Gazi University Journal of Science
GU J Sci
29(4): 769-781 (2016)

A Study on Tests of Hypothesis Based on Ridge Estimator

Esra GÖKPINAR[1,♠], Meral EBEGİL[1]

[1]*Department of Statistics, Gazi University, Ankara, Turkey*

Received: 08/12/2014 *Accepted: 06/06/2016*

ABSTRACT

The literature of Ridge regression include many articles that deal with point estimation of the coefficients vector . However, few of them tackle the statistical inference problem about or some of its components. One of them is introduced by Halawa and Basuiouni[1] who present non-exact tests based on Ridge regression by using two different biasing parameters (k) which are proposed by Hoerl and Kennard [2] and Hoerl et al. [3]. Thus, we investigate others popular k values used the Ridge regression for testing significance of regression coefficients. We compare tests in terms of type I error rates and powers by using Monte Carlo simulation. In addition, a real data example is presented.

Keywords: *Biasing parameters, Ridge regression, Hypothesis testing, Type I error rate, Power of test*

1. INTRODUCTION

The most common method to estimate the regression coefficients is the Least Squares (LS) method. Some assumptions need to be provided for the LS method to have valid results. One of these assumptions is that there is no relationship among explanatory variables. Nevertheless, this assumption is very difficult to achieve in real world problems. Multicollinearity problem appears in the case of violation of this assumption. When this assumption is not achieved, using the LS estimator leads to mis-modelling.

The Ridge regression is the most common method to overcome this problem. The biasing parameters can provide Ridge estimators with smaller Mean Square Errors (MSEs) than the variances of the LS estimator. For this purpose, there are many different optimum k values described in the literature. Hoerl and Kennard [2] gave an optimum k value to minimize the MSE of Ridge estimator. After this study, many authors obtained the optimum k values according to different criteria. The other popular studies in this area can be given as in

♠Corresponding author, e-mail: eyigit@gazi.edu.tr

Theobald [4], Hoerl et al.[3], Lawless and Wang [5], Dempster et al. [6], Gibbons [7], Saleh and Kibria [8], Kibria [9], Khalafand Shukur [10], Zhang and Ibrahim [11], Saleh [12], Alkhamisi et al. [13], Alkhamisi and Shukur [14] and Muniz and Kibria [15]. Gökpınar and Ebegil [16] compared these popular k values according to their MSE criteria and their acceptance rate of the testing procedure given in Liski [17,18]. Although the literature of Ridge regression include many articles that deal with point estimation of the coefficients vector, few of them tackle the statistical inference problem about or some of its components. Obenchain [19] and Coutsourides and Troskie [20] proved the exact F-test and t-test based on Ridge regression. Ullah, Carter and Srivastova [21] derived an asymptotic expansion of the F-ratio. However, this expansion is not easy to apply in practice. Hoerl and Kennard [2] suggested the analysis of variance table for testing using Ridge regression. Halawa and Basuiouni[1] proposed non-exact tests based on Ridge regression. For computing value of test Halawa and Bassiouni [1] used two k values which is proposed by Hoerl and Kennard [2] and Hoerl et al. [3].

In this study, we compare these popular k values according to type I error rates and powers of tests given in Halawa and Bassiouni [1] to identify which k value is the best. This article is organized as follows. The second section gives the testing procedure with popular optimum k values. The third section compares tests based on these optimum k values in terms of type I error rates and powers using Monte Carlo simulation. The fourth section presents an example to show how to calculate tests based on Ridge estimators. The final section is the conclusion.

2. TEST STATISTICS FOR REGRESSION COEFFICIENTS

We consider the following multiple linear regression model;

$$Y = X\beta + \varepsilon, \qquad \varepsilon \sim N\left(\underline{0},\ \sigma^2 I_n\right), rank\left(X_{n\times q}\right) = q \le n. \tag{1}$$

Y is $(n\times 1)$ dimensional vector of dependent variable centered about their mean; X is $(n\times q)$ dimensional non-stochastic input matrix; centered and scaled such that $X'X$ is in correlation form, β is $(q\times 1)$ dimensional unknown coefficient vector and εis $(n\times 1)$ error vector providing $E(\varepsilon)=0$ and $E(\varepsilon\varepsilon')=\sigma^2 I_n$. The LS estimator can be given as $\hat{\beta} = \left(X'X\right)^{-1} X'Y$, which is known to be the best linear unbiased estimator of β parameter. The problem of interest involves testing

$$H_0 : \beta = 0 \text{ against } \mathrm{H}_1 : \beta \neq 0. \tag{2}$$

In this case, the testing procedure is given in Eq. (3)

$$t = \hat{\beta}_i \Big/ S\left(\hat{\beta}_i\right) \tag{3}$$

where $\hat{\beta}_i$ is the ith component of $\hat{\beta}$ and $S\left(\hat{\beta}_i\right)$ is the square root of the ith diagonal element of

$Var\left(\hat{\beta}\right) = \hat{\sigma}^2\left(X'X\right)^{-1}$ with

$$\hat{\sigma}^2 = \left(Y - X\hat{\beta}\right)'\left(Y - X\hat{\beta}\right) \Big/ \left(n-q-1\right) \tag{4}$$

We call Eq. (3) as the LS test. When $X'X$ is ill-conditioned, adding k to diagonal of $X'X$ improves the ill-conditioned situation. In this case, the Ridge estimator can be described as

$$\tilde{\beta}_{(k)} = \left(X'X + k\mathrm{I}\right)^{-1} X'Y \tag{5}$$

where $k>0$. To test H_0 given in Eq. (2), Halawa and Bassiouni [1] proposed test procedure based on Ridge regression. The test statistic which is non-exact t-type test is given as

$$t_k = \frac{\tilde{\beta}_{i(k)}}{S\left(\tilde{\beta}_{i(k)}\right)} \tag{6}$$

where $\tilde{\beta}_{i(k)}$ is the ith element of $\tilde{\beta}_{(k)}$ and $S^2\left(\tilde{\beta}_{i(k)}\right)$ is an estimate of the variance of $\tilde{\beta}_{(k)}$ given by the ith diagonal element of matrix

$$Var\left(\tilde{\beta}_{(k)}\right) = \sigma^2 \left(X'X + kI_n\right)^{-1} X'X \left(X'X + kI_n\right)^{-1}. \quad (7)$$

When σ^2 is unknown, the residual mean square of the ridge regression $\hat{\sigma}_k^2$ given in Eq. (8) is used instead of σ^2.

$$\hat{\sigma}_k^2 = \left(Y - X\tilde{\beta}_{(k)}\right)' \left(Y - X\tilde{\beta}_{(k)}\right) \Big/ (n - q - 1). \quad (8)$$

Since $\hat{\sigma}_k^2$ is a consistent estimator of σ^2, t_k has asymptotically standard normal null distribution [1]. The biasing factors can provide Ridge estimators with smaller MSEs than the variances of the LS estimator. So $|t_k|$ given in Eq. (6) is greater than $|t|$ given in Eq.(3) and it is expected to be more sensitive t-type statistic. Therefore, a suitable choice of k is important to avoid too much reduction. To compute the test value, Halawa and Bassiouni [1] used two k values proposed by Hoerl and Kennard [1] and Hoerl et al. [3]. In this study, we consider other popular k values described in the literature to test H_0.

2.1. The Optimum k Values of Ridge Estimator

This section gives the popular *k* values for Ridge estimator in the literature. The description of each popular *k* values is given in Table 1.

Table 1. The popular *k* values used in the Ridge estimator

Authors	*k* values
Hoerl and Kennard (1970)	$\hat{k}_{HK} = \hat{\sigma}^2 / \hat{\alpha}^2_{\max}$
Theobald (1974)	$\hat{k}_{T} = 2\hat{\sigma}^2 / \hat{\beta}'\hat{\beta}$
Hoerl et. al.(1975)	$\hat{k}_{HKB} = q\hat{\sigma}^2 / \hat{\beta}'_{LS}\hat{\beta}_{LS}$
Lawless and Wang (1976)	$\hat{k}_{LW} = q\hat{\sigma}^2 / \hat{\beta}'X'X\hat{\beta}$
Hocking et al.(1976)	$\hat{k}_{HSL} = \hat{\sigma}^2 \sum_{i=1}^{q}(\lambda_i\hat{\alpha}_i)^2 \Big/ \left(\sum_{i=1}^{q}\lambda_i\hat{\alpha}_i^2\right)^2$
Kibria(2003)	$\hat{k}_{AM} = \sum_{i=1}^{q}\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)\Big/ q$
Kibria(2003)	$\hat{k}_{GM} = \prod_{i=1}^{q}\left(\hat{\sigma}^2/\hat{\alpha}_i^2\right)^{1/q}$
Kibria(2003)	$\hat{k}_{MED} = Median\left\{\hat{\sigma}^2/\hat{\alpha}_i^2 : i=1,...,q\right\}$
Khalaf and Shukur(2005)	$\hat{k}_{KS} = \lambda_{\max}\hat{\sigma}^2 \Big/ \left((n-q)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}^2_{\max}\right)$
Alkhamisi et al. (2006)	$\hat{k}_{AKSMAX} = Max\left\{\lambda_i\hat{\sigma}_i^2 \Big/ \left((n-q)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2\right) : i=1,...,q\right\}$
Alkhamisi et al. (2006)	$\hat{k}_{AKSMED} = Median\left(\lambda_i\hat{\sigma}_i^2 \Big/ \left((n-q)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2\right) : i=1,...,q\right)$
Alkhamisi et al. (2006)	$\hat{k}_{AKSAM} = \sum_{i=1}^{q}\left(\lambda_i\hat{\sigma}_i^2 \Big/ \left((n-q)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2\right)\right) / q$
Muniz and Kibria (2009)	$\hat{k}_{MKGM1} = \left(\prod_{i=1}^{q}\left(\lambda_i\hat{\sigma}_i^2 \Big/ \left((n-q)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2\right)\right)\right)^{1/q}$
Muniz and Kibria (2009)	$\hat{k}_{MKMAX} = Max\left(1\Big/\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2} : i=1,...,q\right)$
Muniz and Kibria (2009)	$\hat{k}_{MKGM2} = \left(\prod_{i=1}^{q}\left(1\Big/\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)\right)^{1/q}$
Muniz and Kibria (2009)	$\hat{k}_{MKGM3} = \left(\prod_{i=1}^{q}\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2}\right)^{1/q}$
Muniz and Kibria (2009)	$\hat{k}_{MKMED} = Median\left\{1\Big/\sqrt{\hat{\sigma}^2/\hat{\alpha}_i^2} : i=1,...,q\right\}$
Muniz et al. (2010)	$\hat{k}_{MKSMAX1} = Max\left\{\left((n-q)\hat{\sigma}_i^2 + \lambda_{\max}\hat{\alpha}_i^2\right)\Big/\left(\lambda_{\max}\hat{\sigma}_i^2\right) : i=1,...,q\right\}$
Muniz et al. (2010)	$\hat{k}_{MKSMAX2} = Max\left\{\left(\lambda_{\max}\hat{\sigma}_i^2\right)\Big/\left((n-q)\hat{\sigma}_i^2 + \lambda_{\max}\hat{\alpha}_i^2\right) : i=1,...,q\right\}$
Muniz et al. (2010)	$\hat{k}_{MKSGM1} = \left(\prod_{i=1}^{q}\left(\left((n-q)\hat{\sigma}_i^2 + \lambda_{\max}\hat{\alpha}_i^2\right)\Big/\left(\lambda_{\max}\hat{\sigma}_i^2\right)\right)\right)^{1/q}$
Muniz et al. (2010)	$\hat{k}_{MKSGM2} = \left(\prod_{i=1}^{q}\left(\left(\lambda_{\max}\hat{\sigma}_i^2\right)\Big/\left((n-q)\hat{\sigma}_i^2 + \lambda_{\max}\hat{\alpha}_i^2\right)\right)\right)^{1/q}$
Muniz et al. (2010)	$\hat{k}_{MKSMED} = Median\left\{\left((n-q)\hat{\sigma}_i^2 + \lambda_{\max}\hat{\alpha}_i^2\right)\Big/\left(\lambda_{\max}\hat{\sigma}_i^2\right) : i=1,...,q\right\}$

Here $\hat{\alpha}$ is defined as

$$\hat{\alpha} = P'\hat{\beta},$$

where P is an orthonormal matrix which satisfies $P'X'XP = \Lambda$ and Λ is a diagonal matrix of eigenvalues (λ_i, i=1,...,q) of X'X.

To calculate test values based on Ridge estimator, we use these biasing factors. The test given in Eq. (6) is called according to each biasing factor, that is, HK, T, HKB, HSL, LW, AM, GM, MED, KS, AKSMAX, AKSMED, AKSAM, MKGM1, MKMAX, MKGM2, MKGM3, MKMED, MKSMAX1, MKSMAX2, MKSGM1, MKSGM2, MKSMED tests. We calculate the type I errors and powers of tests using simulation. However, LW, AM, GM, MED, AKSMAX, AKSAM, MKMAX, MKGM2, MKGM3, MKMED, MKSMAX1, MKSMAX2, MKSGM1, MKSGM2, MKSMED tests have been found not to perform as successfully as the other tests. Therefore, the results of these tests are not presented here.

3. SIMULATION STUDY

In this section, to test H_0 in Eq. (2), we compare the performances of the tests in Eq. (3) and Eq. (6) in terms of their type I error rates and powers under different biasing parameters *k*. For this purpose, we consider three sets of sample sizes *n*=10, 20, 30, 50, 100 and explanatory variables number *q*=2, 4, 7. The explanatory variables are generated by

$$x_{ij} = (1-\rho^2)^{1/2} u_{ij} + \rho u_{iq} \qquad i = 1, 2, ..., n; \; j = 1, 2, ..., q \tag{9}$$

where u_{ij} is the independent standard normal pseudo-random numbers and ρ^2 is the theoretical correlation between any two explanatory variables. We choose two sets of correlations, ρ=0.85 and 0.99. Moreover, each of the vectors given in Eq. (9) is centered and scaled. The dependent variable is generated from

$$y_i = \beta_1 x_{i1} + \beta_1 x_{i2} + ... + \beta_q x_{iq} + \varepsilon_i \;\; i = 1, 2, ..., n$$

where ε_i is independent normal pseudo-numbers with zero mean and standard deviation σ. We choose two sets of standard deviations of errors, σ=0.5, 4. Additionally, each of these vectors is centered about its mean. We generate 5000 sets of explanatory and dependent variables, described above.

The study of Halawa and Bassiouni [1] is based on the most and the least favorable orientations of β given by the normalized eigenvectors corresponding to the largest and the smallest eigenvalues of X'X which is in correlation form. Thus the most favorable (MF) orientation is $\left(1/\sqrt{q}\right)1_q$ and the least favorable (LF) orientation is any normalized vector orthogonal to 1_q. In such a way, all components in the MF orientation are equal, while all of components in the LF orientation are equally likely [1]. Firstly, we calculate type I error rates of the tests under H_0. For this purpose, the *i*th component of the orientation vector of β is replaced by zero; that is, $\beta = \left(1/\sqrt{q}\right)1_q, \; \beta_i = 0, \; i = 1, ..., q.$ The test statistics given in Eq. (3) and Eq. (6) are calculated from the model given in Eq. (1) and type I error rates are estimated by the proportion of the value of test statistics that exceed the critical values calculated from the *t*-distribution with (*n*-*q*-1) degrees of freedom. The numerical results for type I error rates under α=0.05 are given in Tables 2-5.

Table 2. Type I error rates when ρ=0.85 and σ=0.5

Tests	*q*	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.052	0.052	0.047	0.046	0.049	0.045	0.049	0.044	0.054	0.046	0.049	0.049	0.047	0.055	0.051
	LF	0.052	0.049	0.053	0.049	0.054	0.049	0.042	0.047	0.049	0.051	0.052	0.046	0.051	0.048	0.053
HKB	MF	0.093	0.080	0.137	0.111	0.121	0.049	0.065	0.068	0.077	0.081	0.036	0.050	0.054	0.064	0.063
	LF	0.054	0.053	0.067	0.062	0.067	0.043	0.044	0.054	0.049	0.056	0.039	0.040	0.047	0.048	0.055
HK	MF	0.058	0.064	0.072	0.073	0.077	0.047	0.053	0.050	0.058	0.053	0.046	0.052	0.050	0.056	0.057
	LF	0.049	0.049	0.057	0.053	0.057	0.048	0.044	0.053	0.049	0.053	0.050	0.049	0.053	0.050	0.057
HSL	MF	0.083	0.066	0.108	0.083	0.087	0.031	0.051	0.053	0.063	0.064	0.017	0.040	0.046	0.056	0.058
	LF	0.055	0.053	0.072	0.062	0.071	0.033	0.041	0.053	0.051	0.058	0.017	0.033	0.042	0.048	0.054
T	MF	0.093	0.080	0.137	0.111	0.121	0.048	0.054	0.049	0.060	0.055	0.047	0.052	0.050	0.055	0.055
	LF	0.054	0.053	0.067	0.062	0.067	0.048	0.045	0.055	0.049	0.054	0.050	0.049	0.053	0.050	0.055
KS	MF	0.056	0.055	0.052	0.050	0.051	0.049	0.052	0.048	0.054	0.047	0.049	0.052	0.050	0.055	0.054
	LF	0.052	0.050	0.055	0.050	0.054	0.050	0.045	0.052	0.049	0.051	0.052	0.050	0.053	0.050	0.055
AKSMED	MF	0.039	0.050	0.047	0.047	0.050	0.039	0.046	0.045	0.053	0.046	0.017	0.046	0.046	0.054	0.051
	LF	0.027	0.047	0.052	0.049	0.053	0.037	0.042	0.048	0.049	0.052	0.017	0.043	0.050	0.048	0.053
MKGM1	MF	0.047	0.051	0.047	0.047	0.049	0.041	0.046	0.045	0.053	0.046	0.019	0.047	0.046	0.054	0.051
	LF	0.044	0.048	0.053	0.049	0.053	0.038	0.042	0.047	0.049	0.052	0.016	0.045	0.049	0.048	0.053

Table 3.Type I error rates when ρ=0.85 and σ=4

Tests	*q*	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.051	0.049	0.052	0.048	0.046	0.052	0.049	0.050	0.053	0.051	0.051	0.051	0.049	0.048	0.048
	LF	0.051	0.049	0.051	0.053	0.051	0.048	0.051	0.042	0.052	0.048	0.050	0.048	0.048	0.050	0.049
HKB	MF	0.050	0.054	0.058	0.057	0.057	0.045	0.049	0.052	0.057	0.057	0.040	0.043	0.046	0.049	0.049
	LF	0.053	0.052	0.059	0.061	0.061	0.040	0.048	0.046	0.055	0.057	0.038	0.044	0.048	0.050	0.050
HK	MF	0.049	0.049	0.052	0.050	0.049	0.049	0.052	0.053	0.056	0.056	0.050	0.051	0.050	0.052	0.051
	LF	0.050	0.048	0.052	0.055	0.054	0.048	0.052	0.046	0.056	0.054	0.047	0.052	0.052	0.053	0.054
HSL	MF	0.051	0.058	0.066	0.061	0.059	0.035	0.049	0.052	0.062	0.057	0.016	0.037	0.040	0.046	0.050
	LF	0.052	0.056	0.067	0.065	0.066	0.033	0.046	0.046	0.062	0.057	0.018	0.036	0.045	0.049	0.052
T	MF	0.050	0.054	0.058	0.057	0.057	0.049	0.052	0.054	0.056	0.056	0.049	0.050	0.051	0.051	0.050
	LF	0.053	0.052	0.059	0.061	0.061	0.048	0.052	0.046	0.056	0.055	0.048	0.051	0.052	0.053	0.052
KS	MF	0.051	0.049	0.054	0.048	0.045	0.052	0.052	0.053	0.055	0.054	0.050	0.052	0.051	0.051	0.052
	LF	0.052	0.049	0.052	0.054	0.051	0.048	0.053	0.045	0.054	0.051	0.049	0.052	0.053	0.053	0.052
AKSMED	MF	0.038	0.045	0.051	0.048	0.045	0.038	0.049	0.049	0.053	0.051	0.014	0.047	0.046	0.049	0.048
	LF	0.039	0.045	0.051	0.053	0.051	0.034	0.050	0.043	0.052	0.048	0.015	0.047	0.050	0.050	0.049
MKGM1	MF	0.046	0.048	0.053	0.048	0.045	0.039	0.050	0.049	0.053	0.050	0.014	0.047	0.047	0.049	0.048
	LF	0.048	0.048	0.051	0.053	0.051	0.036	0.050	0.043	0.052	0.048	0.013	0.047	0.050	0.050	0.049

Table 4. Type I error rates when ρ=0.99 and σ=0.5

Tests	*q*	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.048	0.049	0.052	0.048	0.051	0.051	0.050	0.046	0.052	0.049	0.047	0.047	0.052	0.054	0.047
	LF	0.052	0.052	0.050	0.049	0.052	0.046	0.052	0.046	0.051	0.048	0.051	0.050	0.049	0.053	0.048
HKB	MF	0.108	0.143	0.151	0.163	0.172	0.047	0.059	0.059	0.065	0.064	0.035	0.043	0.050	0.052	0.050
	LF	0.058	0.060	0.062	0.065	0.067	0.040	0.050	0.052	0.050	0.049	0.038	0.042	0.049	0.048	0.046
HK	MF	0.066	0.096	0.107	0.098	0.116	0.051	0.053	0.053	0.054	0.054	0.044	0.051	0.055	0.055	0.051
	LF	0.052	0.055	0.056	0.056	0.060	0.045	0.054	0.054	0.051	0.050	0.050	0.050	0.053	0.054	0.051
HSL	MF	0.124	0.327	0.392	0.261	0.376	0.061	0.266	0.317	0.432	0.435	0.002	0.127	0.198	0.164	0.304
	LF	0.055	0.090	0.093	0.096	0.103	0.014	0.055	0.062	0.075	0.083	0.004	0.026	0.046	0.056	0.068
T	MF	0.108	0.143	0.151	0.163	0.172	0.050	0.053	0.052	0.054	0.056	0.044	0.050	0.053	0.054	0.050
	LF	0.058	0.060	0.062	0.065	0.067	0.044	0.053	0.054	0.051	0.049	0.049	0.050	0.052	0.053	0.050
KS	MF	0.048	0.052	0.051	0.050	0.051	0.052	0.053	0.052	0.052	0.053	0.045	0.052	0.054	0.055	0.051
	LF	0.052	0.051	0.050	0.049	0.052	0.046	0.054	0.054	0.052	0.050	0.051	0.051	0.054	0.054	0.051
AKSMED	MF	0.033	0.062	0.062	0.047	0.051	0.037	0.048	0.047	0.052	0.049	0.015	0.044	0.050	0.053	0.047
	LF	0.019	0.035	0.041	0.048	0.050	0.029	0.048	0.049	0.051	0.048	0.011	0.045	0.050	0.053	0.049
MKGM1	MF	0.040	0.047	0.049	0.048	0.051	0.036	0.047	0.046	0.052	0.049	0.014	0.043	0.050	0.053	0.047
	LF	0.040	0.048	0.048	0.049	0.052	0.024	0.047	0.049	0.052	0.048	0.007	0.044	0.049	0.053	0.048

Table 5. Type I error rates when ρ=0.99 and σ=4

Tests	*q*	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.051	0.051	0.053	0.048	0.050	0.048	0.049	0.058	0.052	0.050	0.045	0.053	0.048	0.051	0.053
	LF	0.054	0.050	0.055	0.049	0.050	0.050	0.050	0.048	0.050	0.051	0.049	0.052	0.057	0.050	0.049
HKB	MF	0.051	0.057	0.060	0.057	0.062	0.043	0.050	0.060	0.055	0.052	0.034	0.042	0.045	0.055	0.055
	LF	0.056	0.056	0.061	0.057	0.058	0.044	0.051	0.048	0.052	0.054	0.036	0.043	0.052	0.051	0.050
HK	MF	0.048	0.053	0.055	0.053	0.056	0.048	0.050	0.062	0.059	0.053	0.044	0.052	0.052	0.057	0.057
	LF	0.053	0.051	0.056	0.053	0.054	0.051	0.054	0.050	0.056	0.055	0.048	0.055	0.061	0.056	0.054
HSL	MF	0.051	0.073	0.079	0.085	0.085	0.013	0.052	0.072	0.075	0.077	0.006	0.026	0.048	0.066	0.076
	LF	0.050	0.067	0.078	0.080	0.080	0.017	0.046	0.056	0.066	0.073	0.006	0.023	0.040	0.055	0.061
T	MF	0.051	0.057	0.060	0.057	0.062	0.048	0.050	0.061	0.058	0.052	0.044	0.052	0.050	0.056	0.056
	LF	0.056	0.056	0.061	0.057	0.058	0.050	0.054	0.050	0.055	0.055	0.049	0.055	0.060	0.054	0.052
KS	MF	0.049	0.050	0.052	0.049	0.050	0.050	0.051	0.062	0.059	0.051	0.045	0.053	0.051	0.057	0.058
	LF	0.054	0.048	0.054	0.049	0.051	0.051	0.054	0.051	0.056	0.054	0.049	0.057	0.061	0.056	0.055
AKSMED	MF	0.015	0.036	0.043	0.044	0.048	0.037	0.046	0.057	0.052	0.050	0.014	0.045	0.046	0.052	0.052
	LF	0.014	0.033	0.042	0.046	0.050	0.039	0.052	0.049	0.050	0.051	0.014	0.050	0.057	0.049	0.049
MKGM1	MF	0.036	0.047	0.051	0.049	0.050	0.031	0.047	0.056	0.052	0.050	0.010	0.045	0.045	0.053	0.052
	LF	0.039	0.046	0.052	0.049	0.050	0.031	0.049	0.047	0.049	0.051	0.009	0.046	0.058	0.050	0.049

We observe the following from the numerical results in Tables 2-5. In general, the type I error rates of the LS test are close to the nominal level under both orientations. For example, in case of ρ=0.85, q=4, n=10 under MF, when σ=0.5, the type I error rate of the LS is 0.045, when σ=4, the type I error rate of the LS is 0.052. In case of ρ=0.99, q=4, n=10 under MF, when σ=0.5, the type I error rate of the LS is 0.051, when σ=4, the type I error rate of the LS is 0.048.
In general, the type I error rates of HKB, HK, T and KS tests are close to the nominal α. However when q=2, especially under MF, the type I error rates of HKB, HK and T tests are bigger than nominal α. In this case, unlike these tests, KS is close to the nominal α.

While the type I error rates of HSL test are close to the nominal α for ρ=0.85, those of HSL test are bigger than the nominal α for ρ=0.99, that is, this test is negatively affected from the increment of ρ.

We observe that the type I error rates of MKGM1, AKSMED tests are not affected from both ρ and σ values. For example, in case of σ=0.5, q=4, n=10 under MF, when ρ=0.85, the type I error rates of AKSMED and MKGM1 are 0.039 and 0.041, respectively, when ρ=0.99, the type I error rate of AKSMED and MKGM1 are 0.037 and 0.036, respectively. In case of σ=4, q=4, n=10 under MF, when ρ=0.85, the type I error rates of AKSMED and MKGM1 are 0.038 and 0.039, respectively, when ρ=0.99, the type I error rates of AKSMED and MKGM1 are 0.037 and 0.031, respectively. However, when sample sizes are small, the type I error rates of AKSMED and MKGM1 are smaller than the nominal α.

In general, these rates are nearly close to nominal α. However, these rates are bigger than nominal α in some cases. It is seen that these rates are affected when the value of q increases and the value of n decreases.

After calculating the type I error rates of these methods, we calculate the powers of the tests. For this purpose, the ith component of the orientation vector of β is replaced by $w(0)\sigma\beta_i$, where $w^2(0)=\left(\left(1+(\mathrm{p}-2)\rho\right)/(1-\rho)(1+(\mathrm{p}-1)\rho)\right)$ [1]. The test statistics given in Eq. (3) and Eq. (6) are calculated from the model given in Eq. (1). The powers of tests are estimated by the proportion of the value of test statistics that exceed the critical values calculated from the t-distribution with (n-q-1) degrees of freedom. The numerical results for powers of the tests are presented in Tables 6-9.

Table 6. Powers of tests when ρ=0.85 and σ=0.5

Tests	*q*	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.082	0.254	0.164	0.257	0.203	0.082	0.093	0.111	0.106	0.111	0.060	0.064	0.074	0.075	0.075
	LF	0.087	0.239	0.156	0.262	0.202	0.142	0.180	0.199	0.233	0.245	0.078	0.166	0.189	0.207	0.190
HKB	MF	0.340	0.430	0.477	0.463	0.499	0.107	0.192	0.227	0.214	0.247	0.047	0.089	0.115	0.075	0.147
	LF	0.146	0.321	0.287	0.364	0.341	0.168	0.254	0.288	0.313	0.360	0.062	0.206	0.250	0.207	0.254
HK	MF	0.155	0.384	0.370	0.414	0.419	0.094	0.125	0.147	0.154	0.172	0.058	0.073	0.088	0.089	0.094
	LF	0.102	0.290	0.217	0.322	0.276	0.162	0.221	0.247	0.282	0.312	0.077	0.196	0.226	0.229	0.223
HSL	MF	0.243	0.353	0.333	0.375	0.361	0.088	0.151	0.181	0.185	0.201	0.024	0.079	0.107	0.118	0.129
	LF	0.168	0.305	0.270	0.342	0.313	0.149	0.276	0.317	0.352	0.386	0.026	0.211	0.279	0.303	0.311
T	MF	0.340	0.430	0.477	0.463	0.499	0.097	0.133	0.155	0.159	0.180	0.059	0.072	0.087	0.087	0.094
	LF	0.146	0.321	0.287	0.364	0.341	0.165	0.225	0.254	0.283	0.319	0.078	0.193	0.222	0.227	0.221
KS	MF	0.132	0.300	0.214	0.286	0.223	0.095	0.121	0.136	0.133	0.130	0.060	0.074	0.087	0.085	0.086
	LF	0.100	0.262	0.178	0.274	0.213	0.163	0.214	0.235	0.264	0.270	0.080	0.196	0.222	0.224	0.213
AKSMED	MF	0.122	0.285	0.193	0.275	0.214	0.084	0.100	0.117	0.111	0.113	0.024	0.064	0.077	0.076	0.076
	LF	0.094	0.253	0.172	0.270	0.209	0.148	0.194	0.211	0.240	0.249	0.036	0.180	0.201	0.212	0.193
MKGM1	MF	0.104	0.281	0.182	0.271	0.209	0.086	0.099	0.118	0.112	0.114	0.026	0.064	0.078	0.076	0.076
	LF	0.094	0.250	0.165	0.268	0.207	0.152	0.197	0.213	0.241	0.250	0.038	0.180	0.201	0.212	0.193

Table 7. Powers of tests when ρ=0.85 and σ=4

Tests	*q*	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.145	0.139	0.161	0.203	0.142	0.071	0.096	0.096	0.118	0.095	0.058	0.071	0.074	0.084	0.084
	LF	0.150	0.142	0.162	0.200	0.160	0.133	0.180	0.200	0.251	0.185	0.078	0.186	0.200	0.255	0.243
HKB	MF	0.213	0.287	0.320	0.357	0.343	0.081	0.123	0.133	0.157	0.144	0.044	0.071	0.085	0.096	0.094
	LF	0.206	0.261	0.294	0.323	0.320	0.155	0.251	0.286	0.338	0.288	0.066	0.220	0.257	0.305	0.298
HK	MF	0.176	0.200	0.232	0.284	0.233	0.082	0.111	0.119	0.141	0.117	0.055	0.078	0.084	0.091	0.090
	LF	0.176	0.190	0.221	0.268	0.231	0.151	0.215	0.244	0.299	0.234	0.080	0.213	0.240	0.282	0.270
HSL	MF	0.205	0.275	0.298	0.326	0.310	0.064	0.131	0.151	0.171	0.167	0.016	0.061	0.091	0.105	0.114
	LF	0.201	0.257	0.284	0.306	0.307	0.137	0.270	0.318	0.365	0.336	0.027	0.227	0.284	0.336	0.352
T	MF	0.213	0.287	0.320	0.357	0.343	0.082	0.115	0.120	0.144	0.120	0.056	0.076	0.083	0.090	0.088
	LF	0.206	0.261	0.294	0.323	0.320	0.153	0.224	0.249	0.304	0.243	0.080	0.212	0.237	0.280	0.268
KS	MF	0.172	0.171	0.186	0.221	0.154	0.081	0.110	0.111	0.130	0.102	0.057	0.077	0.084	0.089	0.087
	LF	0.174	0.168	0.186	0.217	0.169	0.152	0.210	0.232	0.279	0.206	0.081	0.214	0.235	0.274	0.261
AKSMED	MF	0.167	0.164	0.180	0.215	0.149	0.069	0.099	0.099	0.121	0.096	0.019	0.072	0.076	0.085	0.084
	LF	0.165	0.162	0.179	0.211	0.166	0.138	0.192	0.208	0.257	0.188	0.039	0.197	0.211	0.260	0.247
MKGM1	MF	0.163	0.155	0.173	0.211	0.146	0.071	0.101	0.100	0.122	0.096	0.021	0.071	0.077	0.086	0.084
	LF	0.162	0.155	0.172	0.207	0.164	0.140	0.194	0.209	0.259	0.188	0.042	0.198	0.214	0.260	0.247

Table 8. Powers of tests when ρ=0.99 and σ=0.5

Tests	q	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.126	0.164	0.112	0.175	0.144	0.086	0.099	0.090	0.097	0.132	0.055	0.071	0.074	0.088	0.079
	LF	0.133	0.171	0.116	0.179	0.142	0.158	0.197	0.225	0.191	0.311	0.081	0.189	0.216	0.280	0.227
HKB	MF	0.655	0.792	0.751	0.809	0.798	0.113	0.200	0.219	0.215	0.305	0.044	0.080	0.092	0.116	0.106
	LF	0.573	0.787	0.660	0.828	0.792	0.217	0.345	0.401	0.375	0.539	0.068	0.247	0.286	0.374	0.302
HK	MF	0.339	0.566	0.429	0.675	0.599	0.097	0.130	0.120	0.132	0.185	0.056	0.079	0.080	0.095	0.091
	LF	0.312	0.465	0.382	0.561	0.459	0.188	0.255	0.289	0.261	0.405	0.083	0.228	0.260	0.321	0.264
HSL	MF	0.580	0.637	0.743	0.674	0.724	0.153	0.341	0.374	0.398	0.413	0.006	0.153	0.224	0.248	0.286
	LF	0.699	0.751	0.847	0.775	0.830	0.290	0.563	0.609	0.615	0.683	0.016	0.404	0.519	0.587	0.568
T	MF	0.655	0.792	0.751	0.809	0.798	0.099	0.134	0.128	0.138	0.195	0.055	0.078	0.081	0.094	0.090
	LF	0.573	0.787	0.660	0.828	0.792	0.193	0.263	0.299	0.271	0.418	0.082	0.226	0.258	0.319	0.260
KS	MF	0.325	0.486	0.397	0.436	0.276	0.100	0.130	0.119	0.127	0.177	0.056	0.080	0.082	0.096	0.090
	LF	0.288	0.387	0.298	0.325	0.193	0.192	0.255	0.290	0.252	0.390	0.085	0.227	0.262	0.322	0.260
AKSMED	MF	0.266	0.323	0.302	0.307	0.243	0.085	0.104	0.094	0.100	0.134	0.018	0.071	0.075	0.088	0.081
	LF	0.292	0.360	0.312	0.309	0.227	0.172	0.213	0.239	0.196	0.316	0.041	0.210	0.237	0.285	0.230
MKGM1	MF	0.170	0.209	0.140	0.203	0.159	0.085	0.106	0.096	0.101	0.136	0.021	0.071	0.076	0.089	0.081
	LF	0.165	0.212	0.141	0.206	0.154	0.176	0.220	0.247	0.202	0.322	0.043	0.212	0.240	0.289	0.232

Table 9. Powers of tests when ρ=0.99 and σ=4

Tests	q	2					4					7				
	n	10	20	30	50	100	10	20	30	50	100	10	20	30	50	100
LS	MF	0.172	0.205	0.145	0.135	0.148	0.094	0.096	0.100	0.113	0.089	0.054	0.075	0.073	0.078	0.074
	LF	0.176	0.206	0.147	0.136	0.146	0.151	0.209	0.206	0.239	0.175	0.080	0.182	0.221	0.220	0.182
HKB	MF	0.694	0.833	0.781	0.767	0.811	0.106	0.152	0.170	0.166	0.171	0.041	0.077	0.084	0.083	0.083
	LF	0.682	0.826	0.773	0.753	0.806	0.213	0.347	0.386	0.394	0.393	0.070	0.238	0.292	0.273	0.250
HK	MF	0.370	0.555	0.459	0.445	0.495	0.099	0.121	0.124	0.128	0.122	0.051	0.083	0.084	0.083	0.080
	LF	0.363	0.539	0.445	0.423	0.482	0.186	0.264	0.279	0.277	0.255	0.082	0.220	0.259	0.244	0.212
HSL	MF	0.370	0.725	0.787	0.823	0.817	0.187	0.506	0.536	0.563	0.606	0.002	0.270	0.392	0.467	0.555
	LF	0.363	0.738	0.801	0.833	0.823	0.288	0.571	0.609	0.601	0.619	0.014	0.410	0.503	0.529	0.549
T	MF	0.694	0.833	0.781	0.767	0.811	0.101	0.123	0.129	0.131	0.127	0.052	0.082	0.081	0.081	0.079
	LF	0.682	0.826	0.773	0.753	0.806	0.190	0.272	0.291	0.288	0.265	0.081	0.218	0.258	0.243	0.212
KS	MF	0.322	0.465	0.336	0.229	0.210	0.102	0.119	0.122	0.125	0.116	0.053	0.083	0.082	0.083	0.081
	LF	0.318	0.443	0.312	0.218	0.207	0.186	0.263	0.277	0.274	0.241	0.083	0.221	0.259	0.244	0.212
AKSMED	MF	0.313	0.382	0.324	0.279	0.240	0.086	0.103	0.104	0.114	0.090	0.016	0.075	0.074	0.078	0.075
	LF	0.322	0.381	0.329	0.282	0.236	0.172	0.225	0.217	0.242	0.179	0.040	0.205	0.232	0.225	0.185
MKGM1	MF	0.224	0.254	0.174	0.154	0.161	0.086	0.105	0.110	0.116	0.091	0.015	0.074	0.075	0.078	0.076
	LF	0.229	0.254	0.177	0.156	0.158	0.174	0.229	0.227	0.246	0.185	0.042	0.205	0.234	0.227	0.186

We observe to following results from Tables 6-9. Generally, HKB, HK, HSL, T and KS tests appear to be more powerful than AKSMED and MKGM1 tests.

Since the type I error rates of HKB, HK, HSL and T tests are bigger than the nominal level 0.05, these tests seem to be more powerful than the others, for q=2 and σ=0.5 under MF. For this reason, in these cases, since we do not take these tests into consideration, KS test appears to be more powerful than the others.

HSL test appears to be more powerful than the other tests for ρ=0.85, except case of q=2. However, when ρ=0.99, since the type I error rates of this test have bigger than nominal level 0.05, it seems to be more powerful than the others. For this reason, in these cases we do not take it into consideration. In this cases, that is, when ρ=0.99, HKB test performs better than the other tests.

HK, T and KS tests are more conservative than the other tests for all cases, except case of q=2. Furthermore, their power values seem to be closer to each other.

We observe that all of the tests are slightly more powerful when values of ρ increase. Especially, these increments in LF methods are greater according to MF methods.

4. NUMERICAL EXAMPLE

In this section, we consider the following data given by Woods et al. [23]. This data came from an experimental study into the heat evolved during the setting and hardening of Portland cements of varied composition and dependence of this heat on the percentages of four compounds in the clinkers from which the cement was produced. The four compounds tricalium aluminate(X_1), tricalcium silicate(X_2), tetracalcium aluminaferrite(X_3) and beta-dicalcium silicate(X_4), respectively. The dependent variable Y is the heat evolved in calories per gram of cement after 180 days of curing. Hence, the data are given in Table 10.

Table 10. The data set

Y	78,5	74,3	104,3	87,6	95,9	109,2	102,7	72,5	93,1	115,9	83,8	113,3	109,4
X_1	7	1	11	11	7	11	3	1	2	21	1	11	10
X_2	26	29	56	31	52	55	71	31	54	47	40	66	68
X_3	6	15	8	8	6	9	17	22	18	4	23	9	8
X_4	60	52	20	47	33	22	6	44	22	26	34	12	12

The data are transformed to suit the assumptions of Eq.(1). To detect multicollinearity we obtained the Variance Inflation Factor (VIF) values of explanatory variables given as:

$\mathrm{VIF}_i = \frac{1}{1-R_i^2}$, $i = 1, \dots, 4$, where R_i^2 is the square of the multiple correlation coefficient obtained from regression of the ith explanatory variable on all other explanatory variables. The VIF values are given as:

VIF=(38.496 254.423 46.868 283.513).

Because the VIF values of explanatory variables are greater than 10, there are dependency among explanatory variables. Moreover, to detect multicollinearity we obtained Condition Number (CN) value calculated as $CN = \left(\frac{largest\ eigenvalue(X'X)}{smallest\ eigenvalue(X'X)}\right)^{1/2}$ =37.1063 [24]. Because CN value is greater than 10, there are dependency among explanatory variables. Therefore, ridge estimator should be used instead of the LS estimator. For parameter estimation, we obtained the Ridge estimators used in the simulation study, i.e., LS, HKB, HK, HSL, T, KS, AKSMED and MKGM1 estimators. We want to test the null hypothesis given in Eq. (2) for the significance of the regression coefficients of data. For this purpose, we calculate the p values as $p = 2 \times P\left(t_{n-k-1} > |t_h|\right)$. In the case of p<α, the null hypothesis given in Eq. (2) is rejected. The results are given in Table 11.

Table 11. The results of testing the significance of the regression coefficients

i	LS			HKB			HK			HSL		
	$\hat{\beta}_i$	t ratio	p value	$\hat{\beta}_i$	t ratio	p value	$\hat{\beta}_i$	t ratio	p value	$\hat{\beta}_i$	t ratio	p value

	$\left(S(\hat{\beta}_i)\right)$			$\left(S(\hat{\beta}_i)\right)$			$\left(S(\hat{\beta}_i)\right)$			$\left(S(\hat{\beta}_i)\right)$		
1	31.607 (15.176)	2.083	0.071	26.480 (4.153)	6.377	0.000	27.398 (5.030)	5.447	0.001	27.587 (5.304)	5.200	0.001
2	27.500 (39.015)	0.705	0.501	16.167 (4.512)	3.360	0.009	17.393 (8.616)	2.019	0.078	17.739 (9.672)	1.834	0.104
3	2.261 (16.745)	0.135	0.896	-3.151 (4.084)	-0.771	0.463	-2.288 (5.136)	-0.445	0.668	-2.097 (5.464)	-0.384	0.711
4	-8.353 (41.113)	-0.203	0.844	-20.214 (4.859)	-4.160	0.003	-18.972 (8.972)	-2.118	0.067	-18.613 (10.086)	-1.845	0.102

	T			**KS**			**AKSEMED**			**MKGM1**		
	$\hat{\beta}_i$ $\left(S(\hat{\beta}_i)\right)$	t ratio	p value	$\hat{\beta}_i$ $\left(S(\hat{\beta}_i)\right)$	t ratio	p value	$\hat{\beta}_i$ $\left(S(\hat{\beta}_i)\right)$	t ratio	p value	$\hat{\beta}_i$ $\left(S(\hat{\beta}_i)\right)$	t ratio	p value
1	27.295 (4.894)	5.577	0.001	27.613 (5.345)	5.166	0.001	27.050 (4.613)	5.864	0.000	27.311 (4.915)	5.556	0.000
2	17.212 (8.074)	2.132	0.066	17.789 (9.823)	1.811	0.108	16.833 (6.897)	2.440	0.041	17.243 (8.158)	2.114	0.068
3	-2.390 (4.973)	-0.481	0.644	-2.071 (5.512)	-0.376	0.717	-2.628 (4.635)	-0.567	0.586	-2.374 (4.998)	-0.475	0.648
4	-19.155 (8.379)	-2.286	0.052	-18.561 (10.247)	-1.811	0.108	-19.548 (7.118)	-2.746	0.025	-19.127 (8.470)	-2.258	0.054

The standard errors of ridge estimators are lower than the standard errors of LS estimators. All of the results in Table 11 indicate that β_1 is significant, i.e., X_1 is significant according to results of all the tests except the LS. While β_2 is significant, i.e., X_2 is significant according to the results of HKB and AKSEMED tests, β_2 is not significant, considering those of the others. β_3 is significant, i.e., X_3 is significant compared with the results of all the tests. While β_4 is significant, i.e., X_4 is significant in comparison with the results of HKB and AKSEMED tests, β_4 is not significant considering the results of the other tests.

5. CONCLUSION

In this article we are interested in finding which k value is the best for statistical inference of β in case of multicollinearity. Therefore we compared tests based on different k values according to type I error rates and powers of tests. It could be observed from our simulation results that when HSL test appears to be more powerful than the other tests for moderate multicollinearity, HKB test appears to be more powerful than the other tests for high multicollinearity, except the case of q=2. If the case of q=2, KS test appears to be more powerful than the others.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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