models generally include parameters such as a, b, c. In these models, the parameters do not make sense directly. However, mechanical models include biologically meaningful parameters such as A, μ_m, λ where A is the maximum growth value, μ_m is maximum specific growth rate, λ is the lag time. Mechanical models also give information about the estimation of actual system characteristics such as growth rate, maximum size, displacement time, initial size (Zwietering et al., 1990).

If researchers do not have knowledge a lot about biological growth, the initial values for the parameters are difficult to estimate. For this reason, it is believed that they want to prefer mechanical models in their work. Thus, all growth models are rewritten with mathematical parameters such as A, μ_m and λ. This process is done by deriving the expression of the biological meaningful parameters as a function of the basic parameters and then placing them in the form. As a result, a biologically meaningful model has been obtained with these 3-parameters (Zwietering et al., 1990).

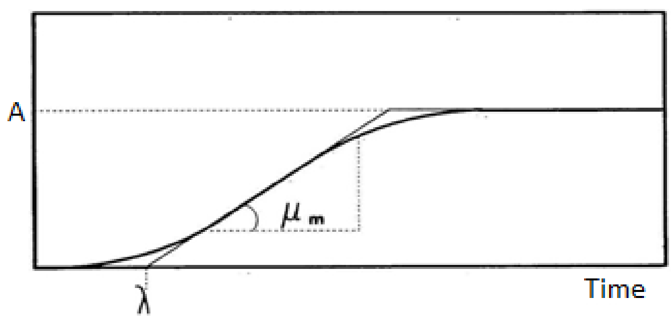

Figure 1. A Growth Curve (Zwietering et al., 1990)

In the literature, studies involving the comparison of the Richards model, which contains 4 parameters, with 3-parameter models (Gompertz, Logistic, etc.) are quite limited (Şenol et al., 2021). Therefore, the aim of this study was to compare the prediction performance of the 4-parameter Richards model with the prediction performance of the 3-parameter Modified Gompertz, Logistic and Bertalanffy models.

Material and Methods

In this study, Richards model (Richards, 1959) having 4-parameters was used to convert into mechanical models. In addition, Gompertz (Winsor,1932), Logistic (Ricker, 1979), Bertalanffy (Bertalanffy Von, 1957)] models were used to investigate the effect of the number of parameters. The calculations were done with the MAPLE package program. As a data set, the length values of E. Camaldulensis Dehn trees were used. Calculated values are shown in Table 1.

In this study, the data, the height growth values according to year, taken from the trees of E. Camaldulensis Dehn were used in Table 1. The dataset was taken from Yıldızbakan's (2005) study.

Table 1. The Height Growth Value of The Trees (*E. Camaldulensis* Dehn) according to Year

Planting Age (year)	0	1	2	3	4	5	6	7	8	9
Height Growth (m)	0.41	3.23	7.45	11.41	14.83	18.11	18.95	19.69	21.50	23.40

To determine a compatible model, the following statistical indicators were determined and compared: the coefficient of determination (R^2) Eq.1, error sum of squares (SSE) Eq.2 (Draper et al., 2014) the second-order Akaike information criterion (AIC) test Eq.3 (Akaike, 1974).

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{\sum_{i=1}^{n}(y_i-\bar{y}_i)^2} \quad (1)$$

$$SSE = \sum_{k=1}^{N}(y_i - \widehat{y_i})^2 \quad (2)$$

$$AIC = \begin{cases} N\ln\left(\frac{SSE}{N}\right) + 2K, & \text{when } \frac{N}{K} \geq 40 \\ N\ln\left(\frac{SSE}{N}\right) + 2K + \frac{2K(K+1)}{N-K-1}, & \text{when } \frac{N}{K} < 40 \end{cases} \quad (3)$$

where y_i is measured values, $\widehat{y_i}$ is estimated values, N is number of data point, K is number of model parameters and $\bar{y}_i$ is the mean value.

Results and Discussion

Most of the equations describing sigmoidal growth curves include mathematical parameters (a, b, c,…) rather than the biological meaningful parameters (A, μ_m, and λ). It is difficult to estimate initial values for the parameters if they have no biological meaning. Thus, the growth models have been rewritten to substitute the mathematical parameters with A, μ_m, and λ. This was done by deriving the equations of the parameters as the biological meaningful parameters and then substituting them in the model. In the literature, Logistic model, Gompertz model, Bertalanffy model were converted step by step into the mechanistic model by Korkmaz, (2017), Oda et al. (2016), and Zwietering et al. (1990) respectively.

In this study, the mechanical model transformation of Richards model having 4-parameters will be given step by step. For the modification of the Richards equation, firstly Richards model is given as Eq. 4:

$$y = a[1 + ve^{b(c-t)}]^{\frac{-1}{v}} \quad (4)$$

where y is growth value, t is time and a,b,c and v are the parameters of the model.

To obtain the inflection point of the model, the first and the second derivatives of the model with respect to t is calculated in Eq. 5 and. 6, respectively:

$$\frac{dy}{dt}=\frac{a[1+ve^{b(c-t)}]^{\frac{-1}{v}}\ b\ e^{b(c-t)}}{1+ve^{b(c-t)}} \quad (5)$$

$$\frac{d^2y}{dt^2}=\frac{a[1+ve^{b(c-t)}]^{\frac{-1}{v}}b^2\,(e^{b(c-t)})^2}{(1+ve^{b(c-t)})^2}(1+v)-\frac{a[1+ve^{b(c-t)}]^{\frac{-1}{v}}b^2e^{b(c-t)}}{1+ve^{b(c-t)}} \quad (6)$$

At the inflection point, where t = t_i, the second derivative is equal to zero in Eq. 7:

$$\frac{d^2y}{dt^2}=0 \rightarrow t_i=c \quad (7)$$

Now an expression for the maximum specific growth rate can be derived as follows by calculating the first derivative at the inflection point in Eq. 8:

$$\mu_m=\left(\frac{dy}{dt}\right)_{t_i}=ab(1+v)^{-(1+\frac{1}{v})} \quad (8)$$

The parameter b in the Richards equation can be substituted as follows by using Eq. 8.

$$b=\frac{\mu_m(1+v)^{(1+\frac{1}{v})}}{a}$$

Also, in Eq. 4 if t = c then $y_i=a(1+v)^{(-\frac{1}{v})}$.

The description of the tangent line through the inflection point is:

$$y=\mu_m.t+a(1+v)^{(-\frac{1}{v})}-\mu_m.t_i \quad (9)$$

The lag time is defined as the t-axis intercept of the tangent through the inflection point:

y=0 and t=λ are written in Eq. 9, so we obtain Eq. 10.

$$0=\mu_m.\lambda+a(1+v)^{(-\frac{1}{v})}-\mu_m.t_i \quad (10)$$

Using Eq. 7, 8, and 10, we get Eq. 11:

$$\lambda=c-\frac{1+v}{b} \quad (11)$$

The parameter c in the Richards equation can be substituted by Eq. 12:

$$c=\lambda+\frac{1+v}{b} \quad (12)$$

The asymptotic value is reached for t approaching infinity in Eq. 13:

$$t\longrightarrow\infty:\ y\longrightarrow a \quad \Rightarrow \quad A=a \quad (13)$$

The parameter a in the Richards equation can be substituted by A, yielding the modified Richards equation:

If all the resulting values are written in their places; we have the following Eq. 14:

$$y=a\left[1+ve^{b(c-t)}\right]^{\frac{-1}{v}}$$

$$y=a\left[1+ve^{b\left(\lambda+\frac{1+v}{b}-t\right)}\right]^{\frac{-1}{v}}$$

$$y=a\left[1+ve^{b(\lambda-t)}e^{(1+v)}\right]^{\frac{-1}{v}}$$

$$y=A[1+ve^{(1+v)}.e^{[\frac{\mu_m(1+v)^{(1+\frac{1}{v})}(\lambda-t)}{A}]}]^{\frac{-1}{v}} \tag{14}$$

The fourth parameter in models having 4-parameters is a shape parameter and it is hard to explain biologically. In some cases, a model having 4- parameters could be much better; thus, it is recommended that the process given in this study can be carried out with some sets of data to find the best model that describes the specific datasets.

The models with 4-parameters also include a shape parameter (v). The mechanical equations obtained from the equations of sigmoidal growth model and containing biologically meaningful parameters are presented in Table 2.

Table 2. Mechanical Equations of Models

Models	Equation	Modified Equation
Gompertz	$y = ae^{-e^{(b-ct)}}$	$y = Ae^{(-e^{[\frac{\mu_m e(\lambda-t)}{A}+1]})}$
Logistic	$y = \frac{a}{1+e^{(b-ct)}}$	$y = \frac{A}{[1+e^{\frac{4\mu_m(\lambda-t)}{A}+2}]}$
Bertalanffy	$y = a[1-be^{-ct}]^3$	$y = -\frac{A}{27}[-3+e^{\frac{2}{3}+\frac{9\mu_m(\lambda-t)}{4A}}]^3$
Richards	$y=a[1+ve^{b(c-t)}]^{\frac{-1}{v}}$	$y=A[1+ve^{(1+v)}.e^{[\frac{\mu_m(1+v)^{(1+\frac{1}{v})}(\lambda-t)}{A}]}]^{\frac{-1}{v}}$

Figure 2 shows the measured values of the E.Camaldulensis Dehn trees and estimated values from the models. Standard error values were expressed by bars.

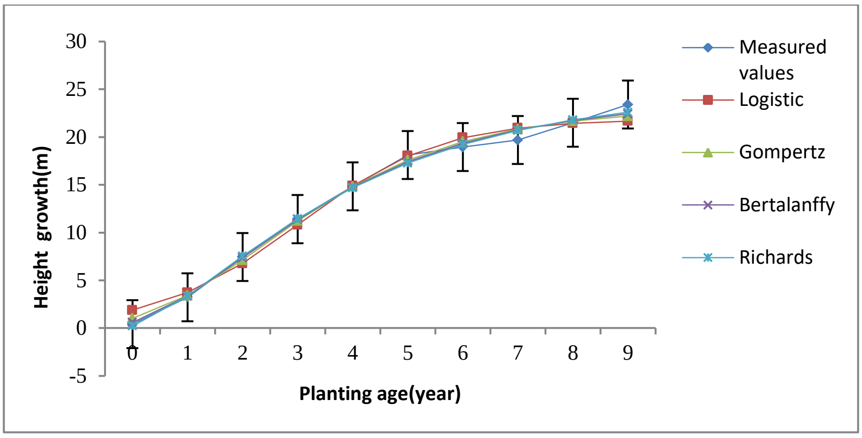

Figure 2. Growth Curve of *E. Camaldulensis* Dehn Trees

Table 1 shows the growth values of the *E. Camaldulensis* Dehn trees for the models used in this study. The model parameters calculated are given in Table 3. Table 3, Table 4 and Table 6 are calculated according to the values in Table 1.

Table 3. Values of Model Parameters

Models	Equations	a	b	c	v
Gompertz	$y=ae^{-e^{(b-ct)}}$	23.071	1.137	0.488	-
Logistic	$y=\frac{a}{1+e^{(b-ct)}}$	21.887	2.369	0.781	-
Bertanlanffy	$y=a[1-be^{-ct}]^3$	23.897	0.711	0.392	-
Richards	$y=a[1+ve^{b(c-t)}]^{\frac{-1}{v}}$	24.706	0.329	1.582	-0.550

According to Table 4, the SSE are the lowest and the R^2 is the highest in the Richards model. The Bertalanffy model also has the lowest AIC. This may be because the AIC value depends on the number of parameters. The other models used in this study have 3-parameters. Only the Richards model has 4-parameters.

Table 4. Error Sum of Squares (SSE), Coefficients of Determination (R^2) and Akaike Information Criteria (AIC) of the Models Calculated

Models	Error Sum of Squares (SSE)	Determination Coefficients of the models (R^2)	Akaike Information Criteria (AIC)
Gompertz	3.919	0.993	6,633
Logistic	8.713	0.985	14,622
Bertalanffy	2.876	0.995	3,538
Richards	2.631	0.996	11,648

The formulas of the lag time and maximum specific growth rates are given in Table 5. The values calculated according to these formulas are written in Table 6. According to Table 6, the Richards growth model has the lowest lag time and the highest maximum growth value.

Table 5. Lag Time and Maximum Specific Growth Rate of Models

Models	Lag time (λ)	Maximum specific growth rate (μ_m)
Gompertz	$\frac{b-1}{c}$	$\frac{ac}{e}$
Logistic	$\frac{b-2}{c}$	$\frac{ac}{4}$
Bertalanffy	$\frac{\ln(3b)-\frac{2}{3}}{c}$	$\frac{4ac}{9}$
Richards	$c\text{-}\frac{1+v}{b}$	$ab(1+v)^{-(1+\frac{1}{v})}$

Table 6. Biological Meaningful Parameter Values of Modified Models

Modified models	Maximum growth value (A)	Lag time (λ)	Maximum specific growth rate (μ_m)
Gompertz	23.071	0.281	4.142
Logistic	21.887	0.472	4.273
Bertalanffy	23.897	0.231	4.163
Richards	24.706	0.214	4.227

In the literature, it was emphasized that 4-parameter sigmoidal models outperformed 3-parameter sigmoidal models (Korkmaz, 2021). Similarly, in this study, the prediction performance of the modified Richards model, which includes 4 biological parameters, was found to be better than the 3-parameter modified Gompertz, Logistic and Bertalanffy model.

Conclusion

In this study, the transformations of four widely used growth models in scientific researches into mechanical models with biological meaningful parameters are given. The transformation steps of the Richards growth model having 4-parameters were mathematically shown step-by-step.

In addition, the effect of the number of parameters on the model selection was investigated. Using the length growth data of *E. Camaldulensis* Dehn tree, which model was more suitable was investigated. It is preferred that the R^2 of a model is high while the SSE and the AIC of that model is low. According to the SSE and R^2 , Richards model is the most appropriate model. However, considering the AIC, the appropriate model appears to be Bertalanffy growth model. Because, AIC are affected by the number of parameters. Therefore; it should not be preferred in comparison of models with different number of parameters. It is suitable for models with the same number of parameters.

In the model equations while the parameter of initial size, a is biologically meaningful, the random parameters such as b, c, v are not biologically meaningful. As a result, when the equations of the sigmoid growth model are converted into mechanical equations, there is no change in the predicted values of the models. Therefore, mechanical models can be successfully converted to modified models with biologically explainable parameters.

Acknowledgment

The authors would like to thank Yıldızbakan for allowing the use of the data.

Authors' Contributions

Volkan Oda, experimental study, data analysis and writing. *Mehmet Korkmaz*, experimental study and data analysis. *Halil Şenol*, literature research and writing.

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is in accordance with research and publication ethics.

ORCID

Volkan Oda https://orcid.org/0000-0001-5724-7678

Mehmet KORKMAZ https://orcid.org/0000-0002-7488-0552

Halil ŞENOL https://orcid.org/0000-0003-3056-5013

References

Akaike, H. (1974). A new look at the statistical model identification, *IEEE Transactions on Automatic Control, 19*(6), 716-723. http://dx.doi.org/10.1109/TAC.1974.1100705

Bertalanffy, Von L. (1957). Quantitative laws in metabolism and growth, *Quart. Rev. Biol. 32* (3), 217-231. https://doi.org/10.1086/401873

Draper, N. R., & Smith, H. (2014). *Applied regression analysis*, John Wiley & Sons.

Gregorczyk, A. (1998). Richards plant growth model. *J. Agron. Crop. Sci. 181*, 243-247. https://doi.org/10.1111/j.1439-037X.1998.tb00424.x

Korkmaz M. (2017). An applied study on converting some sigmoidal models in empirical form to meaningful parameterized mechanistic models, J. Sci. Eng. Res., *4*(7) 82-92. https://doi.org/10.21597/jist.1095933

Korkmaz, M. (2021). A study over with four-parameter Logistic and Gompertz growth models. *Numerical Methods for Partial Differential Equations, 37(*3), 2023-2030. https://doi.org/10.1002/num.22641

Narinc, D., Aksoy, T., & Karaman, E. (2010) Genetic parameters of growth curve parameters and weekly body weights in Japanese quails (Coturnix coturnix japonica). *J. Anim. Vet. Adv., 9*, 501–507. https://doi.org/10.3923/javaa.2010.501.507

Narinc, D., & Karaman, E., First, M. Z. Aksoy T (2010) Comparison of non-linear growth models to describe the growth in Japanese quail. *J. Anim. Vet. Adv., 9*, 1961–1966. https://doi.org/10.3923/javaa.2010.1961.1966

Oda, V., Korkmaz, M., & Özkurt, E. (2016). Some sigmoidal models used in estimating growth curve and biological parameters obtained : Bertalanffy pattern sample, *Ordu Univ. J. Sci. Tech., 6*(1), 54-66. https://dergipark.org.tr/tr/download/article-file/227497

Richards, J. F. (1959). A flexible growth function for empirical use. *J Exp Bot, 1*(10), 290–310. http://dx.doi.org/10.1093/jxb/10.2.290

Ricker, W. E. (1979). Growth rates and models. *Fish Physiology, 8*, 677-743. http://dx.doi.org/10.1016/S1546-5098(08)60034-5

Şenol, H. (2020). Anaerobic digestion of hazelnut (Corylus colurna) husks after alkaline pretreatment and determination of new important points in Logistic model curves. *Bioresource Technology, 300*, 122660. https://doi.org/10.1016/j.biortech.2019.122660

Şenol, H., Açıkel, Ü., & Oda, V. (2021). Anaerobic digestion of sugar beet pulp after acid thermal and alkali thermal pretreatments. *Biomass Conversion and Biorefinery, 11*(3), 895-905. https://doi.org/10.1007/s13399-019-00539-6

Winsor, C. P. (1932). The Gompertz curve as a growth curve, *Proc. Natl. Acad. Sci. U. S. A., 18*(1), 1-8. https://doi.org/10.1073/pnas.18.1.1

Yıldızbakan, A. (2005). *Analysis on mathematical models of tree growth and comparison of these models* [MSc Thesis]. University of Cukurova.

Zwietering, M. H., Jongenburger, I., Rombouts, F. M., & Riet, K. V. (1990). Modeling of the bacterial growth curve. *Appl. Environ. Microbiol., 56*(6), 1875-1881. https://doi.org/10.1128/aem.56.6.1875-1881.1990