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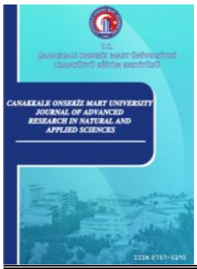

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# Stewart's Theorem and Median Property in the Galilean Plane

Abdulaziz Açıkgöz[1,*], Nilgün Sönmez[2]

[1,2]Department of Mathematic, Faculty of Arts and Sciences, Afyon Kocatepe University, Afyonkarahisar, Türkiye

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***Research Article***

**Abstract** − Galilean plane can be introduced in the affine plane, as in Euclidean plane. This means that the concepts of lines, parallel lines, ratios of collinear segments, and areas of figures are significant not only in Euclidean plane but also in Galilean plane. The Galilean plane $G^2$ is almost the same as the Euclidean plane. The coordinates of a vector $a$ and the coordinates of a point $A$ (defined as the coordinates of $OA$, where $O$ is the fixed origin) are introduced in Galilean plane in the same way as in Euclidean geometry. The galilean lines are the same. All we need add is that we single out special lines with special direction vectors in Galilean plane. We should attention that these two types of galilean lines cannot be compared. The difference between Euclidean plane and Galilean plane is the distance function. Thus, we can compare the many theorems and properties which is included the concept of distance in these geometries. The theorems and the properties of triangles in the Euclidean plane can be studied in the Galilean plane. Therefore, in this study, we give the Galilean-analogues of Stewart's theorem and median property for the triangles whose sides are on ordinary lines.

***Keywords*** *– Galilean distance, Galilean plane , Galilean triangle, Median property, Stewart's theorem*

## 1. Introduction

Galilean geometry is described by (Yaglom,1979). The Galilean plane $G^2$ is represented in the Cartesian plane as in the Euclidean plane. In the Galilean plane, the lines which are parallel to the $y$-axis are called special lines and the lines which are not parallel to $y$-axis are called ordinary lines. The distance function is different from the Euclidean plane. In the Galilean plane, if $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are points, where $x_2 > x_1$, then distance from $P$ to $Q$ is defined by formula as follows (see 1.1):

$$d_G(P, Q) = x_2 - x_1. \tag{1.1}$$

If $x_1 = x_2$ and $y_2 > y_1$, then the points $P$ and $Q$ are belong to same special line. The distance is defined by formula as follows (see 1.2):

$$\delta_{PQ} = y_2 - y_1. \tag{1.2}$$

Since the distance function is different, the properties in the Euclidean plane can be reproduced faithfully in this plane. In the Galilean plane, topics involving distance have been studied by some authors (Kurudirek & Akca,2015; Akar, M., Yüce, S. & Kuruoglu, N.,2013).

Stewart's theorem and the median property are well known in the Euclidean plane. Moreover, this theorem and property has been studied by some outhors (Kaya & Colakoğlu,2006; Gelisgen & Kaya,2013; Gelisgen & Kaya,2009) on other planes.

How can we define this theorem and property in the Galilean plane? In this study, the answer to this question is investigated. Firstly, the definition of the base line is given similar to (Ozcan & Kaya,2003) for triangles in

[1] aziz@aku.edu.tr
[2] nceylan@aku.edu.tr
[*]Corresponding Author

$G^2$. Then, according to the definition of the base line, we give Galilean-analogues of Stewart's theorem and median property for the triangles whose sides are on ordinary lines.

## 2. Meterials and Methods

In this section, the basic information will be given about the triangle which is described by (Yaglom,1979) in the Galilean geometry.

Let $\Delta ABC$ be a triangle whose sides are on ordinary lines and the sides have the lengths $|d_G(B,C)| = a_G$, $|d_G(C,A)| = b_G$, $|d_G(A,B)| = c_G$ (see Figure 1). If $a_G$ is the largest side, then we can write $a_G$ as follows (see 2.1):

$$b_G + c_G = a_G, \qquad \hat{B}_G + \hat{C}_G = \hat{A}_G. \tag{2.1}$$

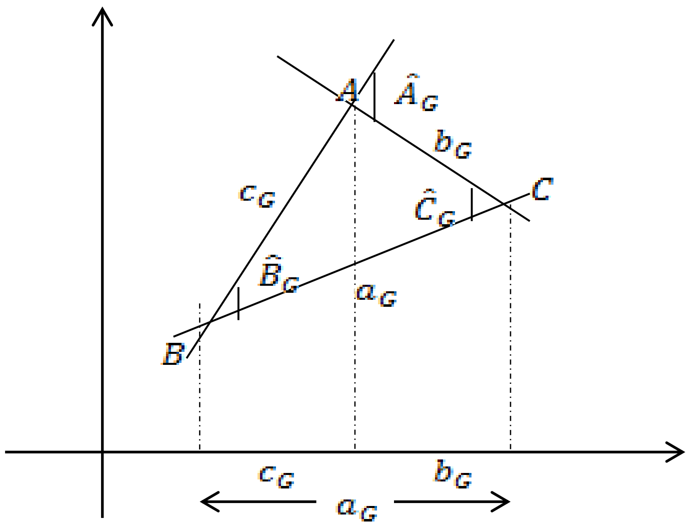


Figure 1 A triangle whose sides are on ordinary lines in $G^2$.

Now, we can show a triangle whose any side is on the special line in $G^2$.

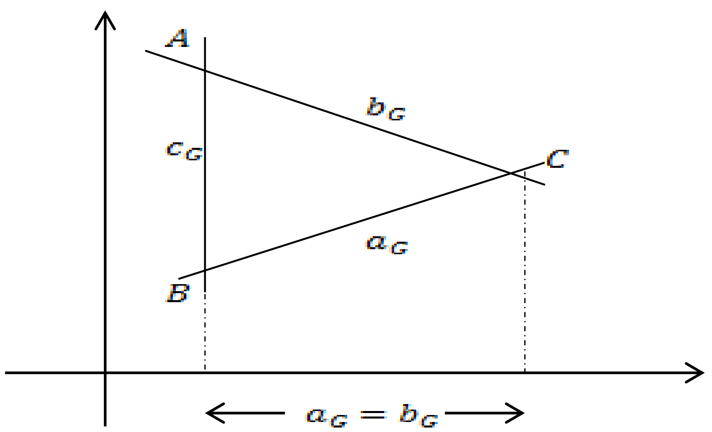


Figure 2 A triangle whose any side is on the special line in $G^2$.

The triangle in figure 2 is an isosceles triangle. Because of the equation (2.1), the side $c_G$ must be equal to the sum of the sides $a_G$ and $b_G$, the angle $\hat{C}_G$ must be the largest angle. But the slope of a special line is equal to the tangent of the angle between special line and the $x - axis$ (i.e., $tan 90°$) and it must be infinite. Therefore the largest angle must be either the angle $\hat{A}_G$ or the angle $\hat{B}_G$. Hence a contradiction is obtained.

Since the two types of Galilean lines are not comparable, it is natural to include in the class of lines of Galilean geometry only ordinary lines (Yaglom,1979). So the sides of a triangle in $G^2$ lie on ordinary lines. Now we use the following definition given in (Ozcan & Kaya,2003) to give a Galilean-analogues of Stewart's theorem and median property.

**Definition 2.1** Let $\Delta ABC$ be any triangle whose sides are on ordinary lines in $G^2$. A line $k$ is said a base line of $\Delta ABC$ if and only if

i) $k$ passes through a vertex,

ii) $k$ is parallel to a coordinate axis,

iii) $k$ intersects the opposite side (as a line segment) to the vertex in Condition 1.

Thus, at least one of the vertices of a triangle always has one or two base lines. Like this a vertex of a triangle is said a basic vertex. A base segment is a line segment on a base line, which is bounded by a basic vertex and its opposite side.

## 3. Results and Discussion

After this preliminary, we can give the following theorem for a triangle whose sides are on ordinary lines.

**Theorem 3.1** Let the sides of a triangle $\Delta PQR$ in $G^2$ have lengths $|d_G(P,Q)| = r_G$, $|d_G(Q,R)| = p_G$, $|d_G(R,P)| = q_G$. If $X \in [QR]$ and $|d_G(Q,X)| = m_G$, $|d_G(X,R)| = n_G$, $|d_G(P,X)| = x_G$, then we can write as follows (see 3.1):

$$x_G = \begin{cases} m_G + r_G, & If\ \Delta PQR\ has\ no\ base\ line\ through\ the\ vertex\ P, \\ n_G + q_G, & If\ \Delta PQR\ has\ only\ base\ line\ through\ the\ vertex\ P, \\ n_G - q_G, & If\ \Delta PQR\ has\ two\ base\ lines\ through\ the\ vertex\ P \\ & and\ X\ is\ between\ the\ intersection\ points\ of\ the\ base\ lines, \\ |n_G - q_G|, & If\ \Delta PQR\ has\ two\ base\ lines\ through\ the\ vertex\ P \\ & and\ X\ is\ not\ between\ the\ intersection\ points\ of\ the\ base\ lines. \end{cases} \tag{3.1}$$

**PROOF :**
Let
$\acute{Q} = Orthogonal\ projection\ of\ Q\ to\ the\ line\ through\ P\ and\ parallel\ to\ y-axis,$
$\acute{R} = Orthogonal\ projection\ of\ R\ to\ the\ line\ Q\acute{Q},$
$\acute{X} = Orthogonal\ projection\ of\ X\ to\ the\ line\ Q\acute{Q}.$

**i)** If the triangle $\Delta PQR$ has no base line through the vertex $P$ (see Figure 3), then (see 3.2)

$$d_G(P,Q) = d_G(\acute{Q},Q), \quad d_G(Q,X) = d_G(Q,\acute{X}). \tag{3.2}$$

Thus, we obtain $d_G(P,X) = x_G$ as follows (see 3.3 and 3.4):

$$d_G(P,X) = d_G(\acute{Q},Q) + d_G(Q,\acute{X}), \tag{3.3}$$

$$x_G = r_G + m_G. \tag{3.4}$$

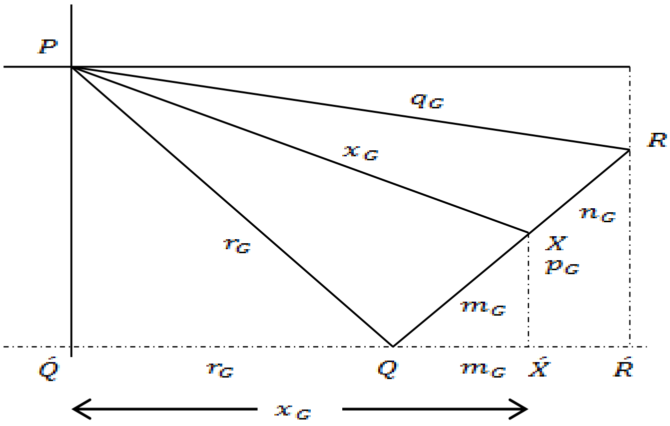


Figure 3 A triangle which has no base line through the vertex $P$.

**ii)** If the triangle $\Delta PQR$ has only base line through the vertex $P$ (see Figure 4), then (see 3.5)

$$d_G(P,R)=d_G(\acute{Q},\acute{R}),\quad d_G(R,X)=d_G(\acute{R},\acute{X}). \tag{3.5}$$

Thus, we obtain $d_G(P,X)=x_G$ as follows (see 3.6 and 3.7):

$$d_G(P,X)=d_G(\acute{Q},\acute{R})+d_G(\acute{R},\acute{X}), \tag{3.6}$$

$$x_G=q_G+n_G. \tag{3.7}$$

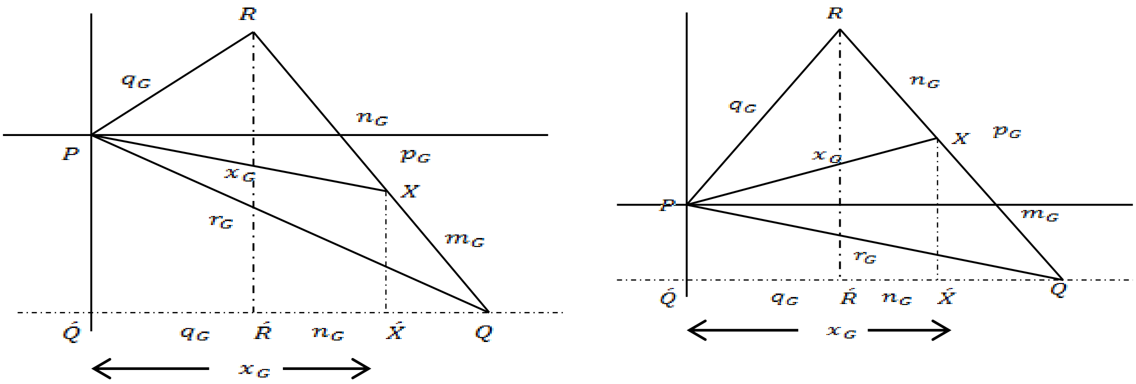


Figure 4 A triangle which has only one base line through the vertex $P$.

**iii)** If the triangle $\Delta PQR$ has two base line through the vertex $P$ and $X$ is between the intersection points of base lines (see Figure 6), then (see 3.8)

$$d_G(R,P)=d_G(\acute{R},\acute{Q}),\quad d_G(R,X)=d_G(\acute{R},\acute{X}). \tag{3.8}$$

Thus, we obtain $d_G(P,X)=x_G$ as follows (see 3.9 and 3.10):

$$d_G(P,X)=d_G(\acute{R},\acute{X})-d_G(\acute{R},\acute{Q}), \tag{3.9}$$

$$x_G=n_G-q_G. \tag{3.10}$$

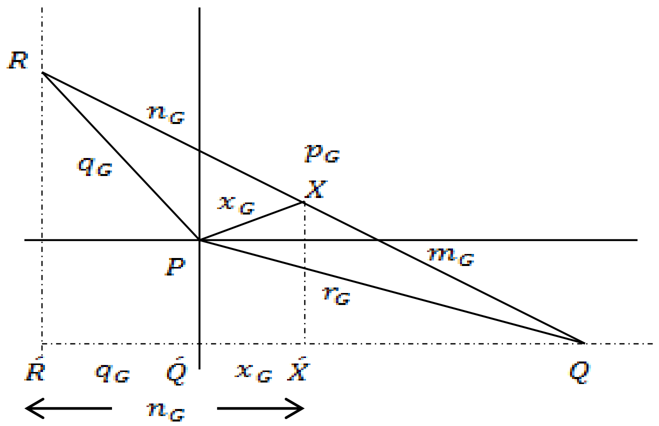


Figure 6 A triangle which has two base lines through the vertex $P$ and $X$ be between the intersection points of base lines.

**iv)** If the triangle $\Delta PQR$ has two base line through the vertex $P$ and $X$ is not between the intersection points of base lines (see Figure 7), then (see 3.11)

$$d_G(R,P) = d_G(\acute{R},\acute{Q}), \quad d_G(R,X) = d_G(\acute{R},\acute{X}). \tag{3.11}$$

Thus, we obtain $d_G(P,X) = x_G$ as follows (see 3.12, 3.13, 3.14, 3.15):

$$d_G(P,X) = d_G(\acute{R},\acute{X}) - d_G(\acute{R},\acute{Q}), \tag{3.12}$$

$$x_G = n_G - q_G. \tag{3.13}$$

or

$$d_G(P,X) = d_G(\acute{R},\acute{Q}) - d_G(\acute{R},\acute{X}), \tag{3.14}$$

$$x_G = q_G - n_G. \tag{3.15}$$

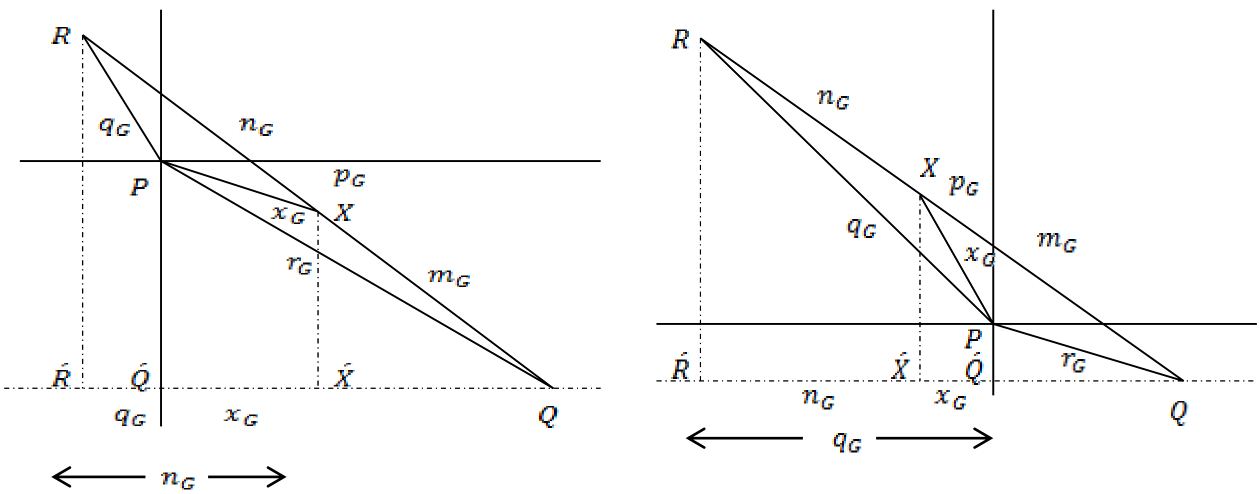


Figure 7 A triangle which has two base lines through the vertex $P$ and $X$ is not between the intersection points of base lines.

As a result we can write as follows (see 3.16):

$$x_G = |n_G - q_G|. \tag{3.16}$$

The following corollary gives a Galilean-analogues of median property in the Euclidean geometry.

**Corollary 3.1** Let the side of a triangle $\Delta PQR$ in $G^2$ have lengths $|d_G(P,Q)| = r_G$, $|d_G(Q,R)| = p_G$, $|d_G(R,P)| = q_G$. If $X$ is midpoint of $[QR]$ and $|d_G(P,X)| = V_{p_G}$, then (see 3.17)

$$2V_{p_G} = \begin{cases} p_G + 2r_G, & If\ \Delta PQR\ has\ no\ base\ line\ through\ the\ vertex\ P, \\ p_G + 2q_G, & If\ \Delta PQR\ has\ only\ base\ line\ through\ the\ vertex\ P, \\ p_G - 2q_G, & If\ \Delta PQR\ has\ two\ base\ lines\ through\ the\ vertex\ P \\ & and\ X\ is\ between\ the\ intersection\ points\ of\ the\ base\ lines, \\ |p_G - 2r_G|, & If\ \Delta PQR\ has\ two\ base\ lines\ through\ the\ vertex\ P \\ & and\ X\ is\ not\ between\ the\ intersection\ points\ of\ the\ base\ lines. \end{cases} \tag{3.17}$$

**Proof :** It can be proved similar to theorem 3.1.

## 4. Conclusion

In this study, Stewart's theorem and median property have been studied for triangles whose sides are on ordinary lines in the Galilean plane $G^2$. In order to achieve this aim, firstly, we give the definition of base line for these triangles in $G^2$. According to the cases of base lines we get Galilean-analogues of Stewart's theorem and median property.

## Author Contributions

Abdulaziz Açıkgöz: Drafting and writing the article.

Nilgün Sönmez: Correcting the deficiencies of any section with important critical revisions.

## Conflicts of Interest

The authors declare no conflict of interest.

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