Research *Öztürk&Bozdoğan/Kırklareli University Journal of Engineering and Science 8-2 (2022) 243-252*

DOI: 10.34186/klujes.1184109 *Geliş Tarihi:04.10.2022* *Kabul Tarihi:02.11.2022*

SLOPE DEFLECTION TRANSFER MATRIX METHOD FOR CONTINOUS BEAMS

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Abstract

The Slope Deflection Method is one of the displacement-based methods developed for structural analysis. In this study, an approach is proposed for the static analysis of continuous beams. The basis of this approach includes the Ricatti Transform, which was used in the literature to get the Transfer Matrix Method by using the Finite Element Method. In this study, Ricatti Transform was applied to the Slope Deflection Method for continuous beams. In the study, firstly, the application of Ricatti Transform to the Slope Deflection Method was shown theoretically, and then two numerical examples were solved to show the application and suitability of the presented approach. With the presented method, the number of system unknowns that are the basis for the solution is reduced, thus saving the analysis time.

Key Words: Slope deflection-transfer matrix method, continuous beam, static analysis.

Öz

Açı yöntemi, yapı analiz için geliştirilmiş deplasman esaslı yöntemlerden birisidir. Bu çalışmada sürekli kirişlerin statik analizi için bir yaklaşım önerilmiştir. Yaklaşımın esası literatürde Sonlu Elemanlar Yönteminden yararlanarak Taşıma Matrisi Yöntemine geçmek için kullanılan ricatti dönüşümünü içermektedir. Bu çalışmada Ricatti dönüşümü sürekli kirişler için Açı Yöntemine uygulanmıştır. Çalışmada önce Ricatti dönüşümünün Açı Yöntemine uygulaması teorik olarak gösterilmiş, sonrasında sunulan yaklaşımın uygulanmasını ve uygunluğunu göstermek üzere iki sayısal örnek çözülmüştür. Sunulan yaklaşım ile çözüme esas sistem bilinmeyenlerinin sayısı azalmakta böylece analiz süresinden tasarruf edilmektedir.

Anahtar Kelime: Açı–taşıma matrisi yöntemi, sürekli kiriş, statik analiz.

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1. INTRODUCTION

The Slope Deflection Method is one of the classical methods used in structural analysis. The method is widely used, especially in civil engineering education, in Structural Analysis courses to better understand the behavior of beams and frames.

Early 20^{th} century, the Slope Deflection Method was introduced by Maney and it was considered to be precursor of the Matrix Stiffness Method [1]. In the method, only the bending deformations are taken into account.

A number of studies have been carried out using this method in the literature. Chi and Lin [2] applied the Slope Deflection Method to the elasto plastic analysis of multistory frames. Ermopoulos [3] used the method for the analysis of variable cross-section beams subjected to stepped loads. Deretić –Stojanovic [4] explained the use of the Slope Deflection Method in composite beams. Backer et al. [5] used the Slope Deflection Method for the analysis of orthotropic plated bridge decks. Deretić –Stojanovic and Kostić [6] adapted the method to the time dependent analysis of composite and prestressed beams. Mirfallah and Bozorgnasab [7] proposed a new Jacobi-based Iterative Method for the solution with the Slope Deflection Method. Husain [8] suggested changes in the equations for more practical use of the Slope Deflection Method. Gholhaki and Sabet [9] applied the Slope Deflection Method for the analysis of steel plate shear wall with Thin Plates. Kotrasova et al. [10] presented their experience by addressing the computer-assisted teaching of the Slope Deflection Method. Husain and Hasan [11] introduced the Slope Deflection Method and applied it to various problems.

On the other hand, the transfer matrix method is an effective method used in the solution of various mechanical problems, and it stands out especially in terms of saving matrix dimensions and thus saving time [12], [13], [14], [15], [16].

In this article, the Ricatti transform, which is known from the literature, was applied to the Slope Deflection Method to obtain Ricatti Transfer Matrix for the static calculation of continuous beams. In the study assumptions have been made:

a) The material obey the hooke's law,

b) The displacements are small enough so that geometric nonlinear effects can be neglected.

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2. Material and Method

2.1. SLOPE DEFLECTION -TRANSFER MATRIX METHOD

The Slope Deflection Method, which is one of the classical methods used in structural analysis, has been taken part in the literature [17] [18] [19] [20].

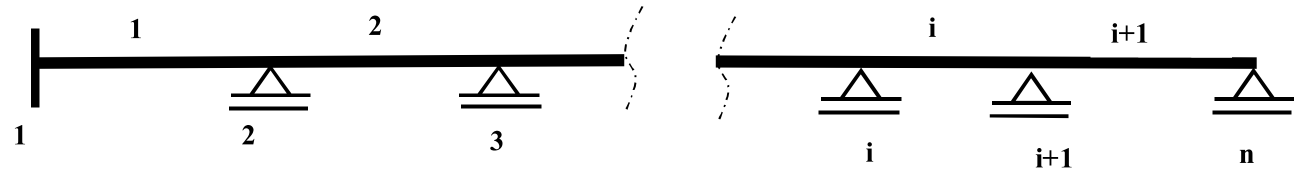

Figure 1. Continuous Beam

The basic equation of the Slope Deflection Method for the i^{th} element of a continuous beam shown in Figure 1, is written as follows

$$\begin{bmatrix} \frac{4EI_i}{L_i} & \frac{2EI_i}{L_i} \\ \frac{2EI_{ii}}{L_i} & \frac{4EI_i}{L_i} \end{bmatrix} \begin{Bmatrix} \theta_{i,i} \\ \theta_{i,i+1} \end{Bmatrix} + \begin{Bmatrix} FM_{i,i} \\ FM_{i,i+1} \end{Bmatrix} = \begin{Bmatrix} -Q_{i,i} \\ Q_{i,i+1} \end{Bmatrix} \tag{1}$$

In this equation, E is the modulus of elasticity, I is the moment of inertia, L is the length, ϴ is the angle of rotation. The above equation can be written as:

$$\begin{bmatrix} 2k_i & k_i \\ k_i & 2k_i \end{bmatrix} \begin{Bmatrix} \theta_{i,i-1} \\ \theta_{i,i+1} \end{Bmatrix} + \begin{Bmatrix} FM_{i,i-1} \\ FM_{i,i+1} \end{Bmatrix} = \begin{Bmatrix} -Q_{i,i-1} \\ Q_{i,i+1} \end{Bmatrix} \tag{2}$$

where kj is defined by Equation (3).

$$k_i = \frac{2EI_i}{L_i} \tag{3}$$

The matrix Equation (2) can be written clearly as in the Equation (4) and Equation (5)

$$2k_i\theta_{i,i} + k_i\theta_{i,i+1} + FM_{i,i} = -Q_{i,i} \tag{4}$$

$$k_i\theta_{i,i} + 2k_i\theta_{i,i+1} + FM_{i,i+1} = Q_{i,i+1} \tag{5}$$

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The Riccati transform, known from the literature [21], [22[, [23], [24], [25], can be applied for Q moments as in the Equation (6) and Equation (7) below.

$$Q_{i,i} = T_{i-1}\theta_{i,i} + R_{i-1} \tag{6}$$

$$Q_{i,i+1} = T_i\theta_{i,i+1} + R_i \tag{7}$$

If the transformations in Equation (6) and Equation (7) are applied to Equation (4) and Equation (5), Equation (8) and Equation (9) are obtained.

$$2k_i\theta_{i,i} + k_i\theta_{i,i+1} + FM_{i,i} = -T_{i-1}\theta_{i,i} - R_{i-1} \tag{8}$$
$$k_i\theta_{i,i} + 2k_i\theta_{i,i+1} + FM_{i,i+1} = T_i\theta_{i,i+1} + R_i \tag{9}$$

If $\theta_{i,i+1}$ taken from the Equation (8), Equation (10) can be written as below

$$\frac{k_i}{[-T_{i-1} + 2k_i]}\theta_{i,i+1} + \frac{FM_{i,i} + R_{i-1}}{[-T_{i-1} + 2k_i]} = \theta_{i,i} \tag{10}$$

If the Equation (10) is written in Equation (9), Equation (11) is obtained.

$$\frac{3k_i^2 + T_{i-1}k_i}{[-T_{i-1} + 2k_i]}\theta_{i,i+1} + \frac{k_i[FM_{i,i} + R_{i-1}]}{[-T_{i-1} + 2k_i]} + FM_{i,i+1} = T_i\theta_{i,i+1} + R_i \tag{11}$$

Equation (12) and Equation (13) are obtained from Equation (11)

$$T_i = \frac{3k_i^2 + T_{i-1}k_i}{[-T_{i-1} + 2k_i]} \tag{12}$$

$$R_i = \frac{k_i[FM_{i,i} + R_{i-1}]}{-[-T_{i-1} + 2k_i]} + FM_{i,i+1} \tag{13}$$

Equation (12) and Equation (13) are the main expressions of the Slope Deflection Transfer Matrix Method.
Before the method can be applied sequentially, T_1 and R_1 must be known.
T_1 and R_1 are computed using the initial support conditions in the continuous beam. In case of fixed support, these expressions are computed as follows

$$T_1 = 2k_1 \tag{14}$$

$$R_1 = FM_{1,2} \tag{15}$$

In case of pinned support, these expressions are computed as follows

$$T_1 = \frac{3k_1}{2} \tag{16}$$

$$R_1 = FM_{1,2} - \frac{FM_{1,1}}{2} \tag{17}$$

As a result of sequential operations, the following relation is written for the last element.

$$Q_{n,n+1} = T_n \theta_{n,n+1} + R_n \tag{18}$$

If the last support of the continuous beam is fixed, $\theta_{n,n+1}$ is equal to zero and the moment is calculated by Equation (19).

$$Q_{n,n+1} = R_n \tag{19}$$

If the last support of the continuous beam is hinged, the moment at the end is equal to zero and for $\theta_{n,n+1}$ Equation (20) is written.

$$\theta_{n,n+1} = -\frac{R_n}{T_n}$$
(20)

After the first and last unknowns (rotation and bending moment) are found, the other unknowns are obtained sequentially with Equation (21) and Equation (22).

$$\theta(i) = -\frac{2k(i)\theta(i+1) + FM_R(i) + M(i+1)}{k(i)} \tag{21}$$

$$M(i) = 2k(i)\theta(i) + k(i)\theta(i+1) + FM_L(i) \tag{22}$$

3. Numerical Example

In this section, two examples were solved to show the suitability of the Slope Deflection Transfer Matrix Method presented in the study, and the results were compared with the Finite Element Method.

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Example 1

Continuous beam with two span shown in Figure 2, was analyzed using Slope Deflection Transfer Matrix Method and the bending moment and rotations at the nodes were calculated. They were compared in Table 1 and Table 2.

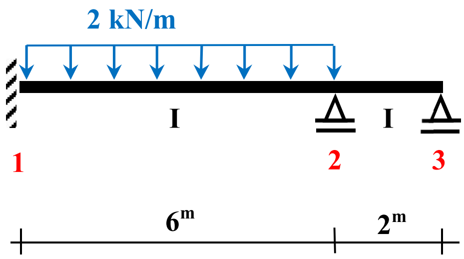

Figure 2. Example Beam

Table 1 shows the flexural moments for the joints.

Table 1. Comparison of Flexural Moment of Example 1(kNm)

	1	2	3
SDTM	-6.9231	-4.1538	0
FE	-6.9231	-4.1538	0

Table 2 shows the rotations for the joints.

Table 2. Comparison of Rotation of Example 1 (radian)

	1	2	3
SDTM	0	-2.76923/EI	1.38462/EI
FE	0	-2.76927/EI	1.38456/EI

As can be seen in Table 1 and Table 2, it has been observed that the Slope Deflection Transfer Matrix Method gives results compatible with the Finite Element Method.

Example 2

For this example, the continuous beam with five span shown in Figure 3, was analyzed using Slope Deflection Transfer Matrix Method . The bending moment and rotations of the joints were calculated and compared in Table 1 and Table 2.

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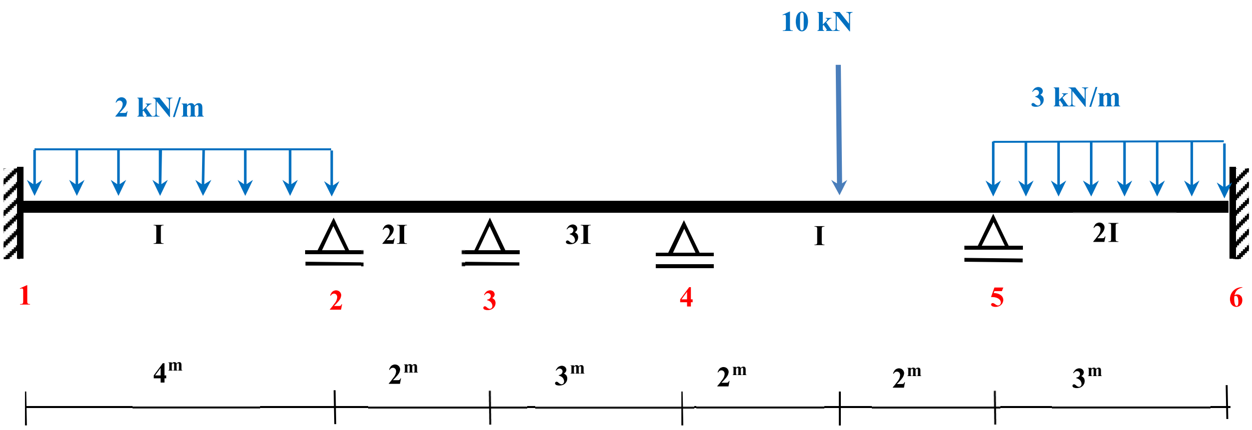

Figure 3. Example Beam

Table 3 shows the flexural moments for the joints.

Table 3. Comparison of Flexural Moment of Example 2(kNm)

	1	2	3	4	5	6
STM	-2.9022	-2.2056	1.6248	-4.2938	-4.672	-1.039
SAP2000	-2.8985	-2.203	1.6241	-4.2936	-4.672	-1.039

Table 4 shows the rotations for the joints.

Table 4. Comparison of Rotation of Example 2 (radian)

	1	2	3	4	5	6
STM	0	-0.46439/EI	-0.17401/EI	1.16045/EI	-0.90841/EI	0
SAP2000	0	-0.46365/EI	-0.17422/EI	1.16052/EI	-0.90826/EI	0

The rotations and bending moment values were compared in Table 3 and Table 4 and found to be compatible.

While in the classical Slope Deflection Method, a set of equations with four unknowns is required for the calculations in the example, the result can be obtained by solving an equation with a single unknown for the Slope Deflection-Transfer Matrix Method presented in this study. This shows the advantage of the presented approach.

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4. CONCLUSION

In this study, an approach was proposed for the static analysis of continuous beams. For this purpose, the Ricatti transform known from the literature was used. In the study, the Ricatti Transfer Matrix was obtained by applying the Ricatti transform in the slope deflection equations written for the continuous beam. At the end of the study, as solved examples showed the suitability of the approach, it was observed that the presented approach gives correct results.

In the presented approach, due to the feature of the Ricatti Transfer Matrix, the set of equations, which are the basis for the solution, is reduced to a single unknown. After the unknowns at the last point are found, the unknowns at other points are easily obtained as a result of sequential operations. In the study, shear deformations have been neglected, but these effects can also be taken into account in the method.

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DOI: 10.34186/klujes.1184109 *Geliş Tarihi:04.10.2022* *Kabul Tarihi:02.11.2022*

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