A note on connectedness on soft multi topological spaces

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Abstract - We show that an alleged theorem stated in [7] is invalid in general, by giving a counter-example.

1   Introduction

The notion of multisets (briefly, msets) was introduced by Yager [10], Blizard [1, 2] and Jena et al. [6] have been mentioned. Moreover, the concept of soft multisets which is combining soft sets and multisets can be used to solve some real life problems. This concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc. The concepts of soft mset, soft multi connectedness, soft multi compactness and soft multi continuous function were introduced by D. Tokat et al. [7, 8, 9]. In addition, the concepts of generalized closed (open) soft msets, semi-compact soft multi spaces, the notion of γ-operation and the notion of γ-continuous soft multi functions are presented by El-Sheikh et al. [3, 4, 5].

2   Preliminaries

Definition 2.1. *[7] Let U be a universal mset, E be a set of parameters and $A \subseteq E$. Then, an ordered pair (F,A) is called a soft mset where F is a mapping given by $F : A \to P^*(U)$. For all $e \in A$, $F(e)$ mset represent by count function $C_{F(e)} : U^* \to N$ where N represents the set of non-negative integers, U^* represents the support set of U and $P^*(U)$ represents the power set of the mset U.*

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Definition 2.2. *[7] For two soft msets (F, A) and (G, B) over U, (F, A) is called a sub soft mset of (G, B) if*

   1. $A \subseteq B$,

   2. $C_{F(e)}(x) \leqslant C_{G(e)}(x)$, $\forall x \in U^*$, $e \in A$.

   It is denoted by $(F, A) \widetilde{\subseteq} (G, B)$.

Definition 2.3. *[7] The union of two soft msets (F, A) and (G, B) over U is the soft mset (H, C), where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall e \in A \cup B$, $\forall x \in U^*$. It is denoted by $(F, A) \widetilde{\cup} (G, B)$.*

Definition 2.4. *[7] The intersection of two soft msets (F, A) and (G, B) over U is the soft mset (H, C), where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall e \in A \cap B$, $\forall x \in U^*$. It is denoted by $(F, A) \widetilde{\cap} (G, B)$.*

Definition 2.5. *[7] The complement of a soft mset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x)$, $\forall x \in U^*$.*

Definition 2.6. *[7] Let (X, τ, E) be a soft multi topological space. A soft multi separation of $\widetilde{X}$ is a pair (F, E), (G, E) of no-null open soft msets in X such that $\widetilde{X} = (F, E) \widetilde{\cup} (G, E)$, $(F, E) \widetilde{\cap} (G, E) = \widetilde{\phi}$.*

Definition 2.7. *[7] A soft multi topological space (X, τ, E) is said to be soft multi connected if there is no a soft multi separation of $\widetilde{X}$. Otherwise, (X, τ, E) is said to be soft multi disconnected.*

Theorem 2.8. *[7] A soft multi topological space (X, τ, E) is soft multi connected if and only if the only soft msets over X that are both soft multi open and soft multi closed in X_E are $\widetilde{\phi}$ and $\widetilde{X}$.*

3   Counter-example

The following example shows the necessity of Theorem 2.8 is not satisfied.

Example 3.1. *Let $X = \{2/a, 4/b, 6/c\}$ be a mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E)\}$ be a soft multi topological space on X, where:*
$F_1(e_1) = \{1/a, 2/b, 3/c\}$, $F_1(e_2) = \{1/a, 2/b, 3/c\}$,
$F_2(e_1) = \{1/a\}$, $F_2(e_2) = \{2/b, 3/c\}$.
It's clear that (X, τ, E) is a soft multi connected but (F_1, E) is an open and closed soft mset different from $\widetilde{X}$ and $\widetilde{\phi}$.

The following theorem is the correction form of Theorem 2.8.

Theorem 3.2. *If the only soft msets over X that are both soft multi open and soft multi closed over X are $\widetilde{\phi}$ and $\widetilde{X}$, then the soft multi topological space (X, τ, E) is soft multi connected.*

   Proof. Immediate from the definition.

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