CHAOS Theory and Applications in Applied Sciences and Engineering

e-ISSN: 2687-4539
RESEARCH ARTICLE
Vol.4 / No.4 / 2022 / pp.241-245
Special Issue (EDIESCA 2022)
https://doi.org/10.51537/chaos.1190234

Dynamics of Indoctrination in Small Groups around Three Options

Maria Guadalupe Medina Guevara [*,1], **Evguenii Kurmyshev** [*,2] **and Hector Vargas-Rodriguez** [*,3]

[*]Departamento de Ciencias Exactas y Tecnología, Centro Universitario de los Lagos, Universidad de Guadalajara, Enrique Díaz de Leon 1144, Lagos de Moreno, Jalisco 47460, Mexico.

ABSTRACT In this work, we consider the dynamics of opinion among three parties: two small groups of agents and one very persuasive agent, the indoctrinator. Each party holds a position different from that of the others. In this situation, the opinion space is required to be a circle, on which the agents express their position regarding three different options. Initially, each group supports a unique position, and the indoctrinator tries to convince them to adopt her or his position. The interaction between the agents is in pairs and is modeled through a system of non-linear difference equations. Agents, in both groups, give a high weight to the opinion of the indoctrinator, while they give the same weight to the opinion of their peers. Through several computational experiments, we investigate the times required by the indoctrinator to convince both groups.

KEYWORDS
Opinion dynamics
Non-linear difference-equations
Indoctrination
Agent-based model

INTRODUCTION

The dynamics of opinion attempt to understand the processes of opinion formation in society through the use of different agent-based models, considering different social networks, different opinion updating rules, and different opinion spaces. To date, there are many models, and the topic is far from exhausted. Some reviews of the topic can be found in Noorazar *et al.* (2020); Dong *et al.* (2018). According to Zha *et al.* (2020), these models can be classified into two categories depending on whether opinions are discrete or continuous, and the dynamics associated with them evolve towards three stable states: consensus, polarization, or fragmentation.

Some models of opinion dynamics have used circles and n-dimensional spheres to study the formation of consensus and dissensus (Caponigro *et al.* 2015; Hegarty *et al.* 2016; Zhang *et al.* 2021, 2022). These spaces are very convenient for modeling the evolution of the preferences of a group of agents around a discrete set of options. For example, in Medina-Guevara *et al.* (2017); Medina Guevara *et al.* (2018), the evolution of preferences around three political options is considered.The opinion space is considered being a circle, where the options are separated at the same distance from each other, and the preference of the agents can freely evolve from one of them to any other, without approaching the third of them. For example, assuming that the options are located at the points 0°, 120° and 240°; an agent whose preference is 240° supports completely this option, while an agent whose opinion is 60° is insecure about the options at 0° and 120°, while completely rejects the option at 240°, this last agent has a diametrically opposed opinion (its opinion is in the opposed side of the circle, they are separated 180°). So, in order to give the agents the knowledge about where the options are located, a non linear map is used.

For certain values of the maps's parameter, it introduces three attractors, one for each option, however, for other values of that parameter, the map also offer the agents the possibility to reject all three options, or to manifest doubt when opinion converges with oscillation to the attractor, or a dilemma when the maps has a 2-cycle around the option. In this sense it is considered that this map emulates an internal reflexion process in the agents, allowing them to update their opinion according with their preferences regarding those three options.

In this work we use the model presented in Medina-Guevara *et al.* (2017), to study the process of indoctrination of a polarized group, the group is formed by two factions of equal size that support two different positions, and a highly influential agent, the indoctrinator, who tries to convince the rest of the agents to adopt a position different from theirs. As the interaction of the agents is in pairs, and to have an even number of agents, an additional agent is considered, who is undecided between the two majority options, but openly in opposition to the indoctrinator. In this work we use the term indoctrination in the same way as in Medina-Guevara *et al.*

Manuscript received: 17 October 2022,
Revised: 13 December 2022,
Accepted: 13 December 2022.

[1]maria.mguevara@academicos.udg.mx
[2]evguenii.kourmychev@academicos.udg.mx
[3]hvargas@culagos.udg.mx (**Corresponding author**).

(2019), so it is interpreted as the fact of trying to impose an opinion different from that of others. That the indoctrinated is influential is because the other agents give significant weight to his opinion. While they trust each other equally. Thus, the indoctrinator is an opinion leader like the one discussed in Boccaletti *et al.* (2018).

The work is organized in the following way. Section 1 presents the introduction. Section 2, The mathematical model, introduces the agent based model used in Medina-Guevara *et al.* (2017). Section 3, Numerical Experiment Settings, considers the initial conditions and settings under which the model emulates the dynamics of the indoctrination of two small groups that support opposite positions to those of the indoctrinator. Section 4, Results, presents the required average times to indoctrinate small groups of agents. Section 5 presents the conclusions of the work. For convenience we use both radians and degrees, in this sense radians are used in Section 2, while degrees are used in Section 3.

THE MATHEMATICAL MODEL

We employ the agent based model given in Medina-Guevara *et al.* (2017). In that model, a set of N agents manifest their opinion with regard to three options, in this sense their opinion space S^1 is a circle with the options located at the points: 0 rad, $2\pi/3$ rad, $4\pi/3$ rad. Agents have two attributes their opinion or preference x, and a personal parameter $\kappa \in K = [-1.5, 1.5]$, that allows them to have a posture and a behavior regarding those options.

Hence, in order to distinguish these three points in the opinion space, a non-linear function $\Xi : S^1 \times K \to S^1$ is introduced, it is defined as:

$$\Xi(x_n, \kappa) = x_n - \kappa \sin(3x_n). \quad (1)$$

For $0 < \kappa < 2/3$, the map $x_{n+1} = \Xi(x_n, \kappa)$ possesses three attractors in 0, $2\pi/3$, $4\pi/3$, and three repellers in $\pi/3$, π, $5\pi/3$, if $-2/3 < \kappa < 0$ the attracting nature of these fixed points reverses. The map also possess $n-$cycles which after a cascade of bifurcations lead to chaos, see Figure 1.

As it is already mentioned in Medina Guevara *et al.* (2018), the parameter κ can be used to model different behaviors in agents regarding those options. For example, when the preference of the agent is governed only by this map, and his personal parameter satisfies $0 < \kappa < 2/3$, the preference of the agent is attracted to those options, but an agent whose personal parameter satisfies $-2/3 < \kappa < 0$ rejects the options, and become attracted to the intermediate postures at $\pi/3$ rad, π rad and $5\pi/3$ rad; the preference of a secure agent, with $\kappa \in [0, 1/3]$, converges without oscillation to the options; but the preferences of a vacillating agent, the one with $\kappa \in [1/3, 2/3]$ converges with oscillation to the options; the preferences of an agent whose personal parameter satisfies $-1.045 < \kappa < -2/3]$ will evolve into a 2-cycle, a dilemma where the agent is insecure about two options, consider, for example, the case when its preference is initially near π, it will evolve to be jumping from preferences near $2\pi/3$ to preferences near $4\pi/3$.Perhaps, it is even possible to have agents whose preferences evolve chaotically in a scenario where information is changing every moment.

Hence the iterative model is the following:

1. In the first temporal step each agent is assigned an opinion and a personal parameter κ.

2. In the following steps, arbitrarily chosen pairs of agents interact according to the affinity of their opinions. To do this, an affinity parameter ϵ is introduced. Depending on how similar the opinions of the agents are, two different situations are contemplated:

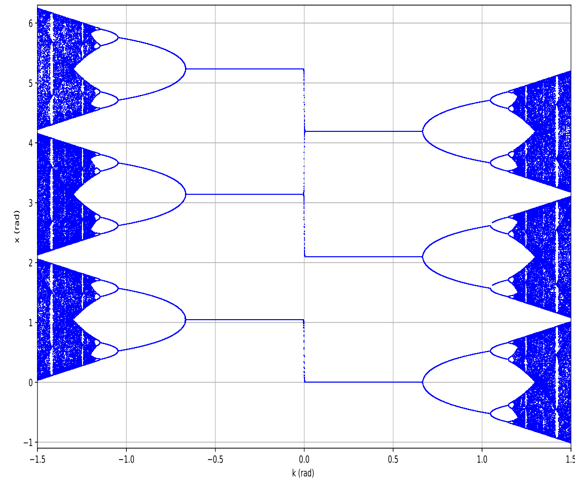

Figure 1 The figure shows the bifurcation diagram corresponding to the iterated map $x_{n+1} = \Xi(x_n, \kappa)$, it can be appreciated the fixed points corresponding to 0, $2\pi/3$ and $4\pi/3$ for $0 < \kappa < 2/3$; and $\pi/6$, π, $5\pi/3$ for $-2/3 < \kappa < 0$. Both axis are in radians. This maps allows the agents to identify three equal options in the opinion space, as well as to have different behaviors to update their opinions.

- **Agents' opinions are affine.** In this case, $|x_n^i - x_n^j| < \epsilon$, the opinions for the $(n+1)$th temporal step will be defined through:

$$\begin{cases} x_{n+1}^i = a_{ii}\Xi(x_n^i, \kappa^i) + a_{ij}x_n^j, \\ x_{n+1}^j = a_{ji}x_n^i + a_{jj}\Xi(x_n^j, \kappa^j), \end{cases} \quad (2)$$

where the coefficients a_{ij} represent the relative weight that agent number i grants to the opinion of agent number j, they satisfied $0 \leq a_{ij} \leq 1$, $a_{ii} + a_{ij} = 1$ and $a_{ji} + a_{jj} = 1$.

- **Agents' opinions are not affine.** In this case, $|x_n^i - x_n^j| > \epsilon$ the agents update their opinions considering only their individual preferences. Thus,

$$\begin{cases} x_{n+1}^i = \Xi(x_n^i, \kappa^i), \\ x_{n+1}^j = \Xi(x_n^j, \kappa^j). \end{cases} \quad (3)$$

NUMERICAL EXPERIMENTS SETTINGS

As the initial conditions we consider

1. A small group of N agents formed by one indoctrinator supporting the choice 240°, two subgroups of equal size, the first one supporting choice 0° and the second one supporting choice 120°, and an indecisive agent with a posture 60° between those of the two subgroups. We consider group sizes $N = 4, 6, 8, ..., 22$.

2. We consider an affinity $\epsilon = 120°$, which prevents the indoctrinator to interact with the indecisive agent, notice that this

last agent has a diametrally opposed preference to that of the indoctrinator.

3. In order to have a very persuasive indoctrinator, agent number 1, we consider that all agents grant to her (or his) opinion a great relative weight,

$$a_{11} = \frac{a}{1+a}, \quad a_{1i} = \frac{1}{1+a}, \tag{4}$$

$$a_{i1} = \frac{a}{1+a}, \quad a_{ii} = \frac{1}{1+a} \tag{5}$$

where $a > 1$ and $i = 2, 3, ..., N$. While among themselves, they give each other the same weight.

$$a_{ii} = \frac{1}{2}, \quad a_{ij} = \frac{1}{2}, \tag{6}$$

$$a_{ji} = \frac{1}{2}, \quad a_{jj} = \frac{1}{2}, \tag{7}$$

here $i, j \neq 1$.

4. To measure the indoctrinator's convincing power we use a quantity defined in Medina-Guevara *et al.* (2019), the charisma, it is defined as in the following manner:

$$\eta_{ij} = \frac{a_{ij}}{a_{ii}}, \tag{8}$$

hence de charisma of the jth-agent as perceived by the ith-agent is simple the ratio of the relative weight of that agent relative to his own weight, for example, the perceived charisma of the indoctrinator is the number $\eta_{i1} = a$ for $i \neq 1$, while that of any other agent is $\eta_{ij} = 1$ for $i, j \neq 1$. We will consider indoctrinators with $a \in \{2, 3, 4, 5, 10\}$.

5. For simplicity, we consider a fixed value of the agents' parameter κ^i, we choose $\kappa^i = 28°$.

RESULTS

After performing 100 computational experiments for each of the above settings, we have the following results:

1. Table 1 presents the average number, $\bar{T}$ and the standard deviation, σ, of the temporal steps (computational cycles) required by a given indoctrinator to convince the groups of agents of her (or his) posture. There were cases when the indoctrinator was unable to convince the whole group, in those cases a dash is reported on the table. In all cases, the average number of cycles grows with the size of the group. Hence relatively large groups of equally trusting agents become immune to be convinced of a different posture.

2. Table 2 reports the minimum number of cycles, found among each series of 100 computational experiments, to indoctrinate the full group, this table shows that for some relatively large groups, and in a few cases, an indoctrinator is still able to convince the group. For example, for a group of 20 agents (including the indoctrinator), an indoctrinator of charisma 10 convinced the group in 55 cycles, although the average number of cycles found for this case was 2638 cycles.

3. Table 3 reports the maximum number of cycles to indoctrinate the full group, these are the maximum number of cycles found among each series of 100 computational experiments. The dashes correspond to the cases when the indoctrinator was unable to convince the group.

4. Table 4 reports the cases when the indoctrinator was convinced of a different posture. For example, an indoctrinator of charisma 2 is always convinced of a different posture by groups of 12 or more agents. While an indoctrinator of charisma 3, was convinced by groups of 12, 14 or 16 agents in few cases; but it always was convinced by groups of 18 or more agents. Indoctrinators with charisma 4 or greater are not persuaded to change their opinion.

In Figures 2(a) and 2(b), just to illustrate the dynamics, we present two examples of the time evolution of the preferences of groups of 16 agents under the presence of an indoctrinator of charisma 10, Figures 2(c) and 2(d) present the evolution of the number of agents in each of the basin options: The number of agents in basin $(-300°, 60°)$ are represented by the red line, agents in basin $(60°, 180°)$ by the blue line, and agents in the basin $(180°, 300°)$ by the green line.

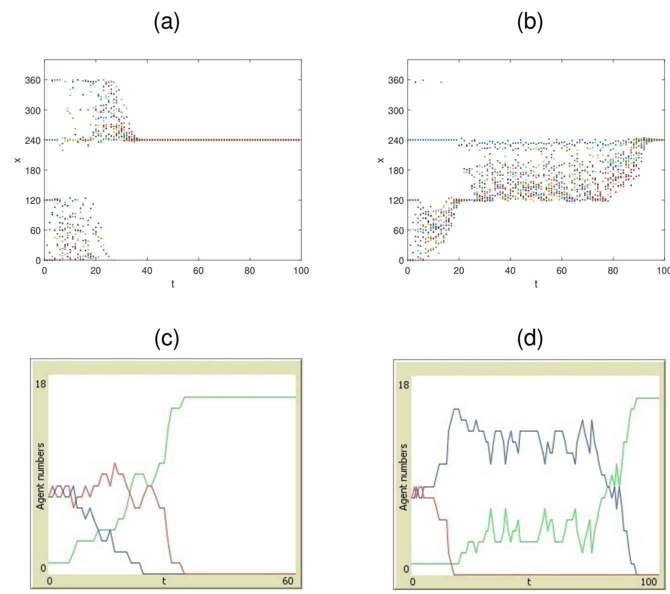

Figure 2 Figures (a) and (b) show the evolution of the opinion preferences of a group of 16 agents. Initially, there are two subgroups each one in a consensus around opinions 0° and 120°, an indecisive agent between those two postures with a 60° opinion, and indoctrinator with charism 10 suporting opinion 240°. Here t denotes the number of cycles, and x the preference regarding the options. In Figure 2(a) the group is indoctrinated after 33 cycles while in Figure 2(b) the group is indoctrinated after 91 cycles. Notice that on figures 2(a) and 2(b), the values 0° and 360° should be identified, the graphic is on a cylinder. Figures 2(c) and 2(d) show the evolution of the agent numbers in each attraction basin. Respectively, red, blue and green lines represents the number of agents in the basin corresponding to the attractors 0°, 120° and 240°. Figure 2(c) correspond to the time series 2(a), while 2(d) to 2(b).

Table 1 **Average indoctrination time and standard deviation vs group size for different charismatic indoctrinators**

charisma	2		3		4		5		10	
Agents	$\bar{T}$	σ	$\bar{T}$	σ	$\bar{T}$	σ	$\bar{T}$	σ	$\bar{T}$	σ
4	17.29	5.56	13.94	3.27	13.8	3.3	13.86	2.88	13.96	5.15
6	22.92	6.29	21.96	5.41	20.75	4.96	19.86	4.62	19.73	3.80
8	33.91	4.56	31.72	6.30	27.90	5.34	27.18	5.40	26.12	4.46
10	30.6	3.61	49.35	12.71	35.82	6.96	32.91	5.62	31.08	5.74
12	-	-	230.63	227.65	55.27	23.39	45.07	13.36	37.38	7.33
14	-	-	-	-	137.89	79.56	68.27	29.86	51.77	14.61
16	-	-	-	-	1412.43	1398.77	252.45	196.98	79.05	36.17
18	-	-	-	-	48676	55111	3604	3854	256.8	203.73
20	-	-	-	-	-	-	114712	101086	2638	2535
22	-	-	-	-	-	-	-	-	63464	54877

Table 2 **Minimum time for indoctrination vs group size for different charismatic indoctrinators**

Agents \ Charisma	2	3	4	5	10
4	8	8	8	7	8
6	14	11	13	11	12
8	27	14	18	14	17
10	25	25	23	24	22
12	-	23	28	20	19
14	-	31	29	29	29
16	-	19347	39	27	26
18	-	-	673	94	45
20	-	-	-	1388	55
22	-	-	-	-	383

Table 3 **Maximum time for indoctrination vs group size for different charismatic indoctrinators**

Agents \ Charisma	2	3	4	5	10
4	32	23	25	23	50
6	47	39	41	32	29
8	43	53	43	43	37
10	35	98	49	51	45
12	-	1614	192	113	58
14	-	-	436	174	104
16	-	-	6448	1088	230
18	-	-	419114	24151	1119
20	-	-	-	291994	11328
22	-	-	-	-	250165

Table 4 **Indoctrinator is convinced by the group in less than 100 cycles**

Agents \ Charisma	2	3
4	6 %	0 %
6	64 %	0 %
8	89 %	0 %
10	95 %	0 %
12	100 %	1 %
14	100 %	3 %
16	100 %	3 %
18	100 %	9 %
20	100 %	9 %
22	100 %	24 %

CONCLUSIONS

In this paper, we have used the model proposed in Medina-Guevara *et al.* (2017) to investigate the influence of an indoctrinator, a very persuasive agent, on a polarized group made up of two factions of equal size. Each faction supports a unique position than the indoctrinator does. In this sense, the opinion space is a circle where the agents express their preferences regarding three equidistant options. To distinguish these options on the circle, the model employs a system of difference equations that introduces three attractors, which are then identified with the options. The interaction between the agents is in pairs, which is why it is necessary to have an even total number of agents, so we introduce an undecided agent between the two factions to complete that even number of agents.

Following Medina-Guevara *et al.* (2019), to ensure that the indoctrinator is a very persuasive agent, we have made all agents in the group give a high weight to his opinion. While, among themselves, the agents give the opinion of their peers the same weight that they give to their own. To measure the persuasive strength of the indoctrinator, the definition of charisma was also adopted, which is the ratio between the weights mentioned, see eq. (8).

In the model, each faction of agents has the possibility to convince its members back, as well as to persuade the others to adopt its position. So groups can offer resistance to changing their minds.

From the results shown in the previous section, it can be seen that:

1. The average number of temporal steps to indoctrinate a group grows with the size of the group, indeed large groups become immune to the indoctrinator's attempts to persuade them, as long as its agents are free to interact with any other agent in the group.

2. Uncharismatic indoctrinators can be convinced to take the stance of the winning faction.

3. Very charismatic indoctrinators are stubborn and cannot be persuaded to adopt a different opinion, no matter how large groups they interact with.

Acknowledgments

The authors appreciate the invitation to participate in EDIESCA 2022.

Availability of data and material

Not applicable.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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How to cite this article: Medina-Guevara, M. G., Kurmyshev, E., and Vargas-Rodriguez, H. Dynamics of Indoctrination in Small Groups around Three Options. *Chaos Theory and Applications*, 4(4), 241-245, 2022.