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Research Article

Natural convection between hot and cold cylinders in enclosed space filled with copper-water nanofluid

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ABSTRACT

The present contribution is a numerical investigation of the natural convection between two circular objects, where the first one is hot and the second is cold. Both cylinders are placed in an enclosed adiabatic cavity filled with copper-water nanofluid. The cylinders are arranged horizontally in the middle of cavity height. The main target of this paper it to examine the effects of geometrical configurations and the thermo-physical characteristics of nanofluid on the fluid motion and heat transfer rates. The study is conducted for various parameters: Rayleigh number ($Ra = 10^3$ to 10^5), nanoparticle volume fraction ($\varphi = 0$ to 10%), the diameter of cylinders ($d/H = 0.2$ to 0.4), and the gap spacing between cylinder ($S/H = 0.25$ to 0.7). Also, two cavity shapes are studied (square and circular). The average Nusselt number of the cylinders is computed and plotted as function of the studied parameters. It was concluded that the cavity shape and the particle volume fraction have a negligible effect on the heat transfer rate, whereas the distance between cylinders and the cylinder diameter have a remarkable effect on the flow patterns and convective heat transfer.

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INTRODUCTION

The buoyancy-driven flow is an inevitable mechanism that is used to transfer the thermal energy between mul-tiple bodies of different temperatures when the fluid is not accelerated by external devices, such pump. This natural mode has many technical embodiments. Indeed, it can be encountered in heat exchangers, cooling towers, cooling the minuscule devices of electronic systems, metallurgies, marine engineering applications, solar collectors, air conditioning systems and so on [1-4]. Therefore, it is necessary to clarify the thermal behavior of such frameworks by determining their relative controlling parameters in order to boost the heat transfer rate of these systems. About this subject, several methods have been elaborated to enhance the heat transfer. Overall, the enhancement of the natural convection mode was achieved by changing the shape of

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the heated objects [5], setting the operating mode [6] and using new kinds of fluids such as non-Newtonian fluid and nanofluids or combining both of them [7-11].

The working fluid in this study is a nanofluid. Therefore, the exact defin tion of these fluids is the improvement of the thermo-physical characteristics of fluid by adding some ultra-fi e solid particles called nanoparticles [12]. Consequently, the nanofluid is a suspension of solid particles of nano-scale in base-fluids of Newtonian [13] or non-Newtonian [14] behaviors. In addition, it was concluded that the presence of nanoparticles in the base-fluid does not provoke any sedimentation, which results an augmentation of pressure drop in the fluid stream [13]. Generally, the nanofluids are characterized by describing quantitatively the thermo-physical characteristics of the base-fluid and nanoparticles. There is also another parameter called the nanoparticle volume fraction.It gives the concentration of the solid particles in the base-fluid.

The recent studies witness extensive researches on enhancement in the convective heat transfer rates by using nanofluids. Indeed, the nanofluids were studied for cases of natural, forced and mixed convections. For the forced convection, therein are some recent examples [15-18]. For the cases of natural and mixed convections, Abu-Nada et al. [19] studied numerically the effects nanofluids on the laminar natural convection in concentric annuli. The work examined the roles of the following parameters: Rayleigh number (from 10^3 to 10^5), the nature of studied solid particles (Ag, Cu, TiO_2 and Al_2O_3), the volume fraction, and the geometrical form of the studied domain. The results showed that the utilization of nanofluids does not enhance the heat transfer rates for all studied cases. Hekmat and Ziarati [20] performed a numerical study on the combined effects of nanofluids and magnetic fi ld on mixed convection in annular space of circular section. It was confi med that the simultaneous effects of nanoparticles volume fraction with the external magnetic fi ld have a significant impact on the fl w motion and thermal patterns. For Ag-water nanofluid, Boutra et al. [21] investigated the natural convection in three-dimensional enclosure of cubical form. The results explained the impacts of volume fraction, Rayleigh number and cube inclination angle on the average Nusselt number. It was noticed that the Ag solid nanoparticles increase the average Nusselt number. Tayebi et al. [22] examined the free convection between two horizontal cylinders of elliptical cross-sectional form. The inner cylinder was heated at fi ed temperature, and the outer enclosure was maintained cold at fi ed temperature. The cu-water nanofluid with different values of volume fraction was used. The Rayleigh number was considered for controlling the strength of induced thermal buoyancy. The results indicated that the copper-water nanofluid enhances the heat transfer of annular surfaces and the rate of heat transfer depends positively on the increment of particle volume fraction. Selimefendigil and Chamkha [23] simulated the mixed convection of CuO-water nanofluid in a vented cavity. Inside the cavity, a rotating circular cylinder was arranged in the center. The enclosure bottom contained some corrugations. The computational domain was considered to be exposed to an external magnetic fi ld. The fi dings of this work showed that the nonfluid behaves like base-fluid. Sheremet and Pop [24] performed a numerical study on the buoyancy-driven fl w in a circular annular space. The domain was considered porous, and it was filled with copper-water nanofluid. As the previous mentioned works, the research looked for the roles of thermo-physical characteristics of porous and nanofluid mediums on heat transfer rate between the annular surfaces. The obtained results revealed that the nanoparticles of nanofluide improve the heat transfer for the well-selected characteristics of the porous domain. For Copper-water nanofluid, Arbabanand Salimpour[25] studied the effects of Rayleigh number, volume fraction, and geometrical configur tion of radial fi s on the laminar free convection between two concentric cylinders. Their results showed that the increase in the volume fraction of solid particles increases the Nusselt number. Shahi et al.[26] studied numerically the copper-water nanofluid fl w through a vented cavity. The mode of heat transfer was assumed to be mixed. Therefore, the following control parameters were studied: volume fraction, Richardson number, and Reynolds number. They found that the heat transfer rate enhances for specifi temperatures.

The annular space with multiple cylinders is an important geometry that can refl ct its existences in many engineering applications such as heat exchangers, devices of food treatment, refi eries, refrigeration systems, condensates and so on. Therefore, several researches have been conducted for studying the free convection heat transfer within these geometries. The challenging point of the seworks is looking for the optimal parameters that improve the heat transfer rate while remaining the fl w more stable [27-29]. It was observed that the fl w motions and the heat transfer rate are significantly influenced by the thermo-physical characteristics of a fluid (such as density, thermal conductivity, specifi heat transfer, etc.) and geometrical parameters (such as form of obstacles and cavity, the position of cylinders, and others).

It can be noticed that there is no research about the roles of nanofluid on the natural convection heat transfer between two confi ed cylinders, where one cylinder is hot and the second is cold. Therefore, this paper presents a numerical study of free convection heat transfer between hot and cold obstacles of circular cross-section. Both cylinders are placed in a single cavity of adiabatic walls. The cavity is assumed to be filled with copper-water nanofluid. The hydrodynamic and thermal behaviors of copper-water nanofluid are examined for several values of nanoparticle concentration, cylinders diameter, distance between cylinders, and thermal buoyancy strength. Furthermore, the cross-sectional form of the cavity is studied, the common circular and square forms are chosen.

PHYSICAL MODEL AND MATHEMATICAL FORMULATION

The sketch of the present physical model is schematically presented in Fig. 1. It consists of two circular cylinders confi ed horizontally in an adiabatic enclosure. The cylinders are separated by the distance S. The right cylinder is maintained at fi ed temperature (T_h), which is higher than that for the left cylinder (T_c). Both cylinders have the same diameter (d), which is related to the cavity diameter (H). The following values of the blockage ratio are considered: $B = d/H = 0.2$, 0.3 and 0.4. The cavity is filled with copper-water nanofluid. The study inspects the roles of cylinder diameter, distance between cylinders, form of the cavity (circular (Fig. 1a) and square (Fig. 2b)), buoyancy strength and nanoparticle concentration on the fl w patterns and the irinfluence on the heat transfer rate. The considered problem is two-dimensional, laminar, and steady. The nanofluid density is only the single propriety that is assumed to be dependent on the temperature and it is evaluated according to the Boussinesq approach.

The conserving equations of mass, momentum, and energy in dimensional states are given as follows:

$$\left(\frac{\partial u}{\partial x}\right)+\left(\frac{\partial v}{\partial y}\right)=0 \tag{1}$$

$$u\left(\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)=-\frac{1}{\rho_{nf}}\left(\frac{\partial p}{\partial x}\right)+\upsilon_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\left(\frac{\partial v}{\partial x}\right)+v\left(\frac{\partial v}{\partial y}\right)=-\frac{1}{\rho_{nf}}\left(\frac{\partial p}{\partial y}\right)+\upsilon_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\,\beta_{nf}\left(T-T_c\right) \tag{3}$$

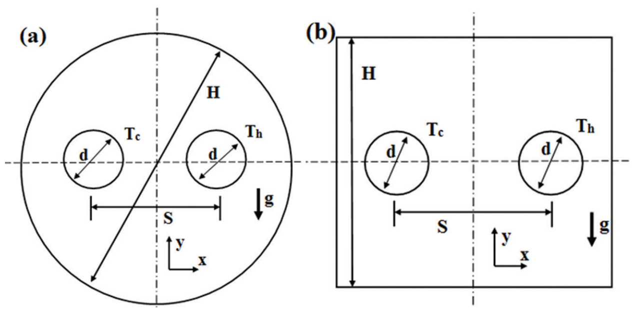

Figure 1. schematic of physical model, (a) circular enclosure, (b) square enclosure.

$$u\left(\frac{\partial T}{\partial x}\right)+v\left(\frac{\partial T}{\partial y}\right)=\ _{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where u, v, p and T are the velocity along the x-direction, velocity along y-direction, pressure and temperature respectively. Also, α, $\rho$and ν refer to the thermal diffusivity, density and kinematic viscosity respectively. It should be mentioned that the last term of the equation (3) is the thermal buoyancy force.

The governing equations (1), (2), (3) and (4) are converted in dimensionless form after considering the following dimensionless parameters:

$$X=\frac{x}{d},Y=\frac{y}{d},U=\frac{ud}{\alpha_f},V=\frac{vd}{\alpha_f},P=\frac{pd^2}{\rho_f{\alpha_f}^2},\theta=\frac{\left(T-T_c\right)}{\left(T_h-T_c\right)},$$
$$Ra=\frac{g\beta_f\left(T_h-T_c\right)d^3}{\upsilon_f\alpha_f},Pr=\frac{\upsilon_f}{\alpha_f} \tag{5}$$

The boundary conditions of dimensionless form are imposed on the extremities of the studied geometry as follows:

- For the right cylinder: $U = 0$, $V = 0$, $\theta = 1$
- For the left cylinder, $U = 0$, $V = 0$, $\theta = 0$
- For the cavity walls, $U = 0$, $V = 0$, $\frac{\partial\theta}{\partial n}=0$.

The thermo-physical characteristics (density, thermal conductivity, kinematic viscosity, etc.) of nanofluid are approximated by the thermo-physical characteristics of the base fluid (water) containing ultra-fi e particles of the copper. The spherical form of nanoparticlesis considered. So, the thermo-characteristics of nanofluid are:

The density:

$$\rho_{nf}=\left(1-\varphi\right)\rho_f+\varphi\rho_s \tag{6}$$

The viscosity is expressed according to [30]:

$$\mu_{nf}=\frac{\mu_f}{\left(1-\varphi\right)^{2.5}} \tag{7}$$

The heat capacity:

$$\left(\rho c_p\right)_{nf}=\left(1-\varphi\right)\left(\rho c_p\right)_f+\varphi\left(\rho c_p\right)_s \tag{8}$$

Thermal expansion coeffici t:

$$\left(\rho\beta\right)_{nf}=\left(1-\varphi\right)\left(\rho\beta\right)_f+\varphi\left(\rho\beta\right)_s \tag{9}$$

Thermal conductivity is given according to the approximation of [31]:

$$\frac{k_{nf}}{k_f}=\left[\frac{\left(k_s-2k_f\right)-2\varphi\left(k_f-k_s\right)}{\left(k_s-2k_f\right)-\varphi\left(k_f-k_s\right)}\right] \tag{10}$$

Thermal diffusivity:

$$(\alpha)_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (11)$$

where f, $s$and nf refer to subscripts of base fluid, nanoparticlesand nanofluid, respectively. The proprieties of base fluid (water) and solid particles of copper are represented in Table 1.

NUMERICAL METHODOLOGY

Solution

The governing equations (1) to (4) are discretized by using the software ANSYS CFX. Th s powerful tool uses the fin te volume method. The present package converts the above equationsfrom the partial differential form to a system of discrete algebraic. The convective term of the system is solved by using the high-resolution scheme, and the SIMPLEC algorithm is used to perform the pressure-velocity coupling.The obtained results are considered converged only when the relative error is less than 10^{-8} for the continuity and momentum equations, and 10^{-6} for the energy equation.

It is very important to mention that the heat transfer rate is evaluated according to the Nusselt number. Therefore, both local and average Nusselt numbers of a hot surface are expressed as:

$$Nu_l = -\frac{k_{nf}}{k_f}\frac{\partial\theta}{\partial n}\bigg|_{wall}, \quad Nu_l = \frac{1}{s}\int_0^s Nu_l ds \quad (12)$$

where n and s are normal vectors to a line and surface respectively.

Mesh Independency

Unstructured mesh of non-uniform distribution is considered for grid generation of all geometrical cases. The grid elements have a triangular form, and they are concentrated in vicinity of cylinders,where the thermal and hydrodynamic layers are thin(Fig. 2).The mesh independency test

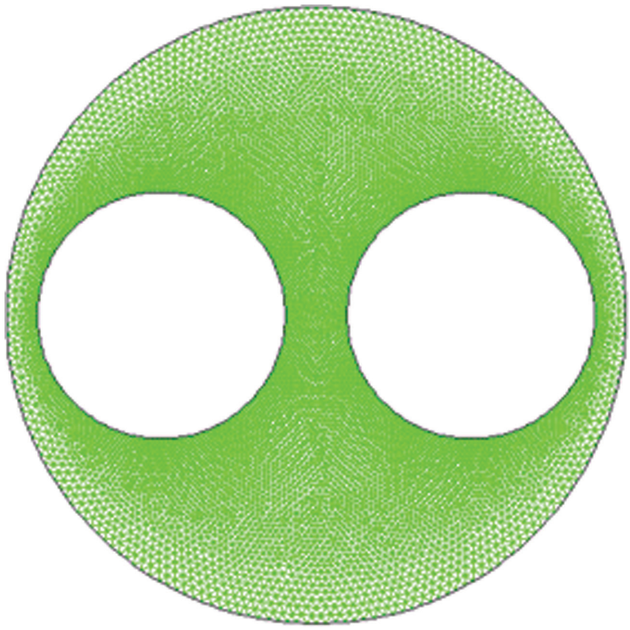

Figure 2. Typical grid for numerical domain.

Table 1. Thermo-physical characteristics of nanoparticles and base fluid

	Pr	c_p(J/kg·K)	ρ(kg/m³)	k(W/m·K)	$\alpha\times10^{-5}$ (m²/s)	$B\times10^{-5}$ (K⁻¹)
Nanoparticles (copper, Cu)	-	385	8933	401	11.7	1.67
Base fluid (water)	6.2	4179	997.1	0.613	-	21

Table 2. Grid independency test

For square cavity				For circular cavity		
B = d/H	**Elements**	**Nu**	**Error %**	**Elements**	**Nu**	**Error %**
	29682	2.75414	0.66	13440	2.76074	0.60
0.2	66714	2.73578	0.82	28246	2.74417	0.22
	155940	2.71343	-	48922	2.7381	-
	44932	2.75314	0.40	14000	2.7574	0.30
0.3	79544	2.74188	0.6	24866	2.74897	0.21
	179326	2.72556	-	56166	2.74314	-
	57380	2.23134	1.12	14794	2.06282	0.67
0.4	88750	2.20645	0.22	24866	2.04897	0.31
	127568	2.2015	-	33256	2.04266	-

is fulfilled in such way that the mesh elements are progressively increased until the variation of average Nusselt number on the right cylinder (heated cylinder) becomes negligible(Table 2).The values of average Nusselt number are calculated for different numbers of grid elements and blockage ratio d/H at $Ra = 10^4$ and $\varphi = 0.1$. The variation of average Nusselt number with grid sizes is seen to be negligible, and for each case of blockage ratio, the second grid is seen to be appropriate for the present investigation.

Validation Test

The prior results of experimental and numerical approaches are used to ensure the accuracy of the current software and numerical boundary conditions. Fig. 3 shows comparative results of average Nusselt number of a natural convection between two concentric circular cylinders. The present results are plotted simultaneously with the experimental results of Kuehn and Goldstein [32] and the numerical of Matin and Khan [7]. The investigation was performed for $Pr = 0.7$ and volume fraction of $\varphi = 0$.From Fig. 3, a good agreement is observed between all results. The results of the second validation test are plotted in Fig.4. Indeed, Fig. 4 shows the distribution of dimensionless temperature in gap spacing of two concentric cylinders in horizontal arrangement. The present results are compared with those of Abu-Nada et al. [19] and Matin & Khan [7]. An excellent agreement is also observed between the results.

RESULTS AND DISCUSSION

The present work is a numerical simulation of copper-water nanofluid in a 2D adiabatic cavity of two different shapes (circular and square), where two identical cylinders of circular cross-section are horizontally placed. One cylinder is hot with constant uniform temperature, while the second one is cold with constant uniform temperature. The main purpose of this research is to studythe natural convective mode of heat transfer between those cylinders. In addition, the research focuses on the interaction between the thermal buoyancy and thermo-physical characteristics of Cu-water nanofluid coupled with the geometrical configur tions of the studied domain. The computational fi dings of this investigation may be useful for the optimization and practical development of many thermal engineering applications. The study is conducted for various parameters such as Rayleigh number (between 10^3 and 10^5), volume fraction of solid nanoparticles (between 0 and 0.1), the diameter of cylinders which is controlled by the ratio d/H (between 0.2 and 0.4), and the ratio of gap space between confi ed cylinders S/H (from 0.25 to 0.7).The effects of these parameters on the heat transfer characteristics and hydrodynamics are represented.

The Effect of Enclosure Shape and Rayleigh Number

For $d/H = 0.2$ and $S/H = 0.5$, Figs. 5 and 6 show the effects of enclosure shape and Rayleigh number on the representative isotherms and streamlines within the 2D enclosure, respectively. The volume fraction of solid nanoparticles is taken as $\varphi = 0$. Th ee values of Rayleigh number ($Ra = 10^3$, 10^4 and 10^5) and two shapes of cavity (circular and square) are examined. The Ra value denotes the buoyancy strength. From Fig. 5, it is observed that the isotherms that are around the hot cylinder of the values limited between 0.5 and 1advance towards the upper side. In contrast to the isotherms that are around the cold cylinder, they move towards the bottom. Furthermore, an increase in the isotherms intensity is observed with the rise in Ra. Th s is due to the buoyancy force, i.e. the

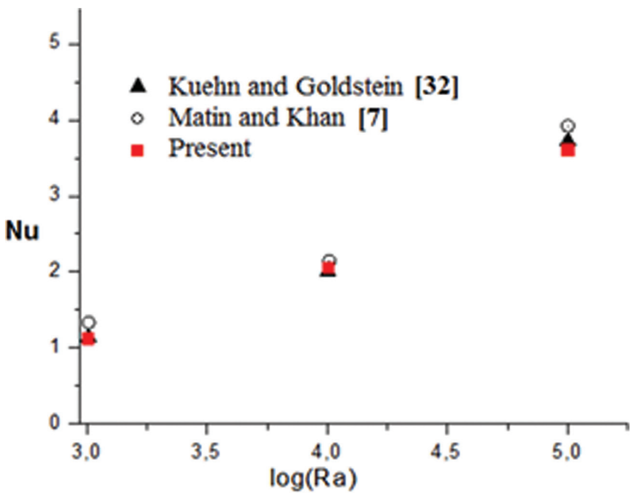

Figure 3. Comparison results of average Nusselt number in concentric annuli for $Pr = 0.7$.

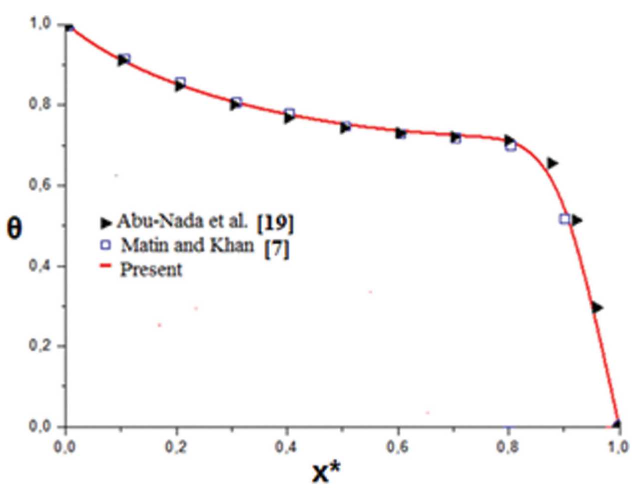

Figure 4. Comparison results of dimensionless temperature in concentric annuli.

heat causes a decrease in density of fluid layers that are around the hot cylinder. As a result, the fluid particles shift upwardly. On the other hand, the same behavior happens inversely around the cold cylinder. It is worth noting that when the isotherms are crowded around the cylinder, the thermal gradient and *Nu* becomes more signifi ant. It is also noticed that the thermal gradient increases with the rise of *Ra*. Also, the thermal gradient is considerable at the bottom of hot cylinder and at the top of cold cylinder. It is also observed that the thermal plume over the hot cylinder shifts towards the cold cylinder, whereas the plume under the cold cylinder shifts towards the hot cylinder. However, no effect of the cavity shape on the isotherms around the cylinders was observed.

The fluid movement inside the cavity at various conditions is analyzed through the streamlines of Fig. 6. For *Ra*= 10^3, the opposite effect of thermal buoyancy in the vicinity of cylinders creates a recirculation fl w in the cavity, and the main center of rotating fl w is located between the cylinders. For the same value of *Ra*, the corners resulting from the square cross-section of the cavity createan earlier counter-rotating zone above the hot cylinder and below the cold cylinder. However, the complexity of fl w patterns is seen to be increased with the gradual increase in *Ra*. Generally, the increased *Ra* (i.e. increased buoyancy force) reduces the main vortex size and increases the extra vortices size. Also, the rise of *Ra* deforms the central vortex of the main counter-rotating fl w from a circular to a stretched form. It is clear that the effect of cavity shape on the streamlines is almost negligible, especially in the internal region between the cylinders.

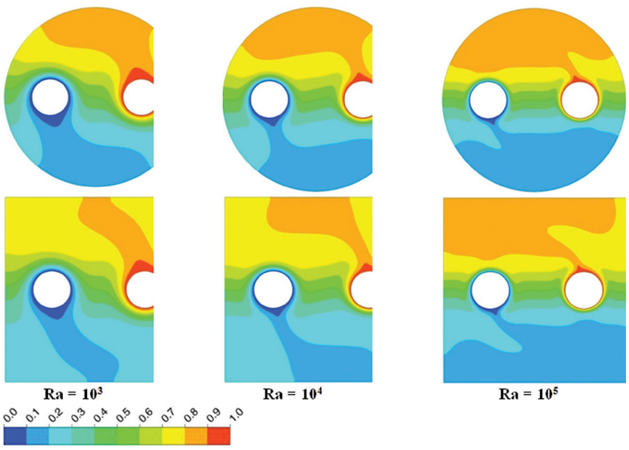

Figure 5. Effect of the enclosure shape and Rayleigh number on isotherms for $d/H = 0.2$, $S/H = 0.5$ and $\varphi = 0$.

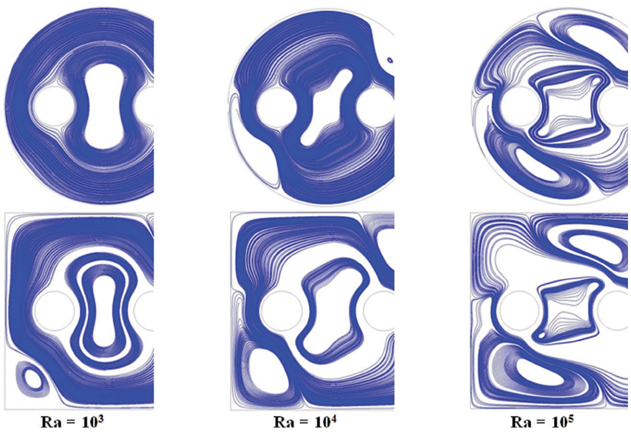

Figure 6. Effect of the enclosure shape and Rayleigh number on streamlines for $d/H = 0.2$, $S/H = 0.5$, and $\varphi = 0$.

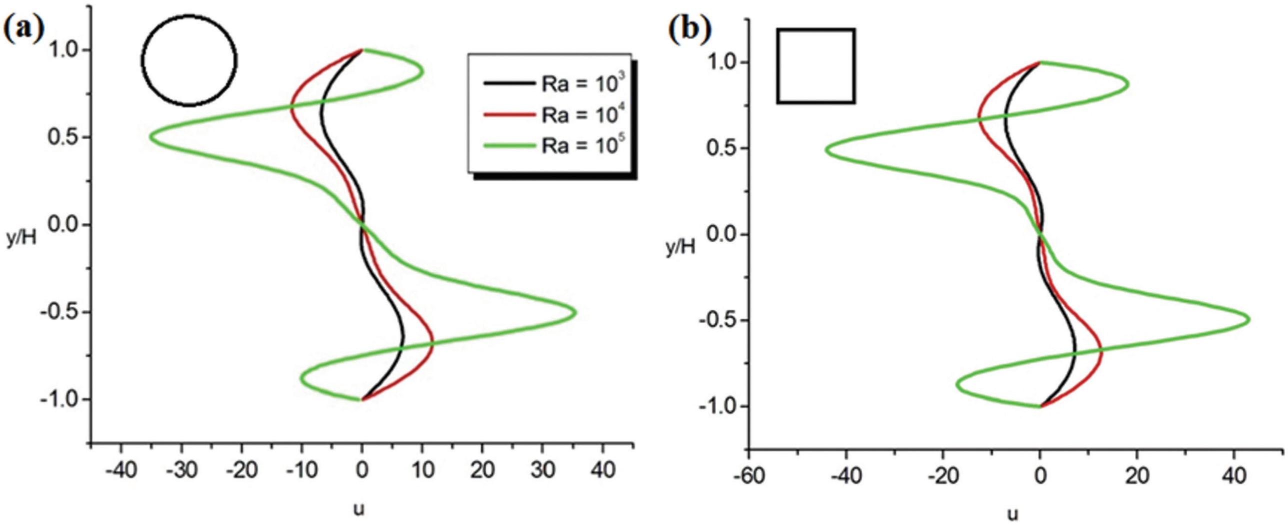

Figure 7. Influence of *Ra* and cavity shape on the dimensionless velocity profile for $d/H = 0.2$, $S/H = 0.5$ and $\varphi = 0$.

Fig. 7 aims to preview the effect of *Ra* and cavity shape on *u*-velocity along the line $x = 0$. It is clear that the effect of cavity shape is negligible along this line. However, the local value of *u*-velocity is increased with the rise of Rayleigh number. Also, all values of *u* are similar with respect to the line $y = 0$. It can be concluded that the increased buoyancy force augments the fl w velocity [33].

Fig. 8 presents the obtained results of average *Nu* number for the hot cylinder at various *Ra* and cavity shapes. These are the geometrical parameters of the domain $d/H = 0.2$, $S/H = 0.5$, and $\varphi = 0$. It is clear that at fi ed *Ra*, the *Nu* values are similar for both shapes of the cavity. However, the average values of *Nu* are seen to be increased with *Ra*. As it was seen earlier, the augmentation of *Ra* increases the fl w velocity, resulting thus in anacceleration of the heat transfer rate from the hot cylinder surface to the fl w.

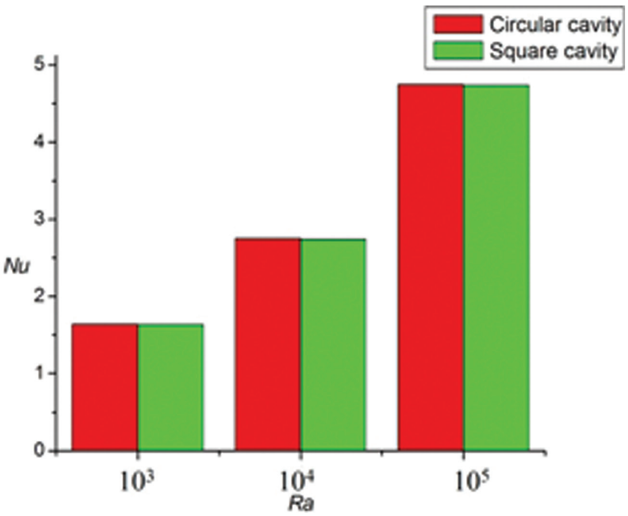

Figure 8. Influence of the cavity shape and *Ra* number on the average *Nu*, for $d/H = 0.2$, $S/H = 0.5$ and $\varphi = 0$.

The Effect of Cylinder Diameter

Fig. 9 shows the influence of cylinders diameter on the isotherm distributions for circular and square cavities, at $Ra = 10^4$, $S/H = 0.5$ and $\varphi = 0$. It is clear that the increase in the ratio d/H reduces the gap distances between the cylinders, as well as the spaces between the cylinders and the cavity. Therefore, the fluid movement becomes more difficult and the isotherm distributions are gradually diminished. Also, the increased d/H decreases the thermal gradient around the cylinders, especially in the space between the cylinder and cavity. Basing on this, it may be deduced that the rise of the cylinders diameter affects negatively the heat transfer rate. Moreover, the effect of cylinders diameter on the isotherms is similar for both considered shapes of cavity.

Fig. 10 refl cts the fluid movement within the cavity under the same studied parameters $Ra = 10^4$, $S/H = 0.5$, and $\varphi = 0$. It is observed that the increase in the cylinder size within the cavity affects the main vortex, as well as the extra small zones of counter-rotating fl ws, i.e. the presence of hot surface of the cylinder under the extra vortex redirects the fl w towards the vortex and according the suppression of the counter-rotating zone. The same thing happens in the cold region of the cavity where the buoyancy force interferes in the opposite direction. Furthermore, the reduction of the gap space between the cold and hot cylinders divides the central vortex into two small loops.

Fig. 11 illustrates the variation of average *Nu* at the hot cylinder vs. the ratio d/H and the cavity form at $Ra = 10^4$, $S/H = 0.5$, and $\varphi = 0$. The effect of cavity shape on *Nu* is almost similar for all studied values of d/H. Furthermore, for both studied shapes of cavity, an increase in the ratio d/H decreases the *Nu* number. For example, the increasing d/H from the value 0.2 to 0.4 yields a decrease in *Nu* by 25.45% for the circular form of cavity and by 7.27% for the square form.

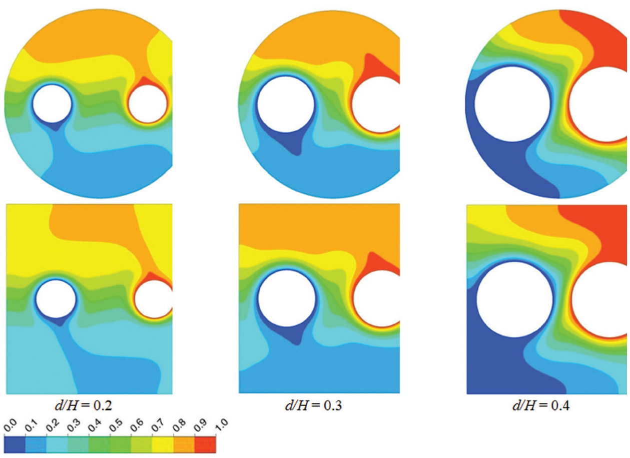

Figure 9. Effect of the cylindersdiameter on isotherms for $Ra = 10^4$, $S/H = 0.5$, and $\varphi = 0$.

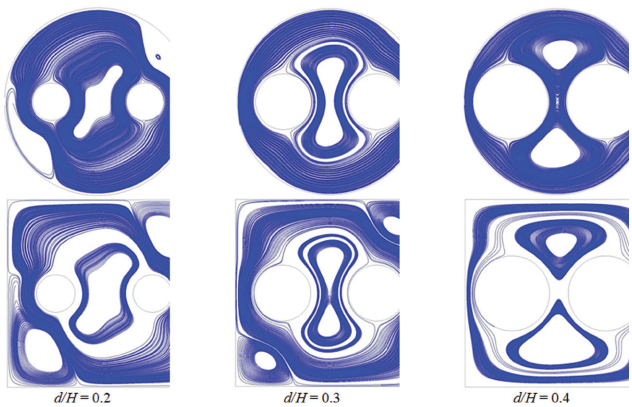

Figure 10. Effect of the cylinders diameter on streamlines for $Ra = 10^4$, $S/H = 0.5$, and $\varphi = 0$.

Fig. 12 shows the effect of the ratio *d/H* on the contours of *v*-velocity. It is clear that *v*-velocityhas the similar magnitude around the cylinders with two opposite signs. Also, Fig. 12 really makes sure that the increased ratio *d/H* decreases the velocity magnitude inside the cavity.

The Effect of Volume Fraction of Nanoparticles

This section is dedicated to illustrate the effect of vol-ume fraction of solid nanoparticles of copper on the hydro-dynamic characteristics and convective heat transfer.

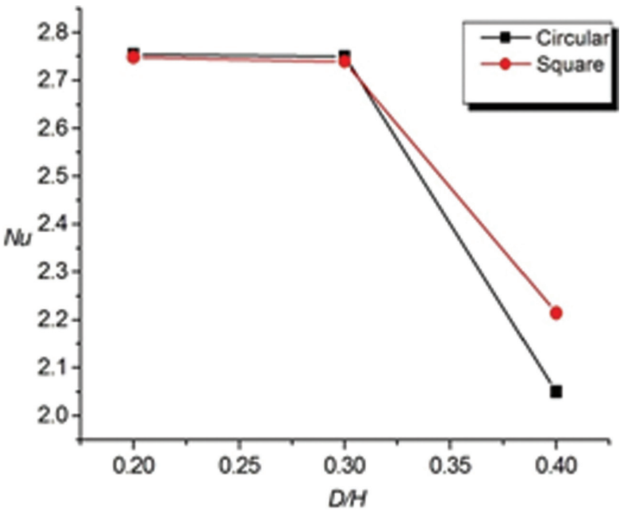

Figure 11. Effects of the cylinders diameter on the average *Nu* at the cylinder for $Ra = 10^4$, $S/H = 0.5$, and $\varphi = 0$.

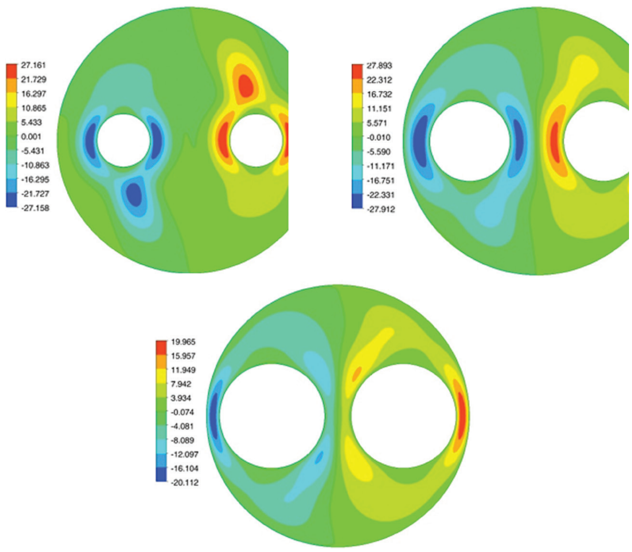

Figure 12. Contours of the dimensionless *v*-velocity in the circular cavity for different values of *d/H*, at $Ra = 10^4$, $S/H = 0.5$ and $\varphi = 0$.

Fig.13 shows the isotherms and streamlines in the circular cavity with two percentages of nanoparticles 0% and 0.1%and at fi ed parameters: $Ra = 10^4$, $S/H = 0.5$ and $d/H= 0$. It is well observed that the effect of volume fraction φ on both isotherms and streamlines is negligible. Therefore, it is predictable that the added nanoparticles in the base fluid (water) do not affect the heat transfer rate in the device. These fi dings agree well with those of different authors [34-38].

Fig. 14 describes graphically the development in average *Nu* at the cylinder under the change of studied parameters: $Ra = 10^3$, 10^4 and 10^5, $d/H = 0.2$, 0.3 and 0.4, $\varphi = 0$ to 0.1 at fi ed $S/H = 0.5$. From Fig. 14 and for all values of *d/H*, the increase in *Ra* augments the *Nu* value due to the fl w velocity that increases with the gradual increase in the buoyancy force, as depicted in Fig. 12. Also, for the values of *Ra* and *d/H*, the influence of volume fraction on *Nu* is completely negligible. Furthermore, the increase of cylindrical diameter (*d/H*) affects negatively on *Nu*. Therefore, it can be concluded that the small sizes of inner cylinder are useful for cooling applications, whereas, the increase in the size of cylinders is optimal for insulating applications. The negligible effect of nanoparticles density on the studied geometry can be explained as: adding small particles to the water raises the thermal transfer properties of the fluid, and this raises the Prandtl value, but based on the results obtained from a considerable number of researches such as [7], [39] and [40], it is confi med that the *Pr* number has a

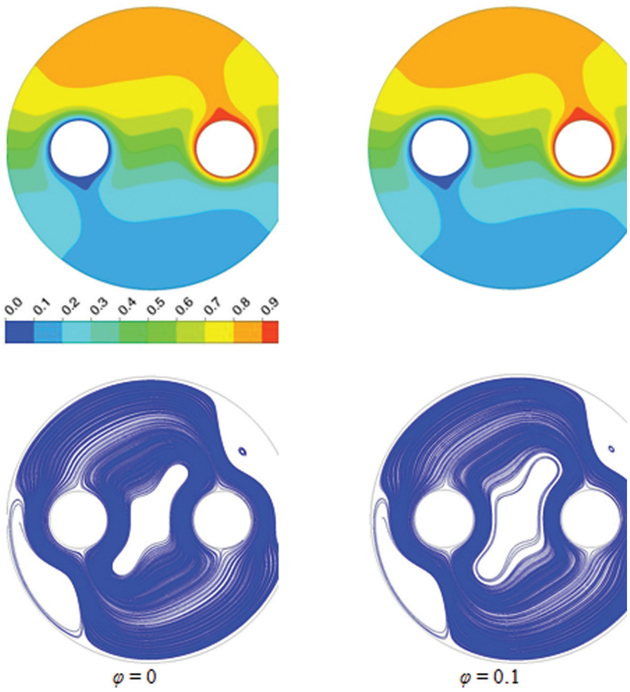

Figure 13. Isotherms and streamlines in the circular cavity for two values of $\varphi = 0$ and 0.1 at fi ed$Ra = 10^4$, $S/H = 0.5$ and $d/H= 0.2$.

limited effect on thermal transfer starting from the value 7 for free convection in annular spaces.

The Effect of Distance Between Cylinders

Th s section is incorporated to examine the impact of the distance between cylinders on the global characteristics of hydrodynamics and convective heat transfer.

Fig. 15 demonstrates the impact of the ratio *S/H* (gap between the cylinders) on the isotherms and streamlines in a circular cavityat $Ra = 10^4$, $\varphi = 0$ and $d/H = 0.2$. It is observed that the thermal gradient around the cylinder surfacedecreases with the rise of *S/H* from 0.25 to 0.7, which indicates that the convective heat transfer decreases with increasing distance between the cylinders. While the impact of *S/H* on the streamlines is summarized in the following point: a decreased *S/H* divides the central vortices into two small parallel vortices and increases the size of extra vortices that are located above and below the hot and cold cylinders, respectively. Finally, it can be recommended that the present system may be used as a new technique for mixing processes of fluids.

At $d/H = 0.2$ and for different values of *S/H* (0.2 to 0.7), *Ra* (10^3 and 10^4), and φ (0 and 0.1), Fig. 16 characterizes the variation of average *Nu* at the hot cylinder in the circular cavity. It is clear that the value of *Nu* decreases with increasing *S/H* for the same values of *Ra* and φ. Also, the effect of φ on *Nu* is also negligible for the studied parameters of *S/H* and *Ra*.

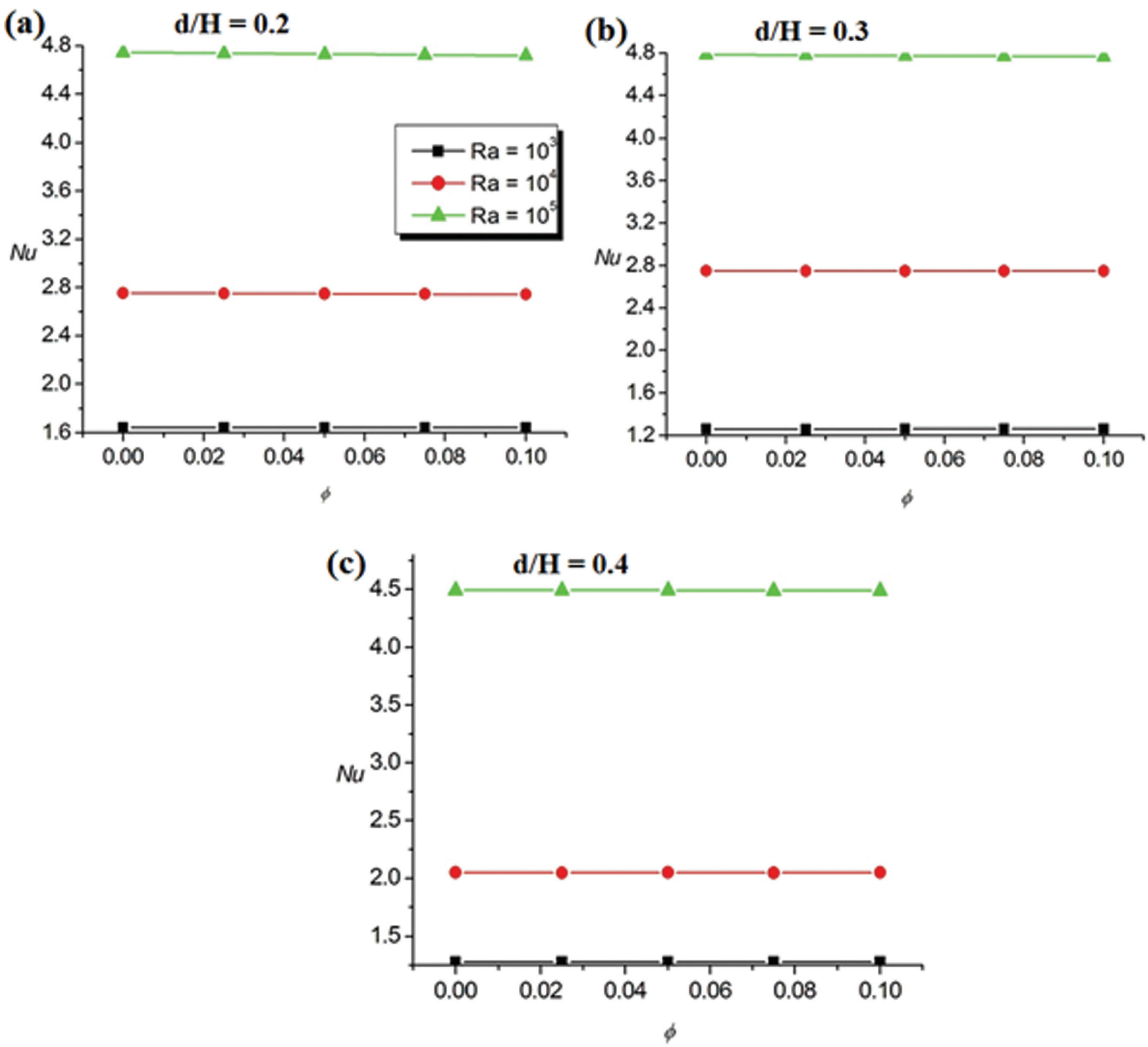

Figure 14. Average *Nu* versus volume fraction of nanoparticles with different *Ra* number and*d/H*, at *S/H* = 0.5.

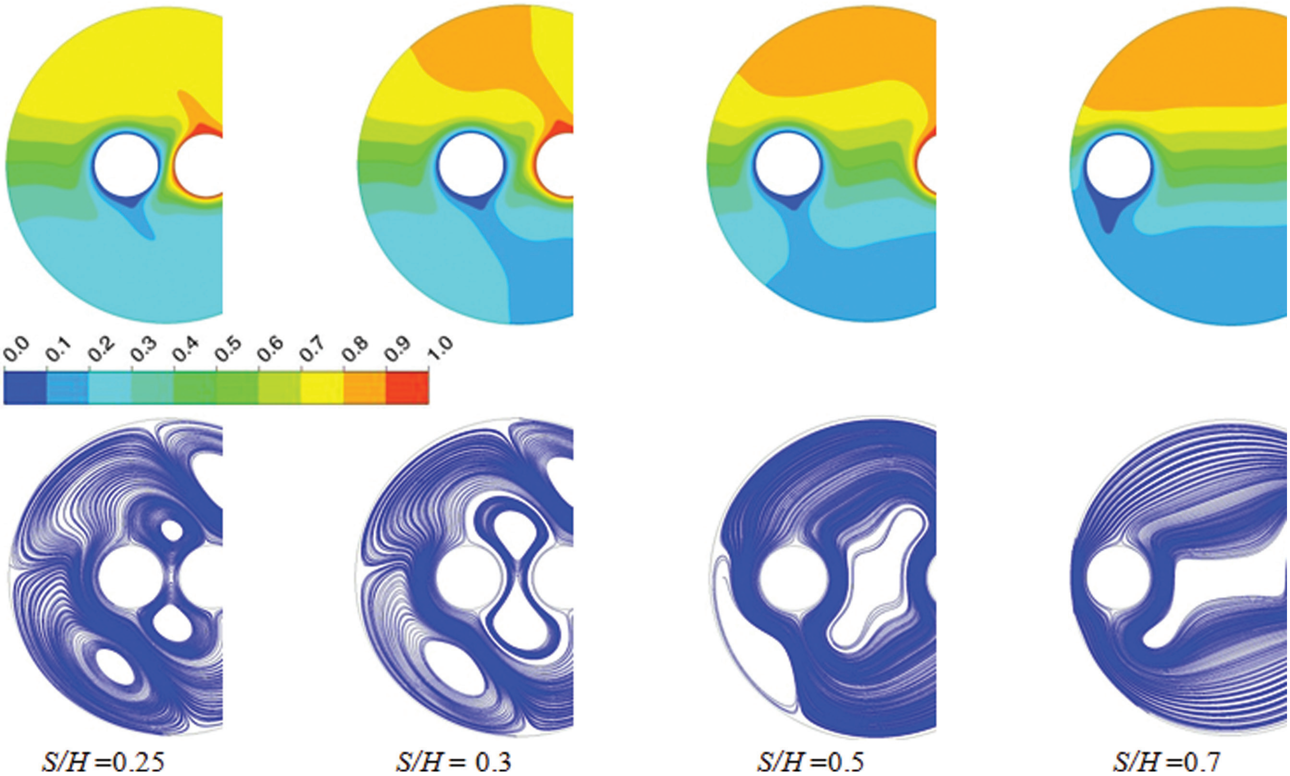

Figure 15. Isotherms and streamlines in circular cavity for different values of S/H at fi ed $Ra = 10^4$, $\varphi = 0$ and $d/H= 0.2$.

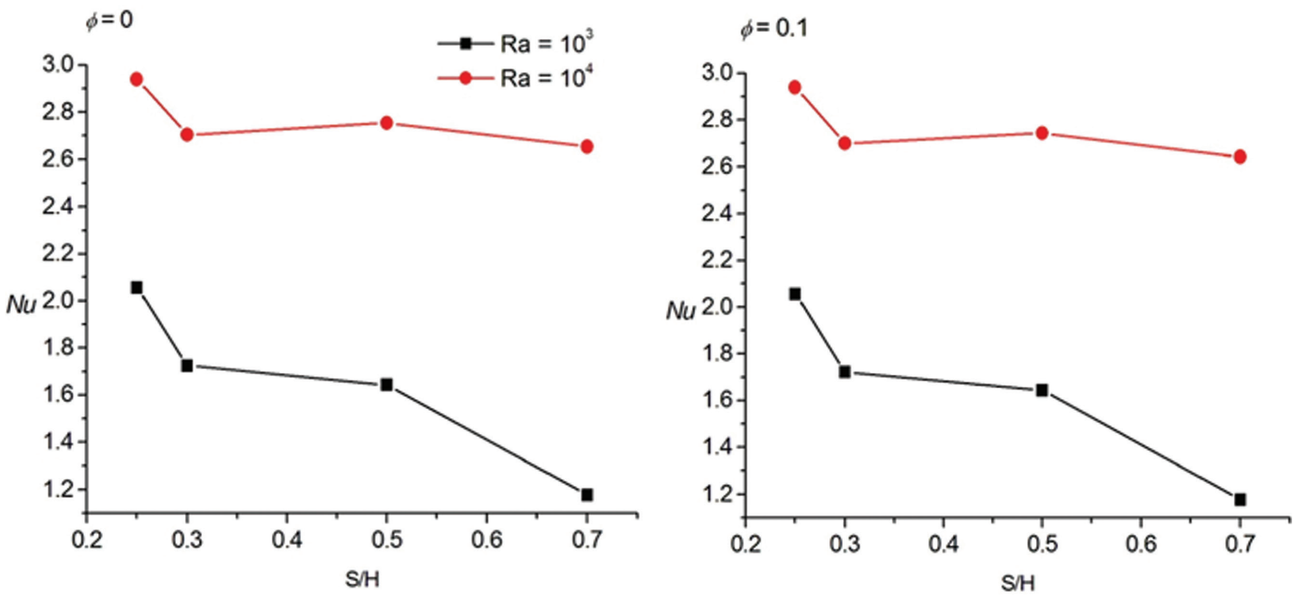

Figure 16. Values of *Nu*of hot cylinder for different values of *S/H* and *Ra* and φ, at$d/H= 0.2$.

CONCLUSION

The free convection of copper-water nanofluid in a 2D adiabatic cavity with two inner circular cylinders was numerically investigated. The cylinders were horizontally placed in the cavity center. The cylinders had different temperatures (one was hot and the second was cold). The effects of volume fraction of nanoparticles, *Ra* number, cavity shape, cylinder diameter, and the distance between thecylinders on the global characteristics of fluid hydrodynamics and convective heat transfer where examined.

The obtained findings revealed the formation of a rotating flow in the cavity due to the opposite effect of buoyancy force from the hot and cold cylinders. The interactions between the hot and cold sources generated two extra loops; one was seen above the hot cylinder and the second was below the cold cylinder. Also, the size of the principal vortex and the additional loops were seen to be dependent on *Ra*, the diameter of the cylinders and the distance between the inner cylinders. In addition, the average *Nu* was seen to be increased with increasing *Ra* and/or decreasing distance between the cylinders.

Regarding the nanoparticles volume fraction, it was proofed that this parameter (when varying from 0 to 0.1) does not have any effect on the fluid flow and heat transfer behaviors. Finally, the circular and square shapes of the cavity had almost a negligible influence on the convective heat transfer. Decreasing the distance between the cylinders increases the heat transfer which indicating the effectiveness of this for cooling applications.

NOMENCLATURE

B	Blockage ratio (=d/H)
d	Diameter of cylinder (m)
H	Diameter of cavity (m)
g	Gravity acceleration ($m{\cdot}s^{-2}$)
Nu	Average Nusselt number (-)
Nu_l	Local Nusselt number (-)
p	Dimensionless pressure (-)
Ra	Rayleigh number (-)
S	Distance between cylinders (-)
T	Dimensionless temperature (-)
u, v	Dimensionless velocity components (-)
x, y	Dimensionless Cartesian directions (-)
Greek symbols	
α	Thermal diffusivity (m^2/s)
β	Volume expansion coefficient (K^{-1})
μ	Dynamic viscosity (Pa·s)
ρ	Fluid density (kg/m^3)
ϕ	Volume fraction (-)

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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