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Fractional Model Reference Adaptive PI$^{\lambda}$D$^{\mu}$ Control

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Abstract – This study explains the auto-adjustable fractional order the proportional–integral–derivative (PID) controller design methodology by using the technique of fractional model reference adaptive control (fractional MRAC). This method purposes a fractional order model reference adaptive PID (fractional MRAPI$^{\lambda}$D$^{\mu}$) control structure by applying the MIT rule to an adaptive control system which is including fractional order PID controller. This structure uses fractional order derivative, integral operators and fractional order reference model. Through this method, fractional order PID controller will be faster and more robustness.Respectively using integer PID and fractional order PID, the coefficients of them are determined with Zeigler-Nichols technique, simulation applications of the fractional MRAPI$^{\lambda}$D$^{\mu}$ control and the model reference adaptive PID (MRAPID) control performed. By means of the results obtained from simulation applications, fractional MRAPI$^{\lambda}$D$^{\mu}$ control is compared to integer MRAPID control in terms of performance, speed and robustness.

Keywords - *Fractional calculus, fractional PID (PI$^{\lambda}$D$^{\mu}$), Model Reference Adaptive Control, Fractional Model Reference Adaptive Control, Zeigler-Nichols, Performance and Robustness Comparison*

1. Introduction

The theory of fractional calculus has been first introduced by Leibnitz and L'Hospital in the late 1600s. Then discussions and research done by other scientists focused on this subject have increased attention more and many studies have been done in this field. These scientific studies benefits have shown the importance and of fractional calculus in mathematics, system modeling and control engineering.

The idea of using fractional calculus in feedback control systems goes in the years 1940. The last decade, the use of fractional calculus in the modeling of physical systems and in

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application areas such as controlling of some systems is increasing more and more [1-5]. Up to now, in the studies related to control system using fractional calculus, fractional order PID controllers have appeared more. One of the reason is that the performance of fractional PID controller is better than the classical PID and being the basis for some other controllers is the other reason. One of the most important problems is determination of suitable K_P, K_D, K_I, λ and μ for controllable system and different methods have been used for this purpose [6-15].

Environmental effects cause disturbing effects and changes of the system parameters. So, the traditional controllers can not give to the desired performance; therefore, a special class of control system is needed to compensate the unforeseen changes on the system caused by the input signal and system parameters [16, 17]. In such cases, the adaptive controller is one of the best alternatives.

The well known MIT rule for model reference adaptive control (MRAC) which is one of the main approaches for adaptive controller was developed by Whitaker and colleagues at 1960s, [18]. In recent years, there are many studies in the literature about to MRAC and it has become a subject taught in textbooks [19-25]. In these studies, researchers were focused on different issues, these are; control of power and energy systems, control of the quadrotor unmanned aerial vehicle (UAV or drone), control of the motor, etc. The main issue, which is common to all these studies, is the non-linearity situation (and change of the parameters).

The rule of MIT, constitute the most important structure of the MRAC, means the controller parameters are dynamically adjusted by using derivative information. Previously, researchers have studied for dynamically adjustment of coefficients integer order PI or PID by using classical integer order MRAC design [26-29]. In addition, there exist other studies related to tuning fractional order PID coefficients using high gain output feedback and artificial neural networks [30, 31] and fractional order MRAC design [32-34].

In this study, different from the other studies, dynamically adjustment of the fractional order PID coefficients using the structure of fractional order MRAC is investigated. The most important reason for the use of fractional order reference model is to fact that fractional order systems have a much better performance than the others, in designs mentioned above. The use of fractional order reference model in adaptive methods causes especially increase in the velocity of output response.

The most important contribution of this study is to offer a fractional MRAP$I^{\lambda}D^{\mu}$ control approach by using derivative and integration of the fractional order error on the feedback path obtained by applying the rule of MIT to the transfer function of the system involving fractional order PID controller. By this approach, it may be possible to obtain a more robust and fast feedback control system involving fractional order PID with coefficients determined by the Ziegler-Nichols method. The results obtained from fractional MRAP$I^{\lambda}D^{\mu}$ control are compared with the results obtained by MRAPID control.

The remaining of the paper is organized as follows. Section 2 introduction to fractional calculus. Section 3 fractional PID (P$I^{\lambda}D^{\mu}$) and tuning methods. Section 4 MRAC, Section 5 fractional MRAP$I^{\lambda}D^{\mu}$, Section 6 examples and simulations of fractional MRAP$I^{\lambda}D^{\mu}$ and comparing the simulation results with MRAPID control. Section 7 comments and conclusions.

2. Fractional Order Calculus

In the process of development of fractional order algebraic operations, a lot of theoretical studies which form the basis of the subject have been done. In this is theoretical studies the Riemann-Liouville and Grunwald-Letnikov definitions of fractional order derivative equations are the most used [35, 36].

For an α order Riemann-Liouville fractional derivative of continuous function $f(t)$ is given by,

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau , \qquad m-1 \le \alpha < m \tag{1} $$

where, the first value refers to a and generally taken as $a = 0$ and, m is an integer. Derivative degree α is between 0 and 1 ($0<\alpha<1$). a and t shows the upper and lower limits of integrals.

The α order fractional derivative definition of Grunwald-Letnikov is;

$$ {}_aD_t^{\alpha} f(t) = \frac{d^{\alpha} f(t)}{dt^{\alpha}} = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{2} $$

where, h is the step size, α is a fractional number, a is the initial value. The [.] symbol used in upper border of the total symbol shows the process of rounding to integer of $(t-a)/h$.

$$ \binom{\alpha}{j} = \frac{\alpha(\alpha-1)........(\alpha-j+1)}{j!} = \frac{\Gamma(\alpha+1)}{j!\,\Gamma(\alpha-j+1)} \tag{3} $$

where, Γ (.) is the Euler Gamma function.

3. Fractional Order PID ($PI^{\lambda}D^{\mu}$)

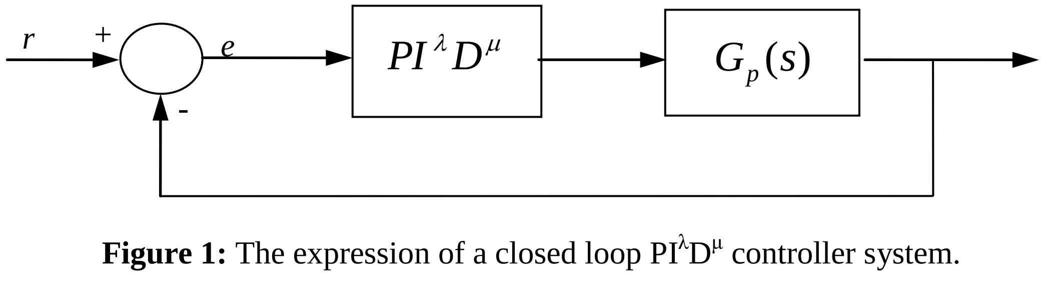

Figure 1: The expression of a closed loop $PI^{\lambda}D^{\mu}$ controller system.

The expression of fractional $PI^{\lambda}D^{\mu}$ controller used in feedback system shown in Figure 1 is given in Equation (4) and the transfer function in s-domain is given in Equation (5). K_P, K_I, and K_D are coefficients and λ, μ are derivative and integral order respectively.

$$u(t) = K_P e(t) + K_I D_t^{-\lambda} e(t) + K_D D_t^{\mu} e(t) \quad (4)$$

$$G_c(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \quad (5)$$

Because of $PI^{\lambda}D^{\mu}$ controller contains fractional integrator with the degree of λ and fractional derivative with the degree of μ it has a more flexible structure with respect to integer order PID controller. In this case, it is possible to control the system more sensitive and faster. But with respect to integer order PID controller, due to the $PI^{\lambda}D^{\mu}$ controller has more parameters (K_P, K_I, K_D, λ and μ) to be adjusted for best controlling, parameter calculation requires more processing and more time. According to the control system to find the most appropriate K_P, K_I, K_D, λ and μ, different techniques have been developed. The first of these techniques is the S- shaped unit step response (S-Shaped Response Based Tuning Rules). In this method, correspond to the unit step input, output response is taken. In this output response, the appeared delay time L and the characteristic time-constant T is determined graphically. According to the L and T, K_P, K_I, K_D, λ and μ parameters are easily calculated from the table [12,13,37]. The second technique used to adjust the $PI^{\lambda}D^{\mu}$ controller parameters based on the adjustment coefficient in the feedback system including plant. This coefficient is adjusted until the system is made oscillation and the situation of the oscillation frequency does not change over time. Then, taking advantage of this coefficient and the oscillation frequency K_P, K_I, K_D, λ and μ parameters are calculated from the table [12,13].

4. Model Reference Adaptive Control

MRAC is one of the most popular approaches of adaptive control, because of its simplicity and its high level of performance [38-40]. In MRAC the desired performance is specified by the choice of a reference model and adjustment of parameters is achieved by taking of error between the output of the plant and the model reference output into account.

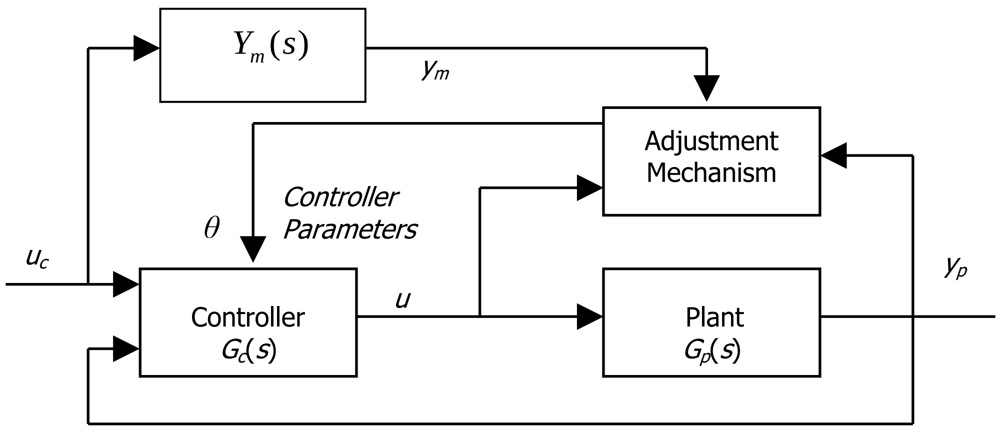

Figure 2: Block diagram of an adaptive control.

In this paper the MIT rule is used to design a Model Reference Adaptive $PI^{\lambda}D^{\mu}$. For this purpose, using the MIT rule, we determined the reference model, using an adjustment mechanism and the controller structure shown in Figure 2 is formed. In this figure $y_{p,}$ and y_m is plant output, model output respectively. Tracking error between the plant output and the reference model output as,

$$e = y_p - y_m \tag{6}$$

Then a cost function of θ, $J(\theta)$ can be formed as:

$$J(\theta) = \frac{1}{2} e^2(\theta) \tag{7}$$

In this equation, J is a function of θ and θ is the parameter that will be adapted inside the controller. Cost function, $J(\theta)$, determine how the parameters are updated. To find out how to update the parameter θ, an equation needs to be formed for change of θ. If the aim is to minimize the cost related to the error, it is reasonable to move in the direction of the the negative gradient of J. The change in J is proportional to the change in θ. Therefore the derivative of θ is equal to the negative change in J.

$$\frac{d\theta}{dt} = -\delta \frac{\partial J}{\partial \theta} \tag{8}$$

The relationship between the change in θ and the cost function is known as MIT rule. In above equation the partial derivative of error with respect to θ, determines how the parameter θ will be updated. If a controller contains several different parameters that require updating, the sensitivity derivative would need to be calculated for each of these parameters.

The output transfer function of the closed-loop system in Figure 2 may be written as,

$$Y_p(s) = \frac{b_m^p s^m + b_{m-1}^p s^{m-1} + \dots + b_1^p s + b_0^p}{a_n^p s^n + a_{n-1}^p s^{n-1} + \dots + a_1^p s + 1} \tag{9}$$

$e(t)$ is the error between the outputs and J is the cost function. For the above transfer function $e(t) = f_1(a_i^p, b_j^p)$ and $J = f_2(a_i^p, b_j^p)$ are functions of a_i^p and b_j^p. To tune the parameters of the controller $G_c(s)$, a_i^p and b_j^p must be tuned with adaptive control. From the equation of MIT rule, (6), (7) and (8), the parameters a_i^p and b_j^p can be derived as:

$$\frac{\partial a_i^p}{\partial t} = -\alpha_i^p \frac{\partial J}{\partial a_i^p} \tag{10}$$

$$\frac{\partial b_j^p}{\partial t} = -\beta_j^p \frac{\partial J}{\partial b_j^p} \tag{11}$$

From Equation (9), using fixed coefficients of the system transfer function a model transfer function can be found as in Equation (12).

$$Y_m(s) = \frac{b_m^m s^m + b_{m-1}^m s^{m-1} + ... + b_1^m s + b_0^m}{a_n^m s^n + a_{n-1}^m s^{n-1} s^{n-1} + ... + a_1^m s + 1} \tag{12}$$

5. Fractional MRAPI$^{\lambda}$D$^{\mu}$ Control

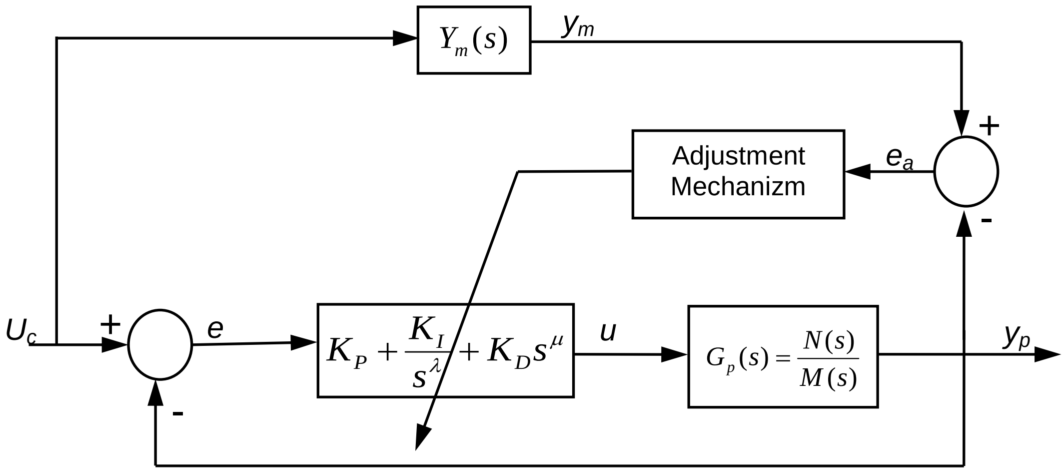

Figure 3: Block diagram of model reference adaptive PI$^{\lambda}$D$^{\mu}$.

To design a Fractional MRAPI$^{\lambda}$D$^{\mu}$ control in Figure 3, let us the controller be:

$$G_c(s) = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \tag{13}$$

The closed-loop plant output is:

$$Y_p(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} U_c(s) \tag{14}$$

$$G_c(s).G_p(s) = (K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}) \frac{N(s)}{M(s)} \tag{15}$$

$$Y_p(s) = \frac{(K_P s^{\lambda} + K_D s^{\lambda+\mu} + K_I) N(s)}{M(s) s^{\lambda} + (K_P s^{\lambda} + K_D s^{\lambda+\mu} + K_I) N(s)} = \frac{Z(s)}{D(s)} \tag{16}$$

Because of $G_c(s)$, the output $Y_p(s)$ is a fractional function.
If we generalize these equations for the parameters of PI$^{\lambda}$D$^{\mu}$, the following equations are obtained.
For K_P, K_I and K_D, using the equations (10) and (11) and using the chain rule of differentiation,

$$\frac{\partial K_P}{\partial t} = -\alpha \frac{\partial J}{\partial K_P} = -\alpha \ \frac{\partial J}{\partial e} \cdot \frac{\partial e}{\partial y_p} \cdot \frac{\partial y_p}{\partial K_P} \tag{17}$$

$$\frac{\partial K_I}{\partial t} = -\beta \frac{\partial J}{\partial K_I} = -\beta \ \frac{\partial J}{\partial e} \cdot \frac{\partial e}{\partial y_p} \cdot \frac{\partial y_p}{\partial K_I} \tag{18}$$

$$\frac{\partial K_D}{\partial t} = -\gamma \frac{\partial J}{\partial K_D} = -\gamma \ \frac{\partial J}{\partial e} \cdot \frac{\partial e}{\partial y_p} \cdot \frac{\partial y_p}{\partial K_D} \tag{19}$$

From equations of error and cost functions (6), (7),

$$\frac{\partial J}{\partial e} = e, \ \frac{\partial e}{\partial y_P} = 1 \tag{20}$$

Putting in place the results of Equation (20) at equations (17), (18) and (19),

$$K_P = -\frac{\alpha}{s} e \frac{\partial y_p}{\partial K_P} \tag{21}$$

$$K_I = -\frac{\beta}{s} e \frac{\partial y_p}{\partial K_I} \tag{22}$$

$$K_D = -\frac{\gamma}{s} e \frac{\partial y_p}{\partial K_D} \tag{23}$$

From Equation (16), taken the derivatives according to K_P, K_I and K_D,

$$\frac{\partial Y_p}{\partial K_P} = s^{\lambda} \ \frac{N(s)}{D(s)} \left(U_c - Y_p\right) \tag{24}$$

$$\frac{\partial Y_p}{\partial K_I} = s^{\lambda+\mu} \ \frac{N(s)}{D(s)} \left(U_c - Y_p\right) \tag{25}$$

$$\frac{\partial Y_p}{\partial K_D} = \frac{N(s)}{D(s)} \left(U_c - Y_p\right) \tag{26}$$

If the results of equations (24), (25), and (26) are put into places in equations (21), (22), and (23) respectively,

$$K_P = -\alpha \ s^{\lambda} \ \frac{e}{s} \ \frac{N(s)}{D(s)} \left(U_c - Y_p\right) \tag{27}$$

$$K_D = -\beta\, s^{\lambda+\mu}\, \frac{e}{s}\, \frac{N(s)}{D(s)} \left(U_c - Y_p\right) \tag{28}$$

$$K_I = -\gamma\, \frac{e}{s}\, \frac{N(s)}{D(s)} \left(U_c - Y_p\right) \tag{29}$$

Using the equations (27), (28), and (29), the designed block diagram of the control system is given in Figure 4.

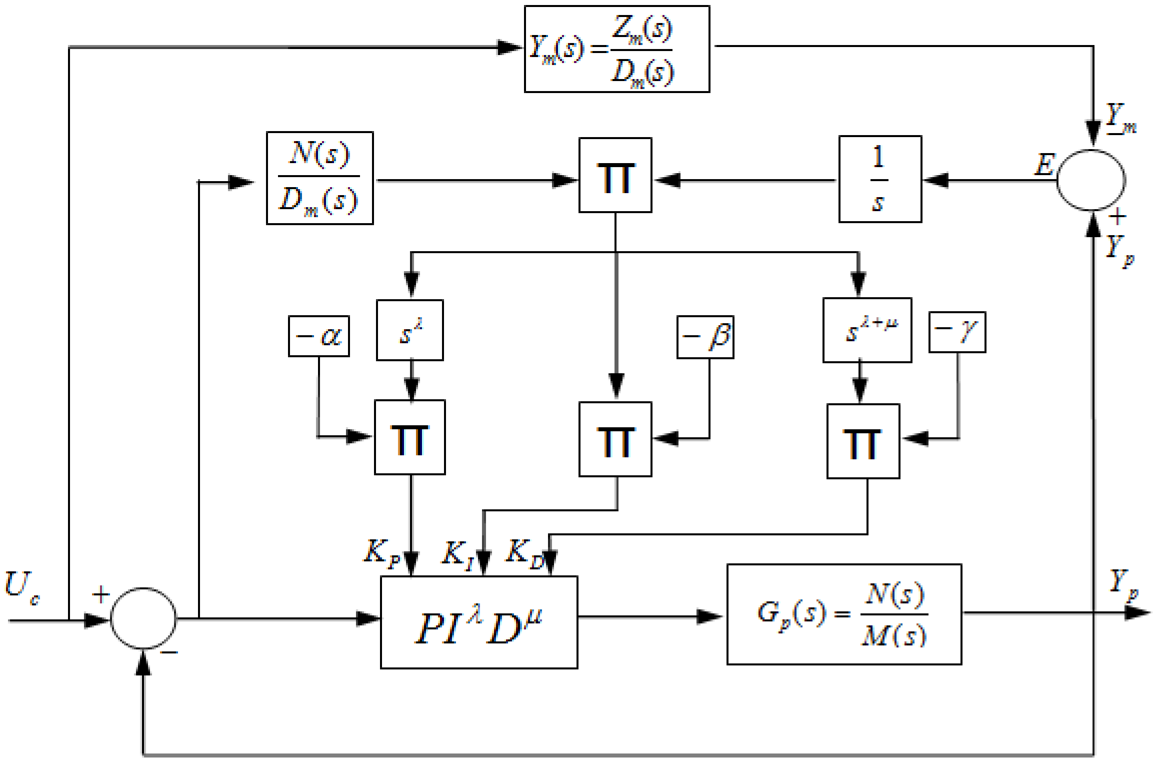

Figure 4: The control system block diagram of fractional MRACPI$^{\lambda}$D$^{\mu}$ control.

6. Examples and Simulations and Comparisons

In this section the rules from Section 4 are applied to two different plants. The fractional MRACPI$^{\lambda}$D$^{\mu}$ and MRAPID control were applied to a second-order and third-order plant, and the results were compared. The used fractional MRAPI$^{\lambda}$D$^{\mu}$ control block diagram in MATLAB SIMULINK is seen in Figure 4. For fractional derivative blocks which are used in SIMULINK diagram the CRONE approximation is used [41].

Example 1.

The transfer function of the plant was taken as in Equation (30) By applying the S-shaped response based tuning rules to this plant the PID and PI$^{\lambda}$D$^{\mu}$ transfer function equations (31), (32) were determined, respectively [12,13].

$$\frac{N(s)}{M(s)}=\frac{K}{4.32s^2+19.1801s+1} \quad (30)$$

$$G(s)=120+\frac{300}{s}+12s \quad (31)$$

$$G(s)=6.9928+\frac{12.4044}{s^{0.6}}+4.1066s^{0.7805} \quad (32)$$

Using equations of (16), (30) and (32), for MRAPI$^{\lambda}$D$^{\mu}$C the model transfer function determined as follows.

$$\frac{Z_m(s)}{D_m(s)}=\frac{6.9928s^{0.6}+4.1066s^{1.3805}+12.4044}{4.32s^{2.6}+19.1801s^{1.6}+4.1066s^{1.3805}+7.9928s^{0.6}+12.4044} \quad (33)$$

Using Oustaloup's filter and high-order approximation, the Equation (33) can be reduced into integer order transfer function such as the following equation [42].

$$\frac{Z_m(s)}{D_m(s)}=\frac{12s^2+120s+300}{4.32s^3+31.1801s^2+121s+300} \quad (34)$$

For PI$^{\lambda}$D$^{\mu}$ controller at Equation (32) and system transfer function at Equation (30), unit step response plotted in Figure 5, and K taken as 1.

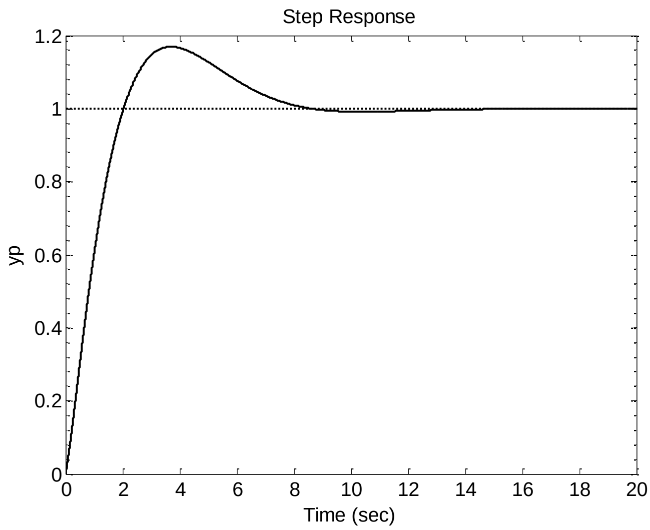

Figure 5: The unit step response of control system for PI$^{\lambda}$D$^{\mu}$ controller.

For classic MRAPID control from equations (27), (28), and (29) after taking $\lambda=\mu=1$, using the equations (30) and (31) the block diagram in Figure 4 was created in MATLAB Simulink and by applying a unit step input the system response was taken (Figure 6). To get the best and fastest answer from the system for MRAPID control, the parameters were selected as $\alpha = 2000$, $\beta = 15$ and $\gamma = 1000$ respectively. Similarly, for fractional MRAPI$^{\lambda}$D$^{\mu}$ control according to the values of equations (30) and (32) , using the equations (27), (28), and (29), the block diagram at Figure 4 was created in MATLAB Simulink and for fractional MRAPI$^{\lambda}$D$^{\mu}$ control the unit step response output was taken (Figure 6). To get the best performance the parameters were selected as $\alpha = 100$, $\beta = 500$ and $\gamma = 600$. In both cases, the plant transfer function gain was taken as $K = 1$.

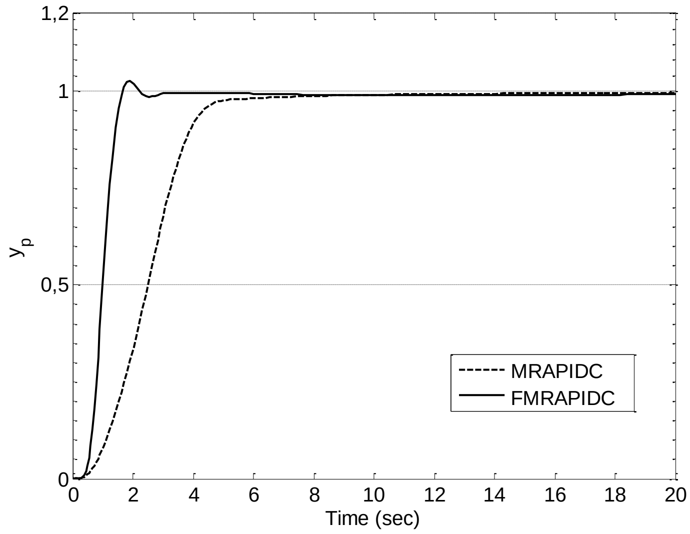

Figure 6: For fractional MRAPI$^{\lambda}$D$^{\mu}$ and MRAPID control the unit step responses.

For comparing the robustness of fractional MRAPI$^{\lambda}$D$^{\mu}$ and MRAPID control, the gain of the plant was taken as K = 4, 2, 1, 1/2, 1/4, 1/8, 1/16, 1/32. For both controllers the output responses of the unit step input are shown in Figure 7 and Figure 8.

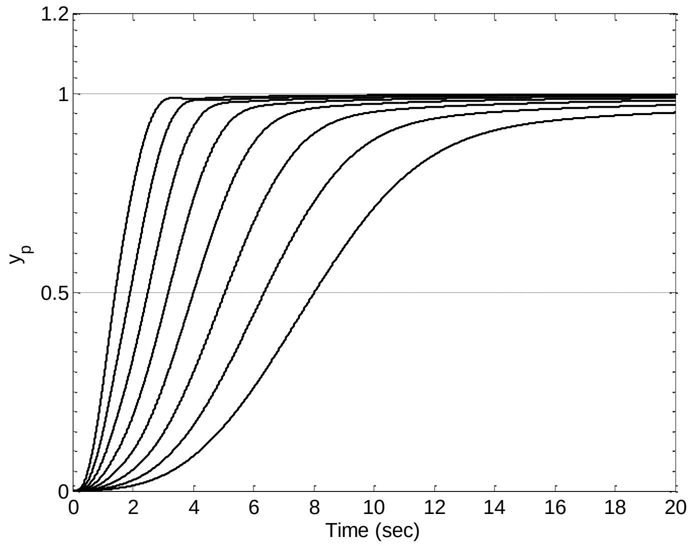

Figure 7: For different K values, unit step responses of the MRAPID control.

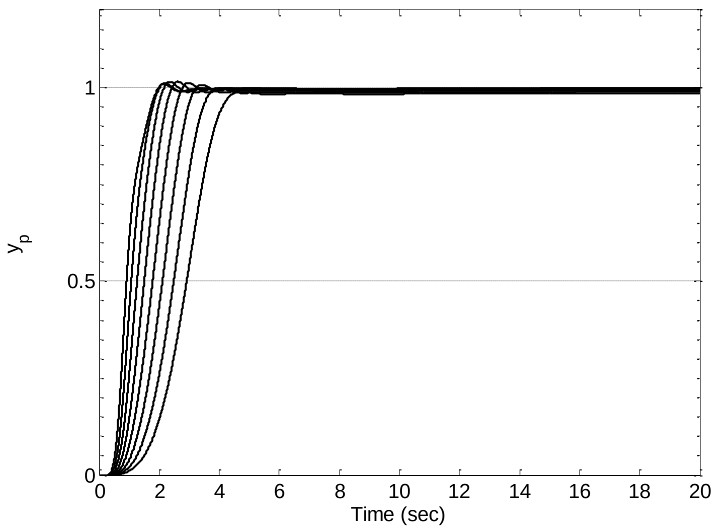

Figure 8: For different K values, unit step responses of the fractional MRAPI$^{\lambda}$D$^{\mu}$ control.

Example 2.

The transfer function of the plant was taken as in Equation (35). By applying "critical gain-based tuning rules" to this plant the transfer function of PID and PI$^{\lambda}$D$^{\mu}$ (36, 37) were determined [12, 13].

$$\frac{N(s)}{M(s)} = \frac{K}{s^3 + 2.539s^2 + 62.15s} \tag{35}$$

$$G(s) = 94.68 + \frac{237.591}{s} + 9.4325s \tag{36}$$

$$G(s) = 0.8271 + \frac{14.3683}{s^{0.5588}} - 1.6866s^{1.2328} \tag{37}$$

$$\frac{Z_m(s)}{D_m(s)} = \frac{9.4325s^2 + 94.68s + 237.591}{s^4 + 2.539s^3 + 71.5825s^2 + 94.68s + 237.591} \tag{38}$$

$$\frac{Z_m(s)}{D_m(s)} = \frac{-1.6866s^{1.7916} + 0.8271s^{0.5588} + 14.3683}{s^{3.5588} + 2.539s^{2.5588} - 1.6866s^{1.7916} + 62.15s^{1.5588} + 0.8271s^{0.5588} + 14.3683} \tag{39}$$

As in Example 1, the same procedures were carried out for this example respectively. Figure 9 shows the unit step response of PI$^{\lambda}$D$^{\mu}$ controller for the plant gain K = 1.

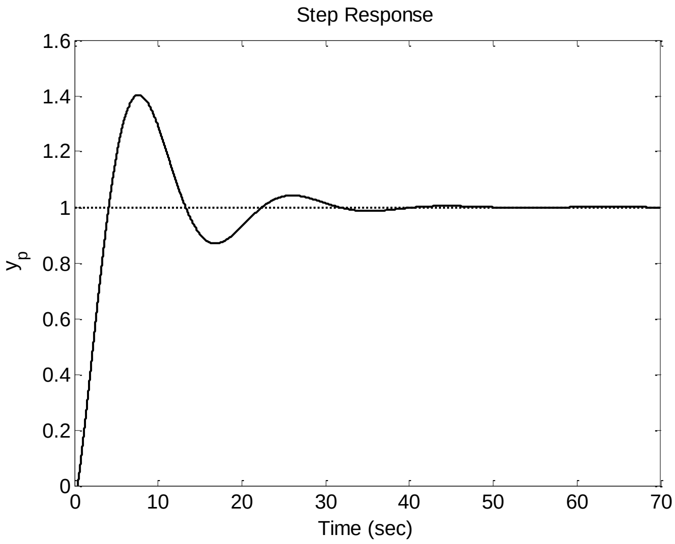

Figure 9: For $PI^{\lambda}D^{\mu}$ controller, the unit step response of control system.

By taking the parameters as α=10, β=200 ve γ=-1 for MRAPID control and α =10, β =390 ve γ =10 for fractional $MRAPI^{\lambda}D^{\mu}$ control, in order to compare the outputs of both control systems Figure 10 was drawn. Here the system gain was taken as K = 1.

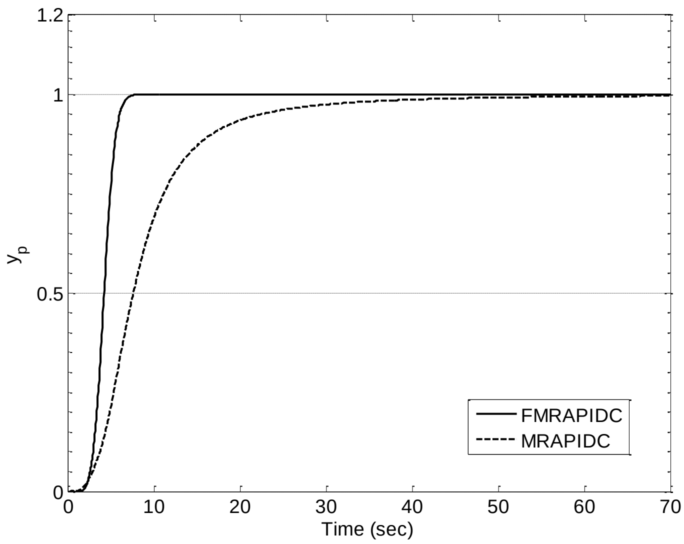

Figure 10: For fractional $MRAPI^{\lambda}D^{\mu}$ and MRAPID control the unit step responses.

To compare the stability of both control systems, by taking K as K = 4, 2, 1, 1/2, 1/4, 1/8, 1/16, 1/32 respectively, for unit step input the curves shown in Figure 11 were obtained for MRAPID control and the curves shown in Figure 12 was obtained for fractional $MRAPI^{\lambda}D^{\mu}$ control as output responses.

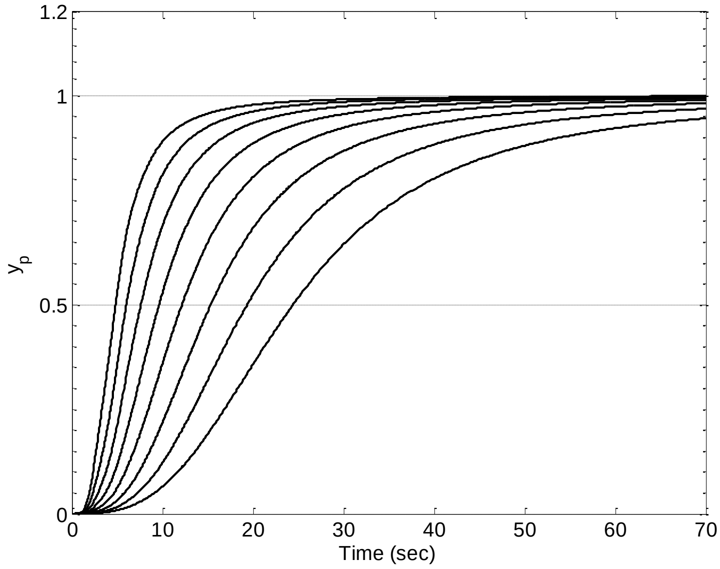

Figure 11: For different *K* values, unit step responses of the MRAPID control.

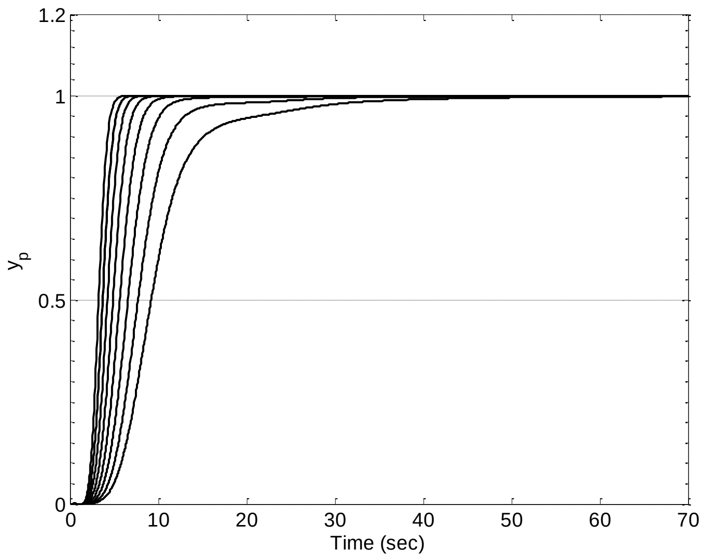

Figure 12: For different *K* values, unit step responses of the fractional MRAPI$^{\lambda}$D$^{\mu}$ control.

7. Comments and Conclusions

The idea of this paper is to enhance a controller from the combination of the PI$^{\lambda}$D$^{\mu}$ controller and the fractional MRAC. The coefficients of PID or PI$^{\lambda}$D$^{\mu}$ controller were set on-line by using the MIT rule. We try to design and testing the performance and robustness of fractional MRAPI$^{\lambda}$D$^{\mu}$ and MRAPID control. The performance of fractional MRAPI$^{\lambda}$D$^{\mu}$ and MRAPID control were tested with two different plants and models.

To see the performance of fractional MRAPI$^{\lambda}$D$^{\mu}$ control, the responses of it were compared with the responses of MRAPID control for the same plants. As seen in the above simulation results, unit step responses of systems were obtained for both control methods, but the response of fractional MRAPI$^{\lambda}$D$^{\mu}$ control is faster than MRAPID control. PI$^{\lambda}$D$^{\mu}$ controller parameters determined by the method of Zeigler-Nichols were used in fractional

MRAPI$^{\lambda}$D$^{\mu}$ control and with this new method the designed controller brought to a faster and more robust position. Although the results obtained from the fractional MRAPI$^{\lambda}$D$^{\mu}$ control were the desired results, in future studies the degree of integrator λ and degree of differentiation μ will be adjusted in the same way by using the MIT rule and by using fractional order plant, the analysis of the results can be done in terms of performance.

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