Gazi University Journal of Science
GU J Sci
29(4): 845-854 (2016)

Supra Soft Separation Axioms and Supra Irresoluteness Based on Supra b-Soft Sets

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Received: 28/01/2016 *Accepted: 30/06/2016*

ABSTRACT

This paper introduces the notion of supra soft b-separation axioms based on the supra b-open soft sets which are more general than supra open soft sets. We investigate the relationships between these supra soft separation axioms. Furthermore, with the help of examples it is established that the converse does not hold. We show that, a supra soft topological space (X,μ,E) is supra soft b-T_1-space, if x_e is supra b-closed soft set in μ for each $x\in X$. For each $i=0,1,2,3,4$, not every supra soft b-open soft subspace of supra soft b-T_i-space is supra soft b-T_i.

Keywords: *Soft sets, Soft topological spaces, Supra soft topological spaces, Supra b-open soft sets, Supra soft b-Ti spaces (i=1,2,3,4), Supra soft continuity, Supra b-irresolute open soft function.*

1. INTRODUCTION

The real world is too complex for our immediate and direct understanding. Several models of reality that are simplifications of aspects of the real word have been established. Unfortunately, these mathematical models are too complicated and we cannot find the exact solutions. The uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical sciences and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world or to the limitations of the means used to measure objects. Thus classical set theory, which is based on the crisp and exact case, may not be fully suitable for handling such problems of uncertainty. There are several theories, for example, theory of fuzzy sets [37], theory of intuitionistic fuzzy sets [4], theory of vague sets, theory of interval mathematics [5] and theory of rough sets [33]. These can be considered as tools for dealing with uncertainties but all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [31]. He initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above.

In his paper [31], he presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly. After presentation of the operations of soft sets [26], the properties and applications of soft set theory have been studied increasingly [23, 32, 34].

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In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [11, 20, 24, 25, 26, 27, 32]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [6].

It got some stability only after the introduction of soft topology [36] in 2011. In [12], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, $\alpha-$open soft and $\beta-$open soft and investigated their properties in detail. the notion of pre open soft sets is extended in [28]. Kandil et al. [19] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al. [15]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al. [13, 14, 16, 17, 18, 21, 29]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [9]. Recently, Kandil et al. [21] introduced the concept of soft supra g-closed soft sets in supra soft topological spaces, which is generalized in [3]. The notion of b-open soft sets was initiated for the first time by El-sheikh and Abd El-latif [8], which is generalized to the supra soft topological spaces in [2, 10]. Properties of b-open soft sets in [35] are discussed.

The main purpose of this paper, is to generalize the notion of supra soft separation axioms [1] by using the notions of supra b-open soft sets.

2. PRELIMINARIES

Definition 2.1 [31] Let X be an initial universe and E be a set of parameters. Let $\mathcal{P}(X)$ denote the power set of X. A pair (F,E) denoted by F is called a soft set over X, where F is a mapping given by $F: E \to \mathcal{P}(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For $e \in E$, $F(e)$ may be considered as the set of $e-$approximate elements of the soft set (F,E) i.e

$$(F,E) = \{(\mathrm{e}, F(e)) : e \in E,\ F: E \to \mathcal{P}(X)\}$$

. If $e \notin E$, then $F(e) = \emptyset$. The set of all soft sets over X will be denoted by $S_E(X)$.

Definition 2.2 *[7] Let $F, G \in S_E(X)$. Then,*

(1)F is said to be null soft set, denoted by $\tilde{\emptyset}$, if $F(e) = \emptyset$ for all $e \in E$.

(2)F is said to be absolute soft set, denoted by $\tilde{X}$, if $F(e) = X$ for all $e \in E$.

(3) F is soft subset of G, denoted by $F \tilde{\subseteq} G$, if $F(e) \subseteq G(e)$ for all $e \in E$.

(4) F and G are soft equal, denoted by $F = G$, if $F \tilde{\subseteq} G$ and $G \tilde{\subseteq} F$.

(5) The soft union of F and G, denoted by $F \tilde{\cup} G$, is a soft set over X and defined by $F \tilde{\cup} G: E \to \mathcal{P}(X)$ such that $(F \tilde{\cup} G)(e) = F(e) \cup G(e)$ for all $e \in E$.

(6) The soft intersection of F and G, denoted by $F \tilde{\cap} G$, is a soft set over X and defined by $F \tilde{\cap} G: E \to \mathcal{P}(X)$ such that $(F \tilde{\cap} G)(e) = F(e) \cap G(e)$ for all $e \in E$.

(7) The soft complement $(\tilde{X} - F)$ of a soft set F is denoted by $F^{\tilde{c}}$ and defined by $F^{\tilde{c}}: E \to \mathcal{P}(X)$ such that $F^{\tilde{c}}(e) = X \backslash F(e)$ for all $e \in E$.

Definition 2.3 *[38] A soft set $F \in S_E(X)$ is called a soft point, denoted by e_F, if there exist an $e \in E$ such that $F(e) \neq \emptyset$ and $F(e') = \emptyset$ for each $e' \in E \backslash \{e\}$. The soft point e_F is said to be in the soft set G, if $F(e) \subseteq G(e)$ for $e \in E$ and we write $e_F \tilde{\in} G$.*

Definition 2.4 *[19, 36] A soft set F over X where $F(e) = \{x\}$, $\forall e \in E$ is called singleton soft point and denoted by x_E or (x,E).*

Definition 2.5 *[36] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \tilde{\subseteq} S_E(X)$ is called a soft topology on X if:*

(1) $\tilde{X}, \tilde{\emptyset} \in \tau$,

(2) The soft union of any number of soft sets in τ belongs to τ,

(3) The soft intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. A soft set F over X is said to be an open soft set in X if $F \in \tau$, and it is called a closed soft set in X, if its relative complement $F^{\tilde{c}}$ is a soft open set and we write F is a soft τ-closed.

Definition 2.6 *[36] Let (X,τ,E) be a soft topological space over X and $F \in S_E(X)$. Then, the soft interior and soft closure of F, denoted by $int(F)$ and $cl(F)$, respectively, are defined as:*

$int(F) = \tilde{\cup}\{G \mid G$ is soft open set and $G \sqsubseteq F\}$,

$cl(F) = \tilde{\cap}\{H \mid H$ is soft closed set and $F \sqsubseteq H\}$.

Definition 2.7 *[22] Let $S_E(X)$ and $S_K(Y)$ be families of soft sets, $u: X \to Y$ and $p: E \to K$ be mappings. Therefore $f_{pu}: S_E(X) \to S_K(Y)$ is called a soft function.*

(1) If $F \in S_E(X)$, then the image of F under f_{pu}, written as $f_{pu}(F)$, is a soft set in $S_K(Y)$ such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F(e)), & p^{-1}(k) \neq \emptyset \\ \emptyset, & \text{otherwise} \end{cases}$$

for each $k \in K$.

(2) If $G \in S_K(Y)$, then the inverse image of G under f_{pu}, written as $f_{pu}^{-1}(G)$, is a soft set in $S_E(X)$ such that

$$f_{pu}^{-1}(G)(e) = \begin{cases} \left(u^{-1}\left(G(p(e))\right)\right), & p(e) \in \mathrm{Y} \\ \emptyset, & \text{otherwise} \end{cases}$$

for each $e \in E$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective. Several properties and characteristics for f_{pu} and f_{pu}^{-1} are reported in detail in [22].

Definition 2.8 *[38] Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces and $f_{pu}: S_E(X) \to S_K(Y)$ be a function. Then, the function f_{pu} is called,*

(1) Continuous soft if $f_{pu}^{-1}(G) \in \tau_1$ for each $G \in \tau_2$.

(2)Open soft if $f_{pu}(F) \in \tau_2$ for each $F \in \tau_1$.

Definition 2.9 *[9] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \tilde{\subseteq} S_E(X)$ is called supra soft topology on X with a fixed set E if*

(1)$\tilde{X}, \tilde{\emptyset} \in \mu$,

(2) The soft union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called supra soft topological space (or supra soft spaces) over X.

Definition 2.10 *[9] Let (X, τ, E) be a soft topological space and (X, μ, E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau \subseteq \mu$.*

Definition 2.11 *[9] Let (X, μ, E) be a supra soft topological space over X, then the members of μ are said to be supra open soft sets in X.*

Definition 2.12 *[9] Let (X, μ, E) be a supra soft topological space over X and $F \in S_E(X)$. Then, the supra soft interior and supra soft closure of F, denoted by $int^s(F)$ and $cl^s(F)$, respectively, are defined as*

$$int^s(F) = \tilde{\bigcup} \{G : G \text{ is supra open soft set and } G \tilde{\subseteq} F\},$$

$$cl^s(F) = \tilde{\bigcap} \{H : H \text{ is supra closed soft set and } F \tilde{\subseteq} H\}.$$

Clearly $int^s(F)$ is the largest supra open soft set over X which contained in F and $cl^s(F)$ is the smallest supra closed soft set over X which contains F.

Definition 2.13 *[2] Let (X, μ, E) be a supra soft topological space and $F \in S_E(X)$. Then, F is called supra b-open soft set if $F \tilde{\subseteq} cl^s int^s(F) \tilde{\cup} int^s cl^s(F)$. We denote the set of all supra b-open soft sets by $SBOS(X, \mu, E)$, or $SBOS_E(X)$ and the set of all supra b-closed soft sets by $SBCS(X, \mu, E)$, or $SBCS_E(X)$.*

Definition 2.14 *[2] Let (X, μ, E) be a supra soft topological space over X and $F \in S_E(X)$. Then, the supra b-soft interior and supra b-soft closure of F, denoted by $int_b^s(F)$ and $cl_b^s(F)$, respectively, are defined as*

$$int_b^s(F) = \tilde{\bigcup} \{G : G \text{ is supra } b-\text{open soft set and } G \tilde{\subseteq} F\}$$

$cl_b^s(F) = \tilde{\bigcap}\{H: H \text{ is supra b-closed soft set and } F \tilde{\subseteq} H\}$.

Definition 2.15 *[1, 9] Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu}: S_E(X) \to S_K(Y)$ is called*

(1) Supra continuous soft function if $f_{pu}^{-1}(F) \in \mu_1$ for each $F \in \tau_2$.

(2) Supra open soft if $f_{pu}(F) \in \mu_1$ for each $F \in \tau_1$.

(3) Supra irresolute soft if $f_{pu}^{-1}(F) \in \mu_1$ for each $F \in \mu_2$.

(4) Supra irresolute open soft if $f_{pu}(F) \in \mu_2$ for each $F \in \mu_1$.

(5) Supra b-continuous soft if $f_{pu}^{-1}(F) \in SBOS(X, \mu_1, E)$ for each $F \in \mu_2$.

3. SUPRA SOFT B-SEPARATION AXIOMS

Definition 3.1 Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Let $x, y \in X$ such that $x \neq y$. Then, (X, μ, E) is called

(1) Supra soft b-T_0-space if there exists a μ-supra b-open soft set F containing one of the points x, y but not the other.

(2) Supra soft b-T_1-space if there exist μ-supra b-open soft sets F and G such that $x \in F$, $y \not\in F$ and $y \in G$, $x \not\in G$.

(3) Supra soft b-Hausdorff space or supra soft b-T_2-space if there exist μ-supra b-open soft sets F and G such that $x \in F$, $y \in G$ and $F \tilde{\cap} G = \tilde{\emptyset}$.

Theorem 3.1 Every supra soft T_i-space is supra soft b-T_i for each $i = 0,1,(2)$

Proof. Since every supra open soft set is supra b-open in due to [2], then for each $i = 0,1,2$, supra soft T_i-space is supra soft b-T_i.

Remark 3.1 The converse of Theorem 3.1 is not true in general, as following examples shall show.

Examples 3.1

(1) Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is a soft set over X defined as follows:

$$F(e_1) = \{h_1\}, \quad F(e_2) = X.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2\}$, where G_1 and G_2 are soft sets over X defined as follows:

$$G_1(e_1) = X, \quad G_1(e_2) = \{h_2\},$$
$$G_2(e_1) = \{h_1\}, \quad G_2(e_2) = X.$$

Therefore, (X, μ, E) is supra soft b-T_2-space, but it is not supra soft T_2.

(2) Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is a soft set over X defined as follows:

$$F(e_1) = \{h_1\}, \quad F(e_2) = X.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2\}$, where G_1 and G_2 are soft sets over X defined as follows:

$$G_1(e_1) = \{h_2\}, \quad G_1(e_2) = \{h_1\},$$
$$G_2(e_1) = \{h_1\}, \quad G_2(e_2) = X.$$

Therefore, (X, μ, E) is supra soft b-T_1-space, but it is not supra soft T_1.

(3) Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is a soft set over X defined as follows by

$$F(e) = \{h_1, h_2, h_3\}.$$

Then, τ defines a soft topology on X. The associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2\}$, where $G_1 and G_2$ are soft sets over X defined as follows:

$$G_1(e) = \{h_1, h_2, h_3\},$$
$$G_2(e) = \{h_1, h_2, h_4\}.$$

Hence, (X, μ, E) is supra soft b-T_0-space, but it is not supra soft T_0.

The proof of the next theorem follows immediately from Definition 3.1.

Theorem 3.2 Every supra soft b-T_i-space is supra soft b-T_{i-1} for each $i = 1,2$.

Remark 3.2 The converse of Theorem 3.2 is not true in general, as following examples shall show.

Examples 3.2 Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is a soft set over X defined as follows:

$$F(e_1) = \{h_1\}, \quad F(e_2) = X.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2, G_3\}$, where G_1, G_2 and G_3 are soft sets over X defined as follows:

$$G_1(e_1) = X, \quad G_1(e_2) = \{h_2\},$$
$$G_2(e_1) = \{h_1\}, \quad G_2(e_2) = X,$$
$$G_3(e_1) = \{h_1\}, \quad G_3(e_2) = \{h_1\}.$$

Therefore, (X, μ, E) is supra soft b-T_1-space, but it is not supra soft b-T_2.

Proposition 3.1 Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. If there exist μ-supra b-open soft sets F and G such that either $x \in F$ and $y \in F^{\tilde{c}}$ or $y \in G$ and $x \in G^{\tilde{c}}$. Then, (X, μ, E) is supra soft b-T_0-space.

Proof. Let $x, y \in X$ such that $x \neq y$. Let F and G be μ-supra b-open soft sets such that either $x \in F$ and $y \in F^{\tilde{c}}$ or $y \in G$ and $x \in G^{\tilde{c}}$. If $x \in F$ and $y \in F^{\tilde{c}}$. Then, $y \in (F(e))^{\tilde{c}}$ for each $e \in E$. This implies that, $y \in F(e)$ for each $e \in E$. Therefore, $y \not\in F$. Similarly, if $y \in G$ and $x \in G^{\tilde{c}}$, then $x \not\in G$. Hence, (X, μ, E) is supra soft b-T_0-space.

Proposition 3.2 Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. If there exist μ-supra b-open soft sets F and G such that $x \in F$ and $y \in F^{\tilde{c}}$ and $y \in G$ and $x \in G^{\tilde{c}}$. Then (X, μ, E) is supra soft b-T_1-space.

Proof. It is similar to the proof of Proposition 3.1.

Theorem 3.3 A supra soft topological space (X, μ, E) is supra soft b-T_0-space if and only if for each pair of distinct points x and y in X, $cl_b^s(x_E) \neq cl_b^s(y_E)$.

Proof. Let (X, μ, E) be a supra soft b-T_0-space and $x, y \in X$ such that $x \neq y$. Then, there exists a μ-supra b-open soft set F such that $x \in F$ and $y \not\in F$. Hence, $F^{\tilde{c}}$ is supra b-closed soft set containing y but not x. It follows that, $cl_b^s(y_E) \tilde{\subseteq} F^{\tilde{c}}$ and so $x \not\in cl_b^s(y_E)$. Thus, $cl_b^s(x_E) \neq cl_b^s(y_E)$. On the other hand, let x, y be two distinct points in X such that $cl_b^s(x_E) \neq cl_b^s(y_E)$. Then, there exists a point z belongs to one of the sets $cl_b^s(x_E), cl_b^s(y_E)$ but not the other. Say, $z \in cl_b^s(x_E)$ and $z \not\in cl_b^s(y_E)$. Now, if $x \in cl_b^s(y_E)$, then, $cl_b^s(x_E) \tilde{\subseteq} cl_b^s(y_E)$, which is a contradiction with $z \not\in cl_b^s(y_E)$. So, $x \not\in cl_b^s(y_E)$. Hence, $[cl_b^s(y_E)]^{\tilde{c}}$ is supra b-open soft set containing x but not y. Thus, (X, μ, E) is supra soft b-T_0-space.

Theorem 3.4 A supra soft topological space (X, μ, E) is supra soft b- T_1, if x_E is supra b-closed soft set in μ for each $x \in X$.

Proof. Suppose that $x \in X$ and x_E is supra b-closed soft set in μ. Then, $x_E^{\tilde{c}}$ is supra b-open soft set in μ. Let $x, y \in X$ such that $x \neq y$. For $x \in X$ and $x_E^{\tilde{c}}$ is supra b-open soft set such that $x \not\in x_E^{\tilde{c}}$ and $y \in x_E^{\tilde{c}}$. Similarly $y_E^{\tilde{c}}$ is supra b-open soft set in μ such that $y \not\in y_E^{\tilde{c}}$ and $x \in y_E^{\tilde{c}}$. Thus, (X, μ, E) is supra soft b-T_1-space over X.

Remark 3.3 The converse of Theorem 3.4 is not true in general, as following examples shall show.

Examples 3.3 Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is a soft sets over X defined as follows:

$$F(e_1) = X, \quad F(e_2) = \{h_2\}.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2, G_3\}$ where G_1, G_2 and G_3 are soft sets over X defined as follows:

$$G_1(e_1) = X, \quad G_1(e_2) = \{h_2\},$$
$$G_2(e_1) = \{h_1\}, \quad G_2(e_2) = X$$
$$G_3(e_1) = \{h_1\}, \quad G_3(e_2) = \{h_1\}.$$

Then, μ defines a supra soft topology on X. Therefore, (X, μ, E) is a supra soft b-T_1-space. On the other hand, we note that for the singleton soft points h_1 and h_2, where

$$h_1(e_1) = \{h_1\}, \quad h_1(e_2) = \{h_1\},$$
$$h_2(e_1) = \{h_2\}, \quad h_2(e_2) = \{h_2\}.$$

The relative complement $h_1^{\tilde{c}}$ and $h_2^{\tilde{c}}$, where

$$h_1^{\tilde{c}}(e_1) = \{h_2\}, \quad h_1^{\tilde{c}}(e_2) = \{h_2\},$$
$$h_2^{\tilde{c}}(e_1) = \{h_1\}, \quad h_2^{\tilde{c}}(e_2) = \{h_1\}.$$

Thus, $h_{2E}^{\tilde{c}}$ is not μ-supra b-open soft set. This shows that, the converse of the above theorem does not hold.

Also, we have

$\mu_{e_1} = \{X, \emptyset, \{h_1\}\}$, and $\mu_{e_2} = \{X, \emptyset, \{h_1\}, \{h_2\}\}$. Therefore,$(X, \mu_{e_1})$ is not a supra b-T_1-space, at the time that (X, μ, E) is a supra soft b-T_1-space.

Definition 3.2 Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Let G be a μ-supra b-closed soft set in X and $x \in X$ such that $x \not\in G$. If there exist μ-supra b-open soft sets F_1 and F_2 such that $x \in F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$, then (X, μ, E) is called supra soft b-regular space. A supra soft b-regular b-T_1-space is called supra soft b-T_3-space.

The proofs of the next propositions are obvious.

Proposition 3.3 Let (X, τ, E) be a soft topological space and μ be an associated supra soft topology with τ. Let G be a μ-supra b-closed soft set in X and $x \in X$ such that $x \not\in G$. If (X, μ, E) is supra soft b-regular space, then there exists a μ-supra b-open soft set F such that $x \in F$ and $F \tilde{\cap} G = \tilde{\emptyset}$.

Proposition 3.4 Let (X, μ, E) be a supra soft topological space, $F \in S_E(X)$ and $x \in X$. Then,

(1) $x \in F$ if and only if $x_E \tilde{\subseteq} F$.

(2) If $x_E \tilde{\cap} F = \tilde{\emptyset}$, then $x \not\in F$.

Proof. Immediate.

Theorem 3.5 Let (X,μ,E) be a supra soft topological space and $x \in X$. If (X,μ,E) is supra soft b-regular space, then

(1) $x \notin F$ if and only if $x_E \tilde{\cap} F = \tilde{\emptyset}$ for every μ-supra b-closed soft set F.

(2) $x \notin G$ if and only if $x_E \tilde{\cap} G = \tilde{\emptyset}$ for every μ-supra b-open soft set G.

Proof.

(1) Let F be a μ-supra b-closed soft set such that $x \notin F$. Since (X,μ,E) is supra soft b-regular space. By Proposition 3.3 there exists a μ-supra b-open soft set G such that $x \in G$ and $F \tilde{\cap} G = \tilde{\emptyset}$. It follows that, $x_E \tilde{\subseteq} G$ from Proposition 3.4 (1). Hence, $x_E \tilde{\cap} F = \tilde{\emptyset}$. Conversely, if $x_E \tilde{\cap} F = \tilde{\emptyset}$, then $x \notin F$ from Proposition 3.4 (2).

(2) Let G be a μ-supra b-open soft set such that $x \notin G$. If $x \notin G(e)$ for some $e \in E$, then we get the proof. If $x \notin G(e_1)$ and $x \in G(e_2)$ for some $e_1, e_2 \in E$, then $x \in G^{\tilde{c}}(e_1)$ and $x \notin G^{\tilde{c}}(e_2)$ for some $e_1, e_2 \in E$. This means that, $x_E \tilde{\cap} G \neq \tilde{\emptyset}$. Hence, $G^{\tilde{c}}$ is μ-supra b-closed soft set such that $x \notin G^{\tilde{c}}$. It follows by (1) $x_E \tilde{\cap} G^{\tilde{c}} = \tilde{\emptyset}$. This implies that, $x_E \tilde{\subseteq} G$ and so $x \in G$, which is contradiction with $x \tilde{\notin} G(e_1)$ for some $e_1 \in E$. Therefore, $x_E \tilde{\cap} G = \tilde{\emptyset}$. Conversely, if $x_E \tilde{\cap} G = \tilde{\emptyset}$, then it is obvious that $x \notin G$. This completes the proof.

The next Corollary follows directly from Theorem 3.5.

Corollary 3.1 Let (X,μ,E) be a supra soft topological space and $x \in X$. If (X,μ,E) is supra soft b-regular space, then the following statements are equivalent:

(1) (X,μ,E) is supra soft b-T_1-space.

(2) $\forall\ x, y \in X$ such that $x \neq y$, there exist μ-supra b-open soft sets F and G such that $x_E \tilde{\subseteq} F$ and $y_E \tilde{\cap} F = \tilde{\emptyset}$ and $y_E \tilde{\subseteq} G$ and $x_E \tilde{\cap} G = \tilde{\emptyset}$.

Theorem 3.6 Let (X,μ,E) be a supra soft topological space and $x \in X$. Then, the following statements are equivalent:

(1) (X,μ,E) is supra soft b-regular space.

(2) For every μ-supra b-closed soft set G such that $x_E \tilde{\cap} G = \tilde{\emptyset}$, there exist μ-supra b-open soft sets F_1 and F_2 such that $x_E \tilde{\subseteq} F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$.

Proof.

(1) $\Rightarrow$ (2) Let G be a μ-supra b-closed soft set such that $x_E \tilde{\cap} G = \tilde{\emptyset}$. Then, $x \notin G$ from Theorem 3.5 (1). It follows by (1), there exist μ-supra b-open soft sets F_1 and F_2 such that $x \in F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. This means that, $x_E \tilde{\subseteq} F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$.

(2) $\Rightarrow$ (1) Let G be a μ-supra b-closed soft set such that $x \notin G$. Then, $x_E \tilde{\cap} G = \tilde{\emptyset}$ from Theorem 3.5 (1). It follows by (2), there exist μ-supra b-open soft sets F_1 and F_2 such that $x_E \tilde{\subseteq} F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Hence, $x \in F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Thus, (X,μ,E) is supra soft b-regular space.

Theorem 3.7 Let (X,μ,E) be a supra soft topological space. If (X,μ,E) is supra soft T_3-space, then $\forall\ x \in X$, x_E is μ-supra b-closed soft set.

Proof. We are going to prove that $x_E^{\tilde{c}}$ is μ-supra b-open soft set for each $y \in \{x\}^{\tilde{c}}$. Since (X,μ,E) is supra soft b-T_3-space. Then, by Theorem 3.6, there exist μ-supra b-open soft sets F_y and G such that $y_E \tilde{\subseteq} F_y$ and $x_E \tilde{\cap} F_y = \tilde{\emptyset}$ and $x_E \tilde{\subseteq} G$ and $y_E \tilde{\cap} G = \tilde{\emptyset}$. It follows that, $\cup_{y \in \{x\}^{\tilde{c}}} F_y \tilde{\subseteq} x_E^{\tilde{c}}$. Now, we want to prove that $x_E^{\tilde{c}} \tilde{\subseteq} \cup_{y \in \{x\}^{\tilde{c}}} F_y$. Let $\cup_{y \in \{x\}^{\tilde{c}}} F_y = H$, where $H(e) = \cup_{y \in \{x\}^{\tilde{c}}} F(e)_y$ for each $e \in E$. Since $x_E^{\tilde{c}}(e) = \{x\}^{\tilde{c}}$ for each $e \in E$ from Definition 2.4. So, for each $y \in \{x\}^{\tilde{c}}$ and $e \in E$, $x_E^{\tilde{c}}(e) = \{x\}^{\tilde{c}} = \cup_{y \in \{x\}^{\tilde{c}}} \{y\} = \cup_{y \in \{x\}^{\tilde{c}}} y_E(e) \tilde{\subseteq} \cup_{y \in \{x\}^{\tilde{c}}} F(e)_y = H(e)$.

Thus, $x_E^{\tilde{c}} \tilde{\subseteq} \cup_{y \in \{x\}^{\tilde{c}}} F_y$, and so $x_E^{\tilde{c}} = \cup_{y \in \{x\}^{\tilde{c}}} F_y$. This means that, $x_E^{\tilde{c}}$ is μ-supra b-open soft set for each $y \in \{x\}^{\tilde{c}}$. Therefore, x_E is μ-supra b-closed soft set.

Theorem 3.8 Every supra soft b-T_3-space is supra soft b-T_2-space.

Proof. Let (X,μ,E) be a supra soft b-T_3-space and $x, y \in X$ such that $x \neq y$. By Theorem 3.7, y_E is μ-supra b-closed soft set and $x \notin y_E$. It follows from the supra soft b-regularity, there exist μ-supra b-open soft sets F_1 and F_2 such that $x \in F_1$, $y_E \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Thus, $x \in F_1$, $y \in y_E \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Therefore, (X,μ,E) is supra soft b-T_2-space.

Corollary 3.2 The following implications hold from Theorem 3.1, Theorem 3.2 and [[1],Corollary 3.2] for a supra soft topological space (X,μ,E).

$$\begin{array}{ccccccc}
\text{soft } T_3 & \rightarrow & \text{soft } T_2 & \rightarrow & \text{soft } T_1 & \rightarrow & \text{soft } T_0 \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{supra soft } T_3 & \rightarrow & \text{supra soft } T_2 & \rightarrow & \text{supra soft } T_1 & \rightarrow & \text{supra soft } T_0 \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{supra soft b} - T_3 & \rightarrow & \text{supra soft b} - T_2 & \rightarrow & \text{supra soft b} - T_1 & \rightarrow & \text{supra soft b} - T_0
\end{array}$$

Definition 3.3 Let (X,μ,E) be a supra soft topological space, F and G be μ-supra b-closed soft sets in X such that $F \tilde{\cap} G=\tilde{\emptyset}$. If there exist μ-supra b-open soft sets F_1 and F_2 such that $F \tilde{\subseteq} F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2=\tilde{\emptyset}$, then (X,μ,E) is called supra soft b-normal space. A supra soft b-normal b-T_1-space is called a supra soft b-T_4-space.

Theorem 3.9 Let (X,μ,E) be a supra soft topological space and $x \in X$. Then, the following statements are equivalent:

(1)(X,μ,E) is supra soft b-normal space.

(2)For every μ-supra b-closed soft set F and μ-supra b-open soft set G such that $F \tilde{\subseteq} G$, there exists a μ-supra b-open soft set F_1 such that $F \tilde{\subseteq} F_1$, $cl_b^s F_1 \tilde{\subseteq} G$.

Proof.

(1) $\Rightarrow$ (2)Let F be a μ-supra b-closed soft set and G be a μ-supra b-open soft set such that $F \tilde{\subseteq} G$. Then, $F, G^{\tilde{c}}$ are μ-supra b-closed soft sets such that $F \tilde{\cap} G^{\tilde{c}}=\tilde{\emptyset}$. It follows by (1), there exist μ-supra b-open soft sets F_1 and F_2 such that $F \tilde{\subseteq} F_1$, $G^{\tilde{c}} \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2=\tilde{\emptyset}$. Now, $F_1 \tilde{\subseteq} F_2^{\tilde{c}}$, so $cl_b^s F_1 \tilde{\subseteq} cl_b^s F_2^{\tilde{c}} = F_2^{\tilde{c}}$, where G is μ-supra b-open soft set. Also, $F_2^{\tilde{c}} \tilde{\subseteq} G$. Hence, $cl_b^s F_1 \tilde{\subseteq} F_2^{\tilde{c}} \tilde{\subseteq} G$. Thus, $F \tilde{\subseteq} F_1$, $cl_b^s F_1 \tilde{\subseteq} G$.

(2) $\Rightarrow$ (1)Let G_1, G_2 be μ-supra b-closed soft sets such that $G_1 \tilde{\cap} G_2=\tilde{\emptyset}$. Then $G_1 \tilde{\subseteq} (G_2)^{\tilde{c}}$, then by hypothesis, there exists a μ-supra b-open soft set F_1 such that $G_1 \tilde{\subseteq} F_1$, $cl_b^s F_1 \tilde{\subseteq} (G_2)^{\tilde{c}}$. So, $G_2 \tilde{\subseteq} [cl_b^s F_1]^{\tilde{c}}$, $G_1 \tilde{\subseteq} F_1$ and $[cl_b^s F_1]^{\tilde{c}} \tilde{\cap} F_1=\tilde{\emptyset}$, where F_1 and $[cl_b^s F_1]^{\tilde{c}}$ are μ-supra b-open soft sets. Thus, (X,μ,E) is supra soft b-normal space.

Obviously, from the fact that, the soft intersection of two supra b-open soft sets need not to be supra b-open soft. Then, not every supra soft b-open soft subspace of supra soft b-T_i-space is supra soft b-T_i-space for each $i = 0,1,2,3,4$.

Theorem 3.10 Let (X,μ,E) be a supra soft topological space. If (X,μ,E) is supra soft b-normal space and x_E is μ-supra b-closed soft set for each $x \in X$, then (X,μ,E) is supra soft b-T_3-sp

Proof. Since x_E is μ-supra b-closed soft set for each $x \in X$, then (X,μ,E) is supra soft b-T_1-space from Theorem 3.4. Also, (X,μ,E) is supra soft b-regular space from Definition 3.3 and Theorem 3.6. Hence, (X,μ,E) is supra soft b-T_3-space

4. SUPRA B-IRRESOLUTE SOFT FUNCTIONS

Definition 4.1 Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu}:S_E(X) \rightarrow S_K(Y)$ is called

(1)Supra b-open soft if $f_{pu}(F) \in SBOS_E(\mu_1)$ for each $F \in \tau_1$.

(2)Supra b-irresolute soft if $f_{pu}^{-1}(F) \in SBOS_E(\mu_1)$ for each $F \in SBOS_K(\mu_2)$.

(3)Supra b-irresolute open soft if $f_{pu}(F) \in SBOS_K(\mu_2)$ for each $F \in SBOS_E(\mu_1)$.

Theorem 4.1 Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively and $f_{pu}:S_E(X) \rightarrow S_K(Y)$ be a soft function which is bijective and supra b-irresolute open soft. If (X,μ_1,E) is supra soft b- T_0-space, then (Y,μ_2,K) is also a supra soft b-T_0-space.

Proof. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then there exist $x_1, x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2)$=y_2 and $x_1 \neq x_2$. By hypothesis, there exist μ_1-supra b-open soft sets F and G in X such that that either $x_1 \in F$ and $x_2 \notin F$, or $x_2 \in G$ and $x_1 \notin G$. So, either $x_1 \in F_E(e)$ and $x_2 \notin F_E(e)$ or $x_2 \in G_E(e)$

and $x_1 \not\in G_E(e)$ for each $e \in E$. This implies that, either y_1=$u(x_1) \in u[F_E(e)]$ and y_2=$u(x_2) \not\in u[F_E(e)]$ or y_2=$u(x_2) \in u[G_E(e)]$ and y_1=$u(x_1) \not\in u[G_E(e)]$ for each $e \in E$. Hence, either $y_1 \in f_{pu}(F)$ and $y_2 \not\in f_{pu}(F)$ or $y_2 \in f_{pu}(G)$ and $y_1 \not\in f_{pu}(G)$. Since f_{pu} is supra b-irresolute open soft function, then $f_{pu}(F), f_{pu}(G)$ are supra b-open soft sets in Y. Hence, (Y, μ_2, K) is also a supra soft b-T_0-space.

Theorem 4.2 Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra b-irresolute open soft. If (X, μ_1, E) is supra soft b-T_1-space, then (Y, μ_2, K) is also a supra soft b-T_1-space.

Proof. It is similar to the proof of Theorem 4.1.

Theorem 4.3 Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra b-irresolute open soft. If (X, μ_1, E) is supra soft b-T_2-space, then (Y, μ_2, K) is also a supra soft b-T_2-space.

Proof. Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then there exist $x_1, x_2 \in X$ such that $u(x_1)$=y_1, $u(x_2)$=y_2 and $x_1 \neq x_2$. By hypothesis, there exist μ_1-supra b-open soft sets F and G in X such that $x_1 \in F$, $x_2 \in G$ and $F \tilde{\cap} G$=$\tilde{\emptyset}_E$. So, $x_1 \in F_E(e)$, $x_2 \in G_E(e)$ and $F_E(e) \cap G_E(e)$=$\emptyset$ for each $e \in E$. This implies that, y_1=$u(x_1) \in u[F_E(e)]$, y_2=$u(x_2) \in u[G_E(e)]$ for each $e \in E$. Hence, $y_1 \in f_{pu}(F)$, $y_2 \in f_{pu}(G)$ and $f_{pu}(F) \tilde{\cap} f_{pu}(G)$=$f_{pu}[F \tilde{\cap} G]$=$f_{pu}[\tilde{\emptyset}_E]$=$\tilde{\emptyset}_K$. Since f_{pu} is supra b-irresolute open soft function, then $f_{pu}(F), f_{pu}(G)$ are supra b-open soft sets in Y. Thus, (Y, μ_2, K) is also a supra soft b-T_2-space.

Theorem 4.4 Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra b-irresolute soft and supra b-irresolute open soft. If (X, μ_1, E) is supra soft b-regular space, then (Y, μ_2, K) is also a supra soft b-regular space.

Proof. Let G be a supra b-closed soft set in Y and $y \in Y$ such that $y \not\in G$. Since f_{pu} is surjective and supra b-irresolute soft, then there exist $x \in X$ such that $u(x)$=y and $f_{pu}^{-1}(G)$ is supra b-closed soft set in X such that $x \not\in f_{pu}^{-1}(G)$. By hypothesis, there exist μ_1-supra b-open soft sets F and H in X such that $x \in F$, $f_{pu}^{-1}(G) \tilde{\subseteq} H$ and $F \tilde{\cap} H$=$\tilde{\emptyset}_E$. It follows that, $x \in F_E(e)$ for each $e \in E$ and G=$f_{pu}[f_{pu}^{-1}(G)] \tilde{\subseteq} f_{pu}(H)$. So, y=$u(x) \in u[F_E(e)]$ for each $e \in E$ and $G \tilde{\subseteq} f_{pu}(H)$. Hence, $y \in f_{pu}(F)$ and $G \tilde{\subseteq} f_{pu}(H)$ and $f_{pu}(F) \tilde{\cap} f_{pu}(H)$=$f_{pu}[F \tilde{\cap} H]$=$f_{pu}[\tilde{\emptyset}_E]$=$\tilde{\emptyset}_K$. Since f_{pu} is supra b-irresolute open soft function. Then, $f_{pu}(F), f_{pu}(H)$ are supra b-open soft sets in Y. Thus, (Y, μ_2, K) is also a supra soft b-regular space.

The next Corollary follows immediately from Theorem 4.2 and Theorem 4.4.

Corollary 4.1 Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra b-irresolute soft and supra b-irresolute open soft. If (X, μ_1, E) is supra soft b-T_3-space, then (Y, μ_2, K) is also a supra soft b-T_3-space.

Theorem 4.5 Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra b-irresolute soft and supra b-irresolute open soft. If (X, μ_1, E) is supra soft b-normal space, then (Y, μ_2, K) is also a supra soft b-normal space.

Proof. Let F, G be supra b-closed soft sets in Y such that $F \tilde{\cap} G$=$\tilde{\emptyset}_K$. Since f_{pu} is supra b-irresolute soft, then $f_{pu}^{-1}(F)$ and $f_{pu}^{-1}(G)$ are supra b-closed soft set in X such that $f_{pu}^{-1}(F) \tilde{\cap} f_{pu}^{-1}(G)$=$f_{pu}^{-1}[F \tilde{\cap} G] = f_{pu}^{-1}[\tilde{\emptyset}_K]$=$\tilde{\emptyset}_E$. By hypothesis, there exist supra b-open soft sets M and H in X such that $f_{pu}^{-1}(F) \tilde{\subseteq} M$, $f_{pu}^{-1}(G) \tilde{\subseteq} H$ and $F \tilde{\cap} H$=$\tilde{\emptyset}_E$. It follows that, F=$f_{pu}[f_{pu}^{-1}(F)] \tilde{\subseteq} f_{pu}(M)$, G=$f_{pu}[f_{pu}^{-1}(G)] \tilde{\subseteq} f_{pu}(H)$ and $f_{pu}(M) \tilde{\cap} f_{pu}(H)$=$f_{pu}[M \tilde{\cap} H]$=$f_{pu}[\tilde{\emptyset}_E] = \tilde{\emptyset}_K$. Since f_{pu} is supra b-irresolute open soft function. Then, $f_{pu}(M), f_{pu}(H)$ are supra b-open soft sets in Y. Thus, (Y, μ_2, K) is also a supra soft b- normal space.

Corollary 4.2 Let (X, τ_1, E) and (Y, τ_2, K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra b-irresolute soft and supra b-irresolute open soft. If (X, μ_1, E) is supra soft b-T_4-space, then (Y, μ_2, K) is also a supra soft b-T_4-space.

Proof. It is obvious from Theorem 4.2 and Theorem 4.5.

5. CONCLUSION

In this paper, we introduced and investigated some soft separation axioms by using the notion of supra b-open soft sets, which is a generalization of the supra soft separation axioms mentioned in [1]. We studied the relationships between these new soft separation axioms. We showed that, some classical results in supra soft topological space are not true. We hope that, the results in this paper will help researcher enhance and promote the further study on supra soft topology to carry out a general framework for their applications in practical life.

ACKNOWLEDGMENT

This paper is supported by *Northern Borders University*.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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