Research Article

A stochastic model for facility locations using the priority of fuzzy AHP

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ABSTRACT

In this study, a production-distribution network system for a company, which is active in producing bottled natural spring water, was established. For this, one of the seven provinces in the Aegean Region was selected as a pilot region. According to the new supply chain network to be established in the region facility location selection model (FLS) decisions were optimized. The production of 0,5 lt bottled water is considered because it is the primary market target of the company. In order to meet the customer demand with a minimum production and distribution cost a network was established with optimum factory capacities and warehouse locations. Accordingly, this study consists of two stages; in the first stage a Fuzzy Analytic Hierarchy Process (FAHP) method was used to choose the most appropriate province. In the second stage the network was established and optimized under stochastic parameters by developing optimization models. The first method, FAHP, is fuzzy, and the second method, stochastic mixed-integer programming (MIP), is also modeled under uncertainty. Thus, the results are realistic and in a range that the company also accepts.

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INTRODUCTION

Companies usually use intuitive and mathematical models together, which results in more useful and realistic results, when new or important decisions need to be taken. The selection of the plant locations, which is the subject of our work, is one of the key and strategic decisions for the firm. For similar strategic decisions, companies use all their means [1, 2]. This includes experienced expert opinion as well as machine output. Thus, the safest measures are taken for fluctuations. In this study, we used both the FAHP method, which includes expert opinion, and the MIP model, which is a mathematical model, to make realistic decisions. Thus, we have put forward a study that adds value to both the literature and the real market.

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In this study, the FLS for a company which is active in the drinkable water branch of the food sector was made. The company wants to form a production and distribution network in a province in the Aegean Region. For this, firstly, one of the current seven provinces were selected. After that, the FLS process for the production and distribution network to be established in this province was carried out. The FAHP method for the first selection process and an MIP model built for the second selection process were used. Thus, it was decided that which facility and which warehouse should be open and how much capacity and how much shipment should be made. In addition to this, the model constructed for the FLS is stochastic. Stochastic modeling was used to obtain more realistic and useful results [3].

In province selection some of the most important criteria are [4, 5, 6] used, such as transportation, labor costs, proximity to suppliers, retail and wholesale customer size in the region, competitor density in the region, environmental, climate factors, etc. These substances vary according to the activities of businesses. However, the criteria, which are not under human control, force companies to make choices under uncertain conditions. Computer programs that can examine the problem under multiple probabilities are useful when making decisions under uncertainty. However, expert opinions with market experience should also be taken into account [7].

METHODOLOGY OF THE STUDY

As stated earlier in the study, first, the province selection was made by FAHP, and then FLS application was made by stochastic MIP method whithin the selected province. Accordingly, Figure 1 shows the methodology visually and comprehensively.

LITERATURE REVIEW

This study includes two methods such as FAHP and MIP. The literature was searched separately for the facility selection studies conducted by these two methods. Thus, a more understandable and analytical perspective was presented for the literature review. Table 1 and Table 2 contain information about the author, application, year, method used and optimization tool used. Accordingly, Table 1 shows the studies using the FAHP method in the FLS. The details of the criteria based search results are provided in Table 4, in the FAHP application section. The criteria used in this study were selected considering Table 4.

The literature review about the methods used in this study is presented above. On the other hand, our study has certain contributions to the FLS literature. FLS studies are generally carried out to establish a network in the selected region and to optimize the established network. However, it is not common to use an additional method when choosing the region where the network will be installed. In our study, the FAHP method was used to select the region where site selection would be made. Thus, a realistic [37], not random, step was taken by considering the regional characteristics. In addition, the fuzzy nature of the method used in region selection increased the realism of the process. This can be considered as a second contribution. Another contribution is the use of stochastic programming for the optimization of the circulation in the network established in the specified
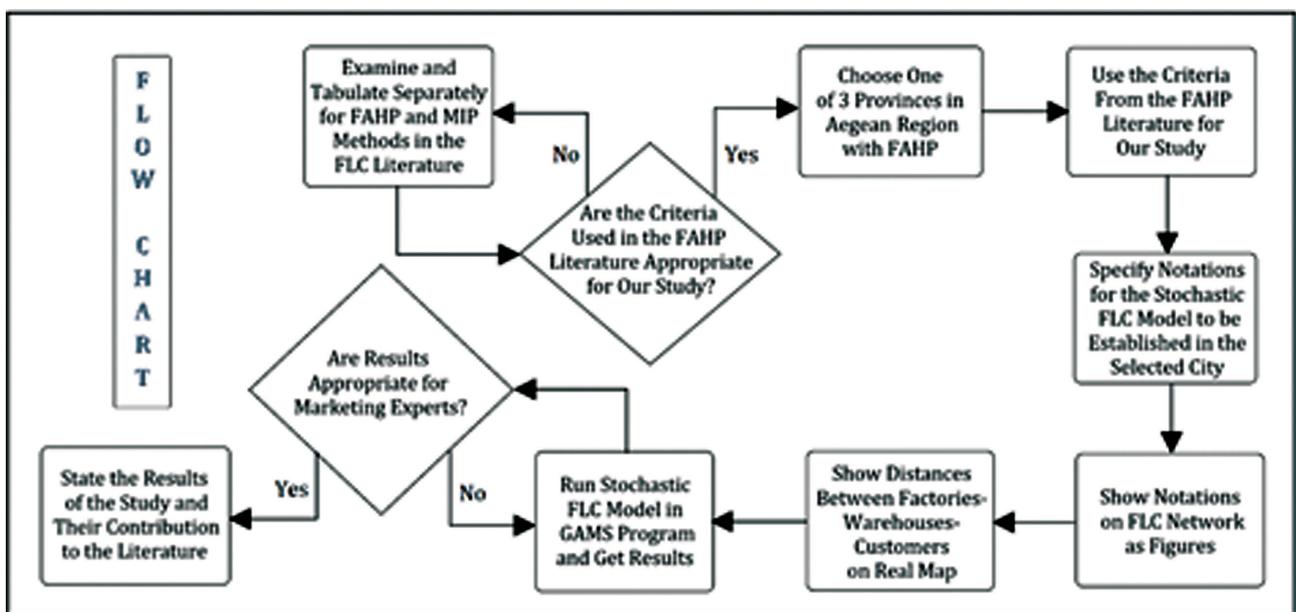
Figure 1. Methodology of the Study.

Table 1. Studies Using FAHP and MCDM* Methods in FLS Literature

Writer(s)	İmplementation	Year	**Method(s)	***Opt. Tool
Ertugrul and Karakasoglu [8]	A Textile Company FLS	2007	-FAHP -FTOPSIS	Unspecified
Vahidnia et al. [9]	Hospital FLS	2009	-FAHP & GIS -Center of Area -(α-cut)	ESRI ArcGIS
Ka [10]	Dry Port FLS	2011	-FAHP -ELECTRE	EXPERT CHOICE
Ömürbek et al. [11]-	Livestock Industry FLS	2013	AHP	EXPERT CHOICE
Belbag et al. [12]	Airport FLS	2013	FTOPSIS & FELECTRE	Unspecified
Yavuz and Deveci [5]	Mall FLS	2014	FTOPSIS-FVIKOR	Unspecified
Kabir and Sumi [13]	Power Substation FLS	2014	FAHP & PROMETHEE	Visual PROMETHEE
Beskese at al. [7]	Landfill Site FLS	2014	FAHP & GIS & TOPSIS	Unspecified
Noorollahi et al. [14]	Solar Farms Exploitation FLS	2016	FAHP & GIS & SAW	SUPER DECISION
Demirel et al [15]	Textile Factory FLS	2016	FTOPSIS-FVIKOR	Unspecified
Darani et al. [16]	Parking Lot Site FLS	2018	FAHP & TOPSIS	Unspecified
Gücer [17]	Glass Factory FLS	2018	AHP	Unspecified
Ammar et al. [18]	PV Water Pumping System FLS	2019	FAHP & GIS	ESRI ArcGIS
Wang et al. [19]	Biomass Energy Power Plants FLS	2019	FAHP & GIS & TOPSIS	Unspecified
Guler and Yomralioglu [6]	Electric Vehicle Fast Charging Station FLS	2020	FAHP & GIS & TOPSIS	ESRI ArcGIS
Anderluh et al. [20]	City Hub FLS	2020	AHP	Unspecified
Deveci et al. [21]	Offshore Wind Farm FLS	2020	FTOPSIS	Unspecified
Komchornrit [22]	Logistic Center FLS	2021	AHP & TOPSIS	Unspecified

*MCDM: Multi-Criteria Decision Making
** "&" is used if methods are integrated. If it is not integrated, itemization is made with the "-" symbol.
**Opt. Tool: Optimization Tool Used

region. Thus, it was possible to make an examination under uncertainty, closer to the truth. Thus, different scenarios can be examined and work can be carried out under uncertainty. Therefore, realistic results were obtained. In this context, this study, the first stage of which is fuzzy and the second stage is multi-scenario, has made a great contribution not only to the literature, but also to the company in question, because both expert judgments and mathematical models were used together for the firm. As a result of this hybrid approach, a certain cost has emerged. This cost is the cost of product circulation in the established network, which was found appropriate by the marketing experts of the current region. This validates the power of the hybrid approach applied.

Table 2 shows the studies using the MIP method in the facility location literature.

STRUCTURE OF FAHP

In the literature, the reason to use FAHP structure is that AHP structure is so useful in the uncertain situations. On the other hand, the uncertainty condition is more dominant in the real market. Therefore, first, the scale of absolute numbers varies unlike the classic AHP. The scale of the fuzzy numbers is illustrated in Table 3.

The steps of FAHP method which was proposed by Buckley [36] and which we used in this study is as follows [37, 38]:

Step 1: "$\tilde{C}^k$" pair-wise comparison matrices are set up and "$\tilde{d}^k_{mn}$" fuzzy values are determined according to experts' judgements

$$\tilde{C}^k = \begin{bmatrix} \tilde{d}^k_{11} & \tilde{d}^k_{12} & \cdots & \tilde{d}^k_{1j} \\ \vdots & \cdots & \ddots & \vdots \\ \tilde{d}^k_{n1} & \tilde{d}^k_{n2} & \cdots & \tilde{d}^k_{nn} \end{bmatrix} \tag{1}$$

If there are more than one expert who will give an idea:

Table 2. Studies Using MIP Method in FLS Literature

Writer(s)	İmplementation	Year	Method(s)	Goal Function	Opt. Tool
Yuan and Saha [23]	Production Inventory Network	2008	DT.* MIP	Min. Cost for Network	LINGO
Mete and Zabinsky [24]	Medical Stuff Storage And Distribution Network for Disaster Management	2010	ST.** MIP	Optimal Storage Locations and İnventory Levels	GAMS
Ahumada and Villalobos. [25]	Production and Distribution Network of Fresh Agricultural Produce	2011	DT. MIP	Max. Revenue for Producer	CPLEX
Zhu and Yu [26]	Medical Supply Location And Distribution Network in Biochemical Attacks	2013	ST. MIP	Optimal Storage Locations and İnventory Levels	Unspecified
Ji et al. [27]	Manufacturing Enterprises Distribution Network	2014	0-1 MIP	Min. Cost for Network	LINGO
Zahrani [28]	Chlorinated Water Distribution Network	2016	Sımulation & DT. MIP	Optimum Chlorine Level for Health	EPANET, CPLEX
Aydin [29]	Hospital	2016	ST. MIP	Min. Distance for Network	MATLAB, CPLEX
Boonmee et al. [3]	Shelter Site Selection for Flood Disaster	2017	ST. MIP	Min. Population Weighted Travel Distance	Gurobi Optimization
Celik et al. [30]	Emergency Supplies Distribution Network	2017	ST. MIP	Min. Cost for Network	CPLEX
Buritica et al. [1]	Non-Alcoholic Beverages Supply Network	2018	Clustering K-Means & DT. MIP	Min. Cost for Network	Orange Canvas, GAMS
Acar [2]	Ammunition Dump Network	2019	DT. MIP	Min. Cost For Network	GAMS
Golpîra [31]	Supply Chain Network	2020	DT. MIP	Min. Cost For Network	GAMS
Guo et al. [32]	Biomass Straw Energy Utilization Engineering Logistic Network	2021	DT. MIP	Min. Cost For Network	LINGO
Soyoz and Ozyoruk [33]	After Disaster Evacuation Network	2021	ST. MIP	Min. Cost For Network	GAMS

*DT: Deterministic, ** ST: Stochastic

Table 3. Dual Scale of Fuzzy Absolute Numbers [34, 35]

Real Number	Triangular Fuzzy Number	Inverse of Triangular Fuzzy Numbers
1	(1,1,1)	(1,1,1)
2	(1,2,3)	(1/3, 1/2, 1)
3	(2,3,4)	(1/4, 1/3, 1/2)
4	(3,4,5)	(1/5, 1/4, 1/3)
5	(4,5,6)	(1/6, 1/5, 1/4)
6	(5,6,7)	(1/7, 1/6, 1/5)
7	(6,7,8)	(1/8, 1/7, 1/6)
8	(7,8,9)	(1/9, 1/8, 1/7)
9	(8,9,9)	(1/9, 1/9, 1/8)

$$\tilde{d}_{ij} = \frac{\sum_{m}^{M} 1\tilde{d}_{ij}^{k}}{K} \tag{2}$$

by using the equation,

$$\tilde{C} = \begin{bmatrix} \tilde{d}_{11} & \cdots & \tilde{d}_{ij} \\ \vdots & \ddots & \vdots \\ \tilde{d}_{n1} & \cdots & \tilde{d}_{nn} \end{bmatrix} \tag{3}$$

the matrix is obtained. In this way, judgements of experts are made singular with arithmetic mean or as in this study, the experts come together to make a common decision and the results is obtained as singular.

Step 2: By taking arithmetic mean of each matrix $\tilde{r}_i$ triangular fuzzy numbers are obtained:

$$\tilde{r}_i = \left(\prod_{j=1}^{n} \tilde{d}_{ij} \right)^{1/n}, \, i = 1, 2, \ldots n \tag{4}$$

Step 3: Fuzzy weights $(\tilde{W}_i)$ and fuzzy numbers composed of low, middle and upper values (l, m, u) are obtained:

$$\tilde{w}_i = \tilde{r}_i \otimes \left(\tilde{r}_i \oplus \tilde{r}_2 \oplus \ldots \oplus \tilde{r}_n\right)^{-1} = l_i, m_i, u_i \qquad (5)$$

Step 4: Center of Average (COA), which is a centroid method, is used for defuzzification of these numbers:

$$D_i = \frac{l_i + m_i + u_i}{3} \qquad (6)$$

Step 5: "D_i" s are normalized and the criteria are arrayed based on these values whose summation is "1".

$$N_i = \frac{D_i}{\sum_{n=1}^{N} D_i} \qquad (7)$$

APPLICATION

APPLICATION OF FAHP

The first step of the integrated application was the choosing of the pilot area in the Aegean region with FAHP method. As choosing the pilot area, the targeted province numbers was reduced from seven to three based on experts` judgements. At this stage, FAHP was used to determine one of the three provinces. FAHP criteria were selected from studies in the literature as previously stated. The most used main and sub-criteria in the literature related to FAHP are shown in Table 4.

The most frequently used criteria in the literature were applied to the FAHP structure that was established for this study. For instance, the SS criterion was directly taken as the main criterion, but the LR criterion was determined as a sub-criterion of SS in the form of Easiness of Employment (EE). Similarly, the TEC criterion was taken as the main criterion of ENSW. According to this, hierarchical structure of FAHP determined with three main criteria and eight sub-criteria. Further, the provinces are shown in Figure 2.

The definition of the criteria is expressed as below:

Location (LOC): It describes the main criterion concerning the geographical location of the facility to be established.

a) *Closeness to Suppliers (CS):* It defines the criterion that determines the closeness to suppliers for PET bottle preform which is the primary raw material in our study.

b) *Closeness to the Market (CM):* Evaluated as drinking water industry market, supermarkets and stores.

Table 4. The Most Commonly Used Main and Sub-Criteria in the FLS Literature Study with the FAHP and MCDM Methods

Writers	Criteria									
	LR	**PM**	**EO**	**SS**	**PSR**	**TR**	**LC**	**PD**	**PE**	**TEC**
Ertugrul and K.	√	√	√	√	√					
Vahidia et al.		√				√	√	√		
Ka			√	√		√	√		√	
Ömürbek et al.	√	√			√	√	√			√
Belbag et al.			√	√		√	√			√
Yavuz and Deveci	√	√	√	√	√					√
Kabir and Sumi		√	√	√	√	√				√
Beskese at al.		√			√	√	√			√
Noorollahi et al.		√			√					√
Demirel et al.	√	√	√	√	√	√				√
Darani et al.		√					√	√		√
Gücer	√	√			√	√				√
Ammar et al.					√			√		√
Wang et al.	√		√							√
Guler and Yomralioglu		√	√		√	√	√	√		
Anderluh et al.			√			√				√
Deveci et al.		√	√	√	√					√
Komchornrit	√		√		√	√	√			
TOTAL	7	12	10	8	12	11	8	4	1	13

*LR: Labor Resource, PM: Proximity to Markets, ES: Economic Obligations, SS: Socioeconomic Status, PSR: Proximity to Suppliers and Resources, TR: Transportation, LC: Land Cost, PD: Population Density, PE: Policy Environment, TEC: Technical Environment and Capacity.

c) *Transportation (T):* Urban transport has been taken into account.

Socioeconomic Situation (SOS): It describes the main criterion concerning the the social and economic situations of the city where the facility will be established.

  a) *Tax (TX):* Tax deduction has been taken into consideration in the production and activities.

  b) *Government Promotion (GP):* Government promotion has been taken into consideration in the production and activities.

  c) *Easiness of Employment (EE):* The quality and number of personnel who could be employed in the city have been taken into account.

Effectiveness of the Natural Spring Water (ENSW): It has been evaluated as the quality composed by the

nutritional values of natural spring water sources that are evaluated as drinking water of the city where the facility will be established and the existing quantity of water.

  a) *Natural Spring Water Quality (NSWQl):* In addition to the characteristics of the various natural spring water sources in the city such as "pH, hardness", organic and inorganic contents of the water have been taken into consideration.

  b) *Natural Spring Water Quantity (NSWQn):* Based on tonnage, water quantity of the springs has been addressed.

The pair-wise comparison built based on this hierarchy and criterion and the weighted charts from which are among the main criteria are illustrated in Table 5. as an example.
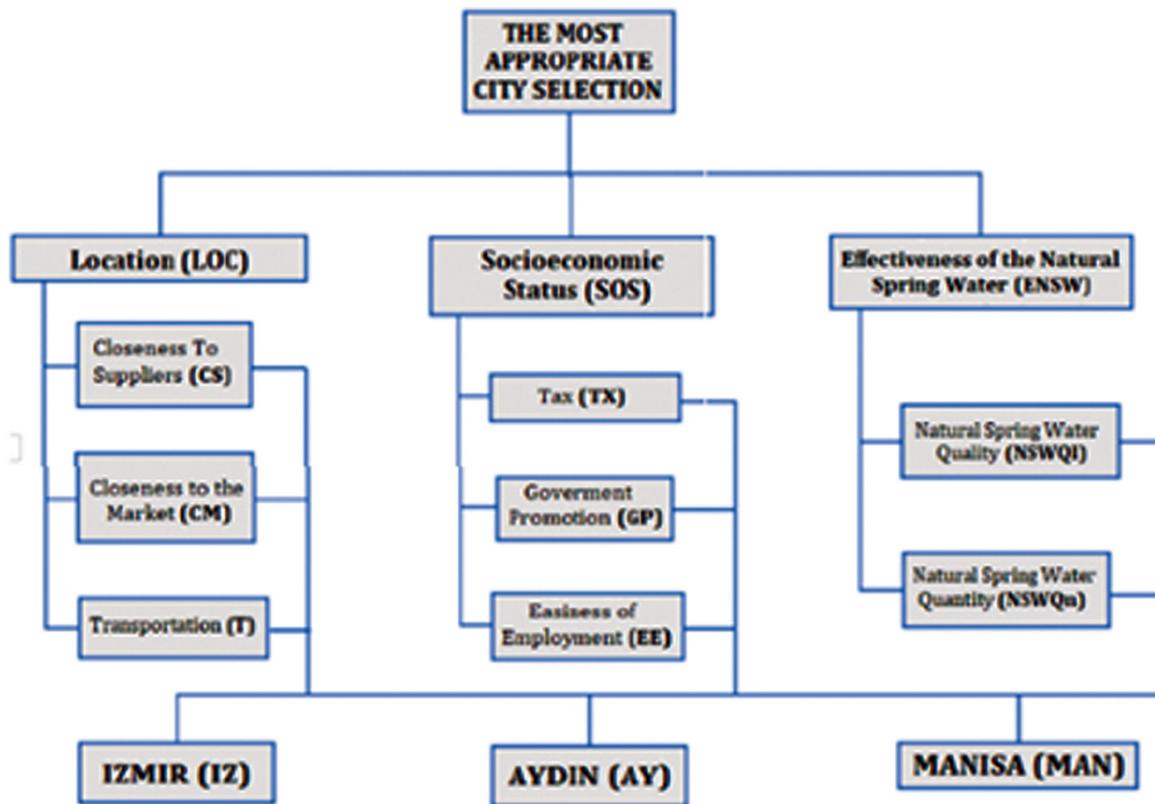
Figure 2. The Hierarchical Structure of the Established FAHP Model [39].

Table 5. Pairwise Comparison Matrix and Weighting of Main Criteria

MAIN	LOC	SOS	ENSW	r_{ij}			w_{ij}			*DEF	NRM
LOC	1,1,1	3,4,5	0,25, 0,33, 0,5	0,908	1,100	1,357	0,189	0,270	0,402	0,287	0,26
SOS	0,2, 0,25, 0,33	1,1,1	0,14, 0,16, 0,2	0,305	0,346	0,405	0,063	0,085	0,120	0,089	0,08
ENSW	2.3.4	5,6,7	1,1,1	2,154	2,620	3,036	0,448	0,644	0,901	0,664	0,66
		TOTAL		3,368	4,068	4,799				1,042	1

*DEF: Defuzzification, NRM: Normalization

Table 6. General Weighting Chart (Total Weights: 1.00)

MAIN	W. MAIN	SUB	W. SUB	IZ	AY	MAN	W. IZ	W. AY	W. MAN
		CS	0,18	0,50	0,24	0,26	0,023	0,011	0,012
LOC	0,26	CM	0,38	0,25	0,58	0,17	0,024	0,057	0,016
		T	0,44	0,54	0,35	0,11	0,061	0,04	0,012
		TX	0,13	0,52	0,26	0,22	0,005	0,002	0,002
SOS	0,08	GP	0,30	0,33	0,33	0,33	0,007	0,00	0,007
		EE	0,57	0,52	0,35	0,13	0,023	0,01	0,005
ENSW	0,66	NSWQl	0,34	0,26	0,57	0,17	0,058	0,127	0,038
		NSWQn	0,66	0,27	0,41	0,32	0,117	0,178	0,139
TOTAL							**0,322**	**0,441**	**0,235**
RANKING							**2.**	**1.**	**3.**

Table 7. Notations of the established stochastic model

Notations	Definitions
Set and Index	
$i \in I, I = \{1,2\}$	Set of factories
$j \in J, J = \{1,2,.,5\}$	Set of warehouses
$k \in C, C = \{1,2,.,6\}$	Set of customers
$a \in A, A = \{1,2,.,4\}$	Set of scenarios
Parameters	
F_{ai}	Production capacity of the factory i based on scenario a
W_{aj}	Capacity of warehouse j based on scenario a
C_{ak}	Demand of customer k based on scenario a
R_i	Operating cost of the factory i
S_j	Usage cost of warehouse j
fw *(skalar)*	Transport cost between factories and warehouses per unit, per km *(with truck)*
MF_{ij}	Distance matrix between factories and warehouses
$TFc_{ij} = fw \times MF_{ij}$	Transport cost between factories and warehouses per unit
wm *(skalar)*	Transport cost between warehouses and customers per unit, per km *(with van)*
MW_{jk}	Distance matrix between warehouses and customers
$TFc_{jk} = wm \times Mw_{jk}$	Transport cost between warehouses and customers per unit
PFc_i	Unit production cost at factory i
SWc_j	Unit storage cost at warehouse j
pc *(skalar)*	Penalty cost for unmet demands
p	Realization probability of each scenario, in stochastic models, the probability value is usually handled in accordance with the continuous distribution [40]. Therefore, we considered it as 1/(scenario number)
Decision Variables	
$h_i \in \{0,1\}$ *(binary)*	Usage decision for the factory i
$g_i \in \{0,1\}$ *(binary)*	Usage decision for the warehouse j
X_{aij}	Quantity moved from factory i to warehouse j based on scenario a
Y_{ajk}	Quantity moved from warehouse j to customer k based on scenario a
Q_{ak}	Unmet demand of customer k based on scenario a
Z	Total production, transportation and storage costs

The weighting score table prepared to select the proper province after finding the weights of targeted states considering main and sub-criteria as shown in Table 6.

According to Table 6, the FLS process should be conducted in Aydın province. In the next stage, the stochastic network structure was built in this province.

EXPLANATION OF THE STOCHASTIC MODEL

Indeterminacy case is dominant in stochastic models [Zhu and Yu] which examines market factors under different possible scenarios and is much close to market conditions. Thus, in the stochastic model we built, factory, warehouse capacities and customer demands were taken different values according to four scenarios. The constructed model is a two-stage stochastic model; in the first stage locations of factory and warehouse were determined and in the second stage, the optimization of distribution were carried out.

NOTATIONS

Notations of the MIP model that reduces the product circulation cost in the established network and determines the number and the location of the factories and warehouses are given below in Table 7:

Numerical data of the parameters in Table 7 are shown below. First of all, since the model is stochastic, demand was treated as uncertain and different demand values were assigned for four scenarios. At this point, an average demand has been determined with the opinions of the existing regional marketing experts and assignments were made to the scenarios based on this value. Then,

warehouse capacities were determined according to demand values, and factory capacities were determined according to warehouse values. Accordingly, the factory capacities changed based on scenarios presented in Table 8.

Table 9 shows the warehouse capacities on the basis of scenarios.

Table 10 shows the varying demands of customers on scenario basis.

Factory operating costs, which are independent from scenarios, are shown in Table 11.

Warehouse usage costs, which are independent from scenarios, are shown in Table 12.

Transportation costs and penalty costs which are independent from scenarios, are shown in Table 13.

Distance matrix between factories and warehouses are shown in Table 14.

Distance matrix between warehouses and customers are shown in Table 15.

The map indicating the distances between the factories-warehouses-customers in Aydın is shown in Figure 3.

Table 8. Factory Capacities Based on Scenario

Scn.	Factory Capacities (monthly-pallet)	
	F_1	F_2
a_1	5.270	4.800
a_2	5.115	4.750
a_3	4.960	4.010
a_4	4.320	3.645

Table 9. Warehouse Capacities Based on Scenario

Scn.	Warehouse Capacities (monthly-pallet)				
	W_1	W_2	W_3	W_4	W_5
a_1	2.150	2.400	1.900	1.525	1.750
a_2	1.995	2.260	1.745	1.340	1.500
a_3	1.620	2.000	1.510	1.100	1.360
a_4	1.350	1.850	1.325	1.055	1.190

Table 10. Customer Demands Based on Scenario

Scn.	Customer Demands (monthly-pallet)					
	C_1	C_2	C_3	C_4	C_5	C_6
a_1	1.620	2.080	1.290	1.450	1.530	1.700
a_2	1.550	1.885	1.010	1.390	1.420	1.625
a_3	1.495	1.670	840	1.220	1.295	1.590
a_4	1.150	1.445	800	1.200	1.160	1.300

Table 11. Factory Operating Costs

R_1 Usage Costs (monthly-TRY)	R_2 Usage Costs (monthly-TRY)
15.400	10.700

Table 12. Warehouse Usage Costs

S_1 (monthly-TRY)	S_2 (monthly-TRY)	S_3 (monthly-TRY)	S_4 (monthly-TRY)	S_5 (monthly-TRY)
5.200	5.600	5.100	4.650	4.800

Table 13. Transportation Costs and Penalty Cost

fw (TRY/km)	wm (TRY/km)	pc (TRY)
5	3	550
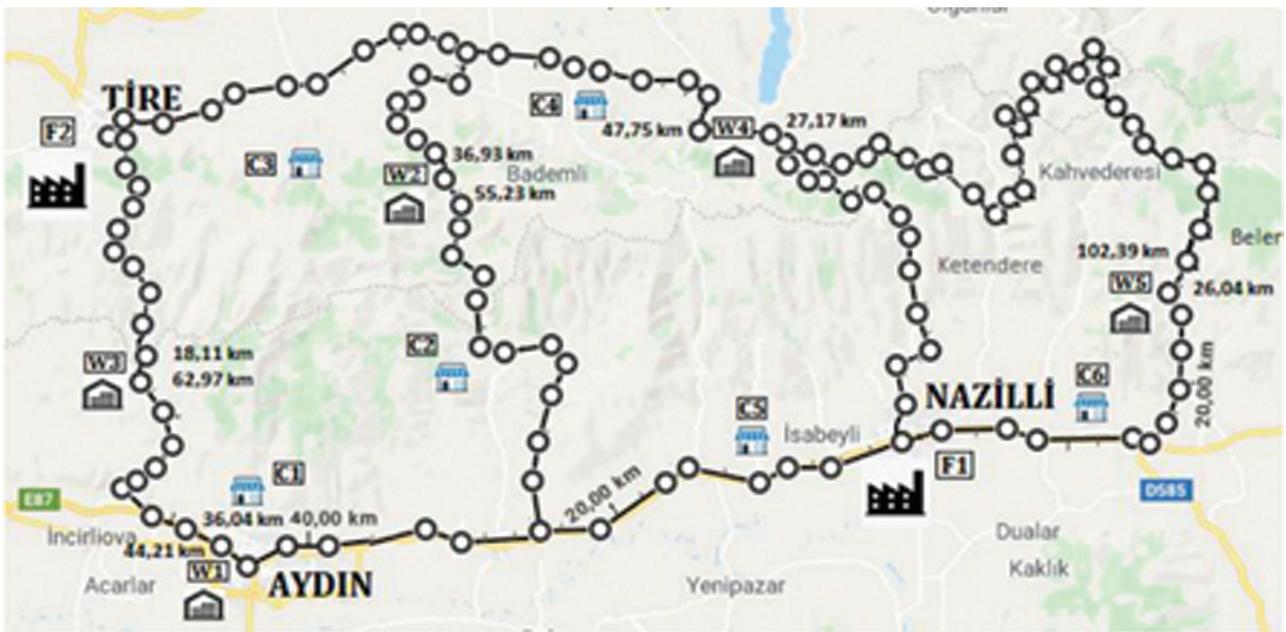
Figure 3. Route Display of Factories-Warehouses-Customers in the Aydın Province.

Table 14. Distance Matrix between Factories and Warehouses

(MF_{ij}) Distance Matrix (km)	Warehouses				
Factories	W_1	W_2	W_3	W_4	W_5
F_1	44,21	55,23	62,97	27,17	26,04
F_2	36,04	18,11	36,93	47,75	102,39

Table 15. Distance Matrix between Warehouses and Customers

(MW_{jk}) Distance Matrix (km)	Customers					
Warehouses	C_1	C_2	C_3	C_4	C_5	C_6
W_1	5	43,5	48,2	68,85	32,6	59,46
W_2	50,05	11,3	13,25	18	29,5	58,2
W_3	14,3	42,45	23	37,5	41,18	56,27
W_4	63,75	60,15	29	9,72	26,53	36,44
W_5	64,7	62,5	82,64	41,5	30,21	8,86

The distances were calculated by taking into account the carrier vehicles tracks on Google Maps. Unit production cost of factories are shown in Table 16.

Unit storage cost of warehouses are shown in Table 17.

Figure 4 shows the circulation in the established network and some notations are also mentioned in the figure. Thus, the structure of the network is better understood.

Table 16. Unit Produced Costs for Factories

PFc_1 (unit-TRY)	PFc_1 (unit-TRY)
65	70

Table 17. Unit Storaged Costs for Warehouses

SWc_1 (unit-TRY)	SWc_2 (unit-TRY)	SWc_3 (unit-TRY)	SWc_4 (unit-TRY)	SWc_5 (unit-TRY)
18	17	20	22	25
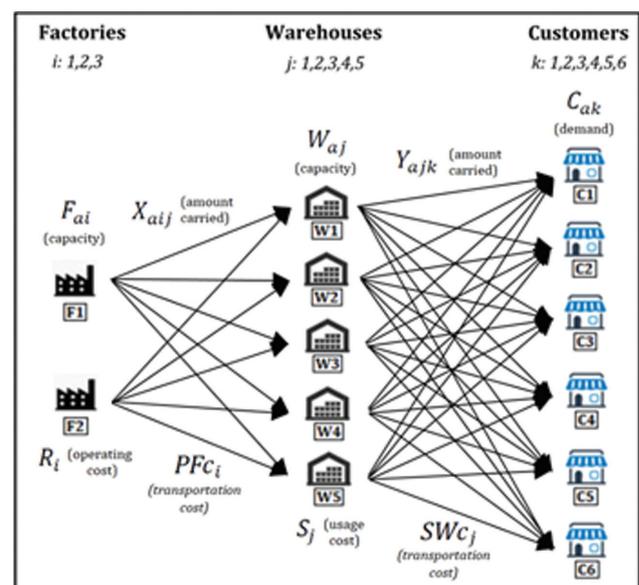
Figure 4. Circulation and Notations in the Established Network.

PROPOSED STOCHASTIC MIP MODEL

In this section, the mathematical model of the objective function and constraints are explained first. Then their definitions are given. Accordingly, the objective function of the model is given as:

Objective Function:

$$\min Z = \sum_{i \in F}(h_i \times R_i) + \sum_{j \in W}(g_j \times S_j) + \left(\sum_{a \in A}\sum_{i \in F}\sum_{j \in W}[(PFc_i \times X_{aij}) + (TFc_{ij} \times X_{aij})] \right.$$
$$\left. + \sum_{a \in A}\sum_{j \in W}\sum_{k \in C}[(SWc_j \times Y_{ajk}) + (TWc_{jk} \times Y_{ajk})]) + [pc \times \sum_{a \in A}\sum_{k \in C}(Q_{ak})] \right) \times p$$

(annotations: obj₂, obj₁, obj₃, obj₄, obj₅, obj₆, obj₇)

Capacity Balance Constraints:

$$\sum_{j \in W} X_{aij} \leq h_i \times F_{ai}, \quad \forall a, i \tag{9}$$

$$\sum_{i \in F} X_{aij} \leq g_i \times W_{aj}, \quad \forall a, j \tag{10}$$

$$\sum_{k \in C} Y_{ajk} \leq g_i \times W_{aj}, \quad \forall a, j \tag{11}$$

Quantity Balance Constraints:

$$\sum_{i \in F} X_{aij} \geq \sum_{k \in C} Y_{ajk}, \quad \forall a, j \tag{12}$$

Demand Balance Constraints:

$$\sum_{j \in W} Y_{ajk} \leq C_{ak}, \quad \forall a, k \tag{13}$$

$$Q_{ak} + \sum_{j \in W} Y_{ajk} = C_{ak}, \quad \forall a, k \tag{14}$$

Positivity Constraints:

$$X_{aij}, Y_{ajk} \geq 0, \quad \forall a, i, j, k \tag{15}$$

Binary Constraints:

$$h_i, g_j \in \{0,1\}, \quad \forall i, j \tag{16}$$

The definitions of the above equations consisting of objective and constraint functions are given below:

Equation 8: This equation recognizes the objective function. The objective function consists of seven cost parts. The sum of these reveals the objective function. The purpose of the problem is to minimize the summation of these functions:

- obj_1: It defines the total operating cost of factories to be opened.
- obj_2: It defines the total cost of using the warehouses to be opened.
- obj_3: It defines the total production cost of the products that produced in the factories and sent to the warehouses.
- obj_4: It defines the total transportation cost of products that transported from factories to warehouses.
- obj_5: It defines the total storage cost of the products that stored in the warehouses and sent to the customers.
- obj_6: It defines the total transportation cost of products that transported from warehouses to customers.
- obj_7: It defines the penalty cost of unmet demands.

Equation 9: The quantity that is going out from factories to be opened should be less or equal to these factories capacity.

Equation 10: The quantity coming into warehouses to be opened should be less or equal to the capacities of these warehouses.

Equation 11: The quantity that is going out from warehouses to be opened should be less or equal to capacities of these warehouses.

Equation 12: The quantity arriving into warehouses to be opened should be more or equal to the quantity going out these warehouses

Equation 13: The quantity going out from warehouses to be opened should be less or equal to the demand of customers.

Equation 14: The summation of the quantity going out from warehouses to be opened and unmet demand of customer should be equal to the demand of customer.

Equatin 15: It shows that the decision variables indicating the amount of transport should be positive.

Equation 16: It is the binary constraint that decides whether factory and warehouses should be used or not.

OUTPUTS OF THE MODEL

The stochastic MIP model mentioned above was applied in GAMS 33.2.0. The results are presented below as in the Table 18. Accordingly, Table 18 shows the scenerio-based quantities transported between factories and warehouses.

Table 19 shows the scenerio-based quantities transported between warehouses and customers.

Tables 18 and 19 show the product flows in network. According to this results, Table 20 shows the unmet demands and the cost to be incurred for them.

As seen in Table 20, the total unmet demand is 845 pallets. This is not a high level. However, the objective function Z, which minimizes the cost of the entire circulation in the network, is obtained as 2.469.132,825 TRY. Costs in the

Table 18. Scenario Based Factories-Warehouses Assignments

Scenerios	Factories	Warehouses				
		W_1	W_2	W_3	W_4	W_5
a_1	F_1	1.595	Close	Close	1.525	1.750
a_2	F_1	1.250	Close	Close	1.340	1.500
a_3	F_1	1.120	Close	Close	1.100	1.360
a_4	F_1	880	Close	Close	1.055	1.190
a_1	F_2	555	2.400	1.845	Close	Close
a_2	F_2	745	2.260	1.745	Close	Close
a_3	F_2	500	2.000	1.510	Close	Close
a_4	F_2	470	1.850	1.325	Close	Close

Table 19. Scenario Based Warehouses-Customers Assignments

Scenerios	Warehouses	Customers					
		C_1	C_2	C_3	C_4	C_5	C_6
a_1	W_1	1.620	Close	Close	Close	530	Close
a_1	W_2	Close	2.080	Close	Close	320	Close
a_1	W_3	Close	Close	Close	Close	555	Close
a_1	W_4	Close	Close	1.290	1.450	75	Close
a_1	W_5	Close	Close	Close	Close	50	1.700
a_2	W_1	1.550	Close	Close	Close	445	Close
a_2	W_2	Close	1.885	Close	50	325	Close
a_2	W_3	Close	Close	Close	Close	650	85
a_2	W_4	Close	Close	1.010	1.340	Close	Close
a_2	W_5	Close	Close	Close	Close	Close	1.500
a_3	W_1	1.495	Close	Close	Close	125	Close
a_3	W_2	Close	1.670	Close	120	210	Close
a_3	W_3	Close	Close	Close	Close	670	Close
a_3	W_4	Close	Close	840	1.100	Close	Close
a_3	W_5	Close	Close	Close	Close	Close	1.360
a_4	W_1	1.150	Close	Close	Close	200	Close
a_4	W_2	Close	1.445	Close	145	260	Close
a_4	W_3	Close	Close	Close	Close	525	Close
a_4	W_4	Close	Close	800	1.055	Close	Close
a_4	W_5	Close	Close	Close	Close	Close	1.190

Table 20. Unmet Customer Demands and the Costs Incurred

Scenerios	Q_1 (pallet)	Q_2 (pallet)	Q_3 (pallet)	Q_4 (pallet)	Q_5 (pallet)	Q_6 (pallet)	TOTAL
a_1	-	-	-	-	-	-	**0**
a_2	-	-	-	-	-	40	**40**
a_3	-	-	-	-	290	230	**520**
a_4	-	-	-	-	175	110	**285**
TOTAL	**0**	**0**	**0**	**0**	**465**	**380**	**845**

network include operating, usage, production, storage and transportation costs.

CONCLUSIONS

New investment decisions are very important for companies. They should act strategically to avoid market fluctuations. For this, they employ several different methods. One of them may be to seek the opinion of an experienced expert, and another one is the use of optimization tools. However, it seems that it is more beneficial to use these methods as a hybrid. The proposed hybrid method can achieve an output closer to the optimum result required for the decision to be taken. In this study, we used FAHP, which is based on expert opinions, and the stochastic MIP method, which is purely mathematical, as a hybrid method. The location selection we perform in this study is way of high added value for both the literature and the company. In addition, two of the logic we applied are fuzziness and stochasticity, which increase the accuracy of the results to be closer to real world [30] applications.

In this study, the facility and warehouse location selection was made for a company being active in the branch of drinking water sector of the food sector, who would like to establish a supply chain network in the Aegean Region. Firstly, FAHP study was conducted in the Aegean Region and Aydın province was found the most appropriate appropriate region. In the next stage, a network structure between factories-warehouses-customers was established in the selected city to select the facility location. In accordance with real market conditions, customer demand is taken as uncertain. Demand average is based on expert opinions. Later, warehouse and factory capacities were determined. The established stochastic MIP model was solved in the GAMS 33.2.0 program. The objective function, which is the sum of all costs, was obtained as the network installation cost of 2,469,132,825 TRY. This result may be more costly than the result of any deterministic model. This is a natural state. Because studies carried out under certainty do not take into account costs beyond the planned in the long run. If the same studies are carried out stochastically, all fluctuations must be taken into account. In this case, the different scenarios considered create additional costs for the model. Thus, the result is more costly.

On the other hand, our study, which was examined under different scenarios, is much closer to real market conditions and results. Companies also want realistic results in their investment plans. Therefore, fuzzy or stochastic models are accepted. Accordingly, the results of our model were shared with senior management and strategy development experts and were deemed appropriate.

This study was established based on real data and it revealed appropriate results for an operating company in reel market. As it stands, this study is a study that contributes to the literature for researchers interested in location selection studies considering stochasticity. Furthermore, this hybrid method chooses not only the facility location but also the region where the network will be established in selected location. Thus it offers more a realistic, comprehensive and alternative way for companies that would like to make a new investment decision.

Our work can be improved. The method can be applied to larger networks and the number of scenarios can be increased. Moreover, different transportation modes can be considered while the distribution decision are taken.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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