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A note on Soft Topology

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Abstract - Cagman et al. (2011) [3] introduced soft topology on a soft set and defined soft topological space. They defined the basic notions of soft topological spaces such as soft open sets, soft closed sets, soft interior, soft closure, soft basis, soft neighborhood, soft limit point, soft boundary, soft subspace and soft Hausdorff space and explored several basic properties of these concepts. But we found that some basic results are not true in general. The main purpose of this note is, with the help of counter examples, to establish that some results do not hold.

Keywords - *Soft sets, soft topology, soft open sets, soft closed sets, soft closure, soft limit point.*

1 Introduction

In 1999, Molodtsov [5] introduced the concept of soft set theory as a mathematical tool for deal with uncertainties while modeling the problems in science and engineering. Many researchers like Maji et al.[4] have further improved the theory of soft sets. Cagman et al.[2] modified the definition of soft sets using approximate function. The notion of soft topology on an initial soft set was introduced by Cagman[3]. They defined soft topology as a collection with soft sets as members and studied the basic notions of soft topological spaces such as soft basis, subspace soft topology, soft interior, soft closure, soft neighborhood, soft limit point and soft boundary. But we found that some results in [3] are incorrect. So the purpose of this note is to point out some errors which have appeared in [[3], theorem 12-15]

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2 Preliminary

Some definitions and results defined and discussed in [1, 2, 3, 4, 5] are included in this section. Throughout this paper U denotes initial universe, E denotes the set of all possible parameters, $\mathcal{P}(U)$ is the power set of U and A is a nonempty subset of E.

Definition 2.1. *A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in \mathcal{P}(U)\}$, where $f_A : E \to \mathcal{P}(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over U will be denoted by $S(U)$.*

Example 2.2. *Suppose that there are six houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration and that $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters. The $x_i (i = 1, 2, 3, 4, 5)$ stand for the parameters "expensive", "beautiful", "wooden", "cheap" and "in green surroundings" respectively.*

Consider the mapping f_A given by "houses (.)", where (.) is to be filled in by one of the parameters $x_i \in E$. For instance, $f_A(x_2)$ means "houses (beautiful)", and its functional value is the set $\{h \in U : h$ is a beautiful house$\}$.

Suppose that $A = \{x_1, x_3, x_4\} \subseteq E$ and $f_A(x_1) = \{h_2, h_4\}$, $f_A(x_3) = U$ and $f_A(x_4) = \{h_1, h_3, h_5\}$. Then, we can view the soft set F_A as consisting of the following collection of approximations: $F_A = \{(x_1, \{h_2, h_4\}), (x_3, U), (x_4, \{h_1, h_3, h_5\})\}$.

Definition 2.3. *Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called an empty soft set, denoted by F_Φ. $f_A(x) = \emptyset$ means that there is no element in U related to the parameter $x \in E$. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.*

Definition 2.4. *Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called an A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{E}}$.*

Definition 2.5. *Let $F_A, F_B \in S(U)$. Then F_A is a soft subset of F_B, denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$, for all $x \in E$.*

Definition 2.6. *Let $F_A, F_B \in S(U)$. Then F_A and F_B are soft equal, denoted by $F_A = F_B$, if and only if $f_A(x) = f_B(x)$, for all $x \in E$*

Definition 2.7. *Let $F_A, F_B \in S(U)$. Then, the soft union $F_A \tilde{\cup} F_B$, the soft intersection $F_A \tilde{\cap} F_B$ and the soft difference $F_A \tilde{\setminus} F_B$ of F_A and F_B are defined by the approximate functions $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$, $f_{A\tilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$ respectively, and the soft complement $F_A^{\tilde{c}}$ of F_A is defined by the approximate function $f_A^{\tilde{c}}(x) = f_A^c(x)$, where $f_A^c(x)$ is the complement of the set $f_A(x)$, i.e., $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$. It is easy to see that $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$ and $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$.*

Proposition 2.8. *Let $F_A \in S(U)$. Then the following hold:*

1. $F_A \tilde{\cup} F_A = F_A, F_A \tilde{\cap} F_A = F_A$

2. $F_A \tilde{\cup} F_\Phi = F_A, F_A \tilde{\cap} F_\Phi = F_\Phi$

3. $F_A \tilde{\cup} F_{\tilde{E}} = F_{\tilde{E}}, F_A \tilde{\cap} F_{\tilde{E}} = F_A$

4. $F_A \tilde{\cup} F_A^{\tilde{c}} = F_{\tilde{E}}, F_A \tilde{\cap} F_A^{\tilde{c}} = F_\Phi$

Proposition 2.9. *Let $F_A, F_B, F_C \in S(U)$. Then,*

1. $F_A \tilde{\cup} F_B = F_B \tilde{\cup} F_A, F_A \tilde{\cap} F_B = F_B \tilde{\cap} F_A$

2. $(F_A \tilde{\cup} F_B)^{\tilde{c}} = F_A^{\tilde{c}} \tilde{\cap} F_B^{\tilde{c}}, (F_A \tilde{\cap} F_B)^{\tilde{c}} = F_A^{\tilde{c}} \tilde{\cup} F_B^{\tilde{c}}$

3. $(F_A \tilde{\cup} F_B) \tilde{\cup} F_C = F_A \tilde{\cup} (F_B \tilde{\cup} F_C), (F_A \tilde{\cap} F_B) \tilde{\cap} F_C = F_A \tilde{\cap} (F_B \tilde{\cap} F_C)$

4. $F_A \tilde{\cup} (F_B \tilde{\cap} F_C) = (F_A \tilde{\cup} F_B) \tilde{\cap} (F_A \tilde{\cup} F_C), F_A \tilde{\cap} (F_B \tilde{\cup} F_C) = (F_A \tilde{\cap} F_B) \tilde{\cup} (F_A \tilde{\cap} F_C)$

Definition 2.10. *Let $F_A \in S(U)$. The soft power set of F_A, denoted by $\tilde{\mathcal{P}}(F_A)$, is defined by $\tilde{\mathcal{P}}(F_A) = \{F_{A_i} | F_{A_i} \tilde{\subseteq} F_A, i \in I \subseteq \mathbb{N}\}$.*

Example 2.11. *Let $U = \{u_1, u_2, u_3\}, E = \{x_1, x_2, x_3\}, A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then*
$F_{A_1} = \{(x_1, \{u_1\})\}$
$F_{A_2} = \{(x_1, \{u_2\})\}$
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$
$F_{A_4} = \{(x_2, \{u_2\})\}$
$F_{A_5} = \{(x_2, \{u_3\})\}$
$F_{A_6} = \{(x_2, \{u_2, u_3\})\}$
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$
$F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$
$F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$
$F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$
$F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$
$F_{A_{15}} = F_A$
$F_{A_{16}} = F_\Phi$ *are all soft subsets of F_A.*

3 Soft Topology

Definition 3.1. *[3] Let $F_A \in S(U)$. A soft topology on a soft set F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having the following three properties:*

1. $F_\Phi, F_A \in \tilde{\tau}$

2. $\{F_{A_i} \tilde{\subseteq} F_A : i \in I \subseteq \mathbb{N}\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcup}_{i \in I} F_{A_i} \in \tilde{\tau}$.

3. $\{F_{A_i} \tilde{\subseteq} F_A : 1 \leq i \leq n, n \in \mathbb{N}\} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcap}_{i=1}^{n} F_{A_i} \in \tilde{\tau}$.

The pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Example 3.2. *[3] Let us consider the soft subsets of F_A that are given in Example 2.11. Then $\tilde{\tau}_1 = \{F_\Phi, F_A\}, \tilde{\tau}_2 = \tilde{\mathcal{P}}(F_A)$ and $\tilde{\tau}_3 = \{F_\Phi, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\}$ are soft topologies on F_A.*

Definition 3.3. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called a soft open set. Clearly F_Φ and F_A are soft open sets.*

Definition 3.4. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space. A sub family $\tilde{\mathcal{B}}$ of $\tilde{\tau}$ is said to be a soft basis for the soft topology $\tilde{\tau}$ if every member of $\tilde{\tau}$ can be expressed as the soft union of some members of $\tilde{\mathcal{B}}$*

Example 3.5. *[3] Let us consider Example 2.11 and 3.2. Then $\tilde{\mathcal{B}} = \{F_\Phi, F_{A_1}, F_{A_2}, F_{A_4}, F_{A_5}\}$ is a soft basis for the soft topology $\tilde{\tau}_2$.*

Theorem 3.6. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space and $\tilde{\mathcal{B}}$ be a soft basis for $\tilde{\tau}$. Then, $\tilde{\tau}$ equals the collection of all soft unions of elements of $\tilde{\mathcal{B}}$.*

Definition 3.7. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then F_B is said to be a soft closed set if its soft complement $F_B^{\tilde{c}}$ is a soft open set.*

Theorem 3.8. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space. Then the following conditions are hold:*

1. *The universal soft set $F_{\tilde{E}}$ and $F_A^{\tilde{c}}$ are soft closed sets.*
2. *Arbitrary soft intersections of the soft closed sets are soft closed.*
3. *Finite soft unions of the soft closed sets are soft closed.*

Definition 3.9. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then the soft closure of F_B, denoted by $\overline{F_B}$ is defined as the soft intersection of all soft closed soft super sets of F_B. Note that $\overline{F_B}$ is the smallest soft closed set that containing F_B.*

Example 3.10. *[3] Consider the soft topology $\tilde{\tau}_3$ that is given in Example 3.2. If $F_B = F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$, then $F_{A_2}^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_2, U), (x_3, U)\}$ and $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$ are soft closed soft super sets of F_B. Hence $\overline{F_B} = F_{A_2}^{\tilde{c}} \tilde{\cap} F_{\tilde{E}} = F_{A_2}^{\tilde{c}}$.*

Theorem 3.11. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. F_B is a soft closed set if and only if $F_B = \overline{F_B}$.*

Theorem 3.12. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_G, F_H \tilde{\subseteq} F_A$. Then the following conditions are hold:*

1. $F_G \tilde{\subseteq} \overline{F_G}$
2. $\overline{\overline{F_G}} = \overline{F_G}$
3. $F_G \tilde{\subseteq} F_H \Rightarrow \overline{F_G} \tilde{\subseteq} \overline{F_H}$
4. $\overline{F_G \tilde{\cap} F_H} \tilde{\subseteq} \overline{F_G} \tilde{\cap} \overline{F_H}$

5. $\overline{F_G\tilde{\cup}F_H} = \overline{F_G}\tilde{\cup}\overline{F_H}$

Theorem 3.13 ([3], theorem 12)**.** *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B, F_C\tilde{\subseteq}F_A$. Then the following conditions are hold:*

1. *$\alpha \in \overline{F_B}$ if and only if every soft open set F_C containing α soft intersects F_B.*

2. *Supposing the soft topology on F_A is given by a soft basis $\tilde{\mathcal{B}}$, then $\alpha \in \overline{F_B}$ if and only if every soft basis element F_D containing α soft intersect F_B.*

The next example shows that the above statements are not always true.

Example 3.14. *Consider the soft topology $\tilde{\tau}_3 = \{F_\Phi, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\}$ on F_A that is given in Example 3.2. The soft closed sets are : $F_\Phi^{\tilde{c}} = F_{\tilde{E}}, F_A^{\tilde{c}} = \{(x_1, \{u_3\}), (x_2, \{u_1\}), (x_3, U)\}, F_{A_2}^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_2, U), (x_3, U)\}, F_{A_{11}}^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_2, \{u_1, u_2\}), (x_3, U)\}, F_{A_{13}}^{\tilde{c}} = \{(x_1, \{u_3\}), (x_2, \{u_1, u_3\}), (x_3, U)\}$. Let $F_B = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$. Then $\overline{F_B} = F_{A_2}^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_2, U), (x_3, U)\}$. Let $F_C = F_{A_{13}}$ and take $\alpha = (x_1, \{u_1, u_2\})$. i.e., F_C is a soft open set containing α. Now $F_C\tilde{\cap}F_B = \{(x_1, \{u_1\}), (x_2, \{u_2\})\} \neq F_\Phi$. But $\alpha \notin \overline{F_B}$. i.e., We can find a soft open set F_C containing α soft intersect F_B and $\alpha \notin \overline{F_B}$. Similarly we can show that the statement (ii) in [5 theorem 12] is not always true.*

Definition 3.15. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space and $\alpha \in F_A$. If there is a soft open set F_B such that $\alpha \in F_B$, then F_B is called a soft open neighborhood (or soft neighborhood) of α. The set of all soft neighborhoods of α, denoted by $\tilde{\nu}(\alpha)$, is called the family of soft neighborhoods of α. i.e., $\tilde{\nu}(\alpha) = \{F_B | F_B \in \tilde{\tau}, \alpha \in F_B\}$.*

Example 3.16. *[3] Let us consider the soft topological space $(F_A, \tilde{\tau}_3)$ given in Example 3.2 and let $\alpha = (x_1, \{u_1, u_2\}) \in F_A$. Then $\tilde{\nu}(\alpha) = \{F_A, F_{A_{13}}\}$.*

Definition 3.17. *[3] Let $(F_A, \tilde{\tau})$ be a soft topological space, $F_B\tilde{\subseteq}F_A$ and $\alpha \in F_A$. If every soft neighborhood of α soft intersects F_B in some points other than α itself, then α is called soft limit point of F_B. The set of all soft limit points of F_B is denoted by F'_B. In other words, if $(F_A, \tilde{\tau})$ be a soft topological space, $F_B, F_C\tilde{\subseteq}F_A$ and $\alpha \in F_A$, then $\alpha \in F'_B$ if and only if $F_C\tilde{\cap}(F_B\tilde{\setminus}\{\alpha\}) \neq F_\Phi$ for all $F_C \in \tilde{\nu}(\alpha)$.*

Example 3.18. *[3] Let us consider the Example 3.16. If $F_B = F_{A_{13}}$ and $\alpha = (x_1, \{u_1, u_2\}) \in F_A$, then $\alpha \in F'_B$, since $F_A\tilde{\cap}(F_B\tilde{\setminus}\{\alpha\}) \neq F_\Phi$ and $F_{A_{13}}\tilde{\cap}(F_B\tilde{\setminus}\{\alpha\}) \neq F_\Phi$.*

Theorem 3.19 ([3], theorem 13)**.** *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B\tilde{\subseteq}F_A$. Then $F_B\tilde{\cup}F'_B = \overline{F_B}$.*

Theorem 3.20 ([3], theorem 15)**.** *Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B\tilde{\subseteq}F_A$. Then $F'_B\tilde{\subseteq}\overline{F_B}$.*

But from the next example we can show that the above statements are not always true.

Example 3.21. *Consider the soft topological space $(F_A, \tilde{\tau})$, where $\tilde{\tau} = \tilde{\tau}_3$ as given in Example 3.2. Let $F_B = F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$. Then $\overline{F_B} = F_{A_2}^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_2, U), (x_3, U)\}$. If $\alpha_1 = (x_1, \{u_1, u_2\})$, then $\alpha_1 \in F_A$ and $(F_B\tilde{\setminus}\{\alpha_1\}) = \{(x_2, \{u_2, u_3\})\}$. $\tilde{\nu}(\alpha_1) = \{F_A, F_{A_{13}}\}$. Since $F_A\tilde{\cap}(F_B\tilde{\setminus}\{\alpha_1\}) \neq F_\Phi$ and $F_{A_{13}}\tilde{\cap}(F_B\tilde{\setminus}\{\alpha_1\}) \neq$*

$F_\Phi, \alpha_1 \in F'_B$. *Similarly, If* $\alpha_2 = (x_2, \{u_2, u_3\})$, *then* $\alpha_2 \in F_A$ *and* $(F_B \tilde{\setminus} \{\alpha_2\}) = \{(x_1, \{u_1\})\}$. $\tilde{\nu}(\alpha_2) = \{F_A\}$. *Since* $F_A \tilde{\cap} (F_B \tilde{\setminus} \{\alpha_2\}) \neq F_\Phi, \alpha_2 \in F'_B$. *Thus* $F'_B = \{\alpha_1, \alpha_2\} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\} = F_A$. $F_B \tilde{\cup} F'_B = F_B \tilde{\cup} F_A = F_A \neq \overline{F_B}$. *Here* $F_B \tilde{\cup} F'_B \nsubseteq \overline{F_B}, \overline{F_B} \nsubseteq F_B \tilde{\cup} F'_B$ *and also* $F'_B \nsubseteq \overline{F_B}$.

Theorem 3.22 ([3], theorem 14)**.** *Let* $(F_A, \tilde{\tau})$ *be a soft topological space and* $F_B \tilde{\subseteq} F_A$. *Then* F_B *is soft closed if and only if* $F'_B \tilde{\subseteq} F_B$.

The example given below shows that the above statement is not true.

Example 3.23. *Consider the soft topological space as in Example 3.2. Let* $F_B = F^{\tilde{c}}_{A_{11}} = \{(x_1, \{u_1, u_3\}), (x_2, \{u_1, u_2\}), (x_3, U)\}$. *Then* F_B *is soft closed, since* $F_B = \overline{F_B}$. *Let* α_1, α_2 *as in Example 3.21. Then* $(F_B \tilde{\setminus} \{\alpha_1\}) = \{(x_1, \{u_3\}), (x_2, \{u_1, u_2\}), (x_3, U)\}$ *and* $(F_B \tilde{\setminus} \{\alpha_2\}) = \{(x_1, \{u_1, u_3\}), (x_2, \{u_1\}), (x_3, U)\}$. *Then* $F'_B = F_A$. *But* $F'_B \nsubseteq F_B$.

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