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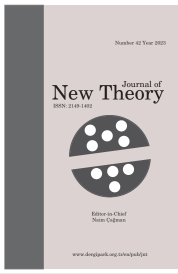

A Hybridization of Modified Rough Bipolar Soft Sets and TOPSIS for MCGDM

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Research Article

Abstract — Uncertain data is a challenge to decision-making (DM) problems. Multi-criteria group decision-making (MCGDM) problems are among these problems that have received much attention. MCGDM is difficult because the existing alternatives frequently conflict with each other. In this article, we suggest a novel hybrid model for an MCGDM approach based on modified rough bipolar soft sets (MRBSs) using a well-known method of technique for order of preference by similarity to ideal solution (TOPSIS), which combines MRBSs theory and TOPSIS for the prioritization of alternatives in an uncertain environment. In this technique, we first introduce an aggregated parameter matrix with the help of modified bipolar soft lower and upper matrices to identify the positive and negative ideal solutions. After that, we define the separation measurements of these two solutions and compute relative closeness to choose the best alternative. Next, an application of the proposed technique in the MCGDM problem is introduced. Afterward, an algorithm for this application is developed, which is illustrated by a case study. The application demonstrates the usefulness and efficiency of the proposal. Compared to some existing studies, we additionally present several merits of our proposed technique. Eventually, the paper handles whether additional studies on these topics are needed.

Keywords *Bipolar soft sets, bipolar soft rough sets, MRBSs, TOPSIS, MCGDM*

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1. Introduction

Various issues in social sciences, engineering, medical sciences, economics, and other domains include uncertainty. It is impossible to address these issues using traditional mathematical approaches. The traditional mathematical model is a rational model of decision-making (DM) that depends on the hypothesis that decision-makers have access to complete knowledge and can make the best decision by weighing every alternative. Due to this, the mathematical model is highly complicated, and an accurate solution cannot be obtained. To overcome this trouble, scholars are endeavoring to discover suitable methodologies and mathematical theories to address data uncertainty. These theories include fuzzy sets (FSs), rough set (RS) theory, vague set theory, automata theory, etc., but they have only partially been successful in solving the problems. These theories diminished the space between traditional mathematical concepts and ambiguous real-world data.

Zadeh [1] developed the FS theory to characterize fuzzy data mathematically. But, in FSs, finding

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the membership function might sometimes be challenging. Consequently, Molodtsov [2] developed the soft set (SS) concept as a new strategy for modeling uncertainty, liberated from this trouble. SS theory needs an approximate description of an object as its initial viewpoint. The selection of suitable parameters like numbers, functions, words, etc., makes SS theory very advantageous and straightforward to use in reality. Maji et al. [3] established several operations on SS. Ali et al. [4] offered a variety of novel operations on SS. Çağman et al. [5] projected the idea of fuzzy SS theory. Al-Shami and Mhemdi [6] offered to belong and non-belong relations on double-framed SS.

The RS theory [7, 8] is an effective mathematical strategy for handling uncertainties. In RSs, uncertainty is characterized by a set's boundary region. Pawlak examined how close a bunch of objects are to the information associated with them using their lower and upper approximations.

The connections between SS theory, RSs, and FSs were provided by Feng et al. [9, 10], leading to three kinds of hybrid models: rough SS (RSS), soft RSs (SRSs), and soft-rough FSs (SRFSs). Shabir et al. [11] redefined a version of an SRS called a modified SRS (MSRS). Shaheen et al. [12] established the concept of dominance-based SRSs.

Bipolarity is critical in various kinds of data when establishing mathematical modeling for specific problems. Bipolarity takes both the positive and negative characteristics of the data into account. The positive data delivers what is conceivable, whereas the negative data emphasizes the impossibility. The idea behind the existence of bipolar information is that a large variety of human DM relies upon bipolar judgmental cognition.

Shabir and Naz [13] put the groundwork for bipolar SSs (BSSs) due to the significance of bipolarity. Following this research, the BSS theory gained much fame among scholars. Karaaslan and Karataş [14] reformulate BSS with a novel approximation, offering a prospect to explore the topological structures of BSS. Mahmood [15] redesigned a form of BSS, known as T-BSS, and employed this concept for DM problems. Moreover, Naz and Shabir [16] established the idea of fuzzy BSS and investigated their algebraic structures. Al-Shami [17] came up with the idea of bipolar soft sets and the relations between them and ordinary points, along with applications.

Karaaslan and Çağman [18] originally suggested bipolar SRSs (BSRSs) tackle the roughness of BSS, which was then changed and improved by establishing the conception of MRBSs by Shabir and Gul [19]. Gul et al. [20] established a new strategy of the roughness for BSS with applications in MCGDM. Gul and Shabir [21] pioneered the concept (α, β)-bipolar fuzzified RS using bipolar fuzzy tolerance relation.

In decision analysis, several multi-criteria DM (MCDM) frameworks have been carried out in the literature. TOPSIS is one of the classical MCDM methods offered by Hwang and Yoon [22] in 1981. The fundamental notion of TOPSIS is to measure the distance between every alternative and ideal solution. The optimal alternative should be the one that has to have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). PIS addresses the scenario for the best possible decision, whereas NIS shows the scenario for the worst. Chen [23] generalized the TOPSIS approach for taking the MCDM problem in a fuzzy context. Afterward, Chen and Tsao [24] proposed the interval-valued fuzzy TOPSIS. Boran et al. [25] fostered the TOPSIS for MCDM problems based on intuitionistic FS. Ali et al. [26] offered the TOPSIS model for probabilistic interval-valued hesitant fuzzy sets with application to healthcare facilities in public hospitals. Eraslan [27] gave a DM method using TOPSIS on SS theory. Eraslan and Karaaslan [28] gave a group DM method based on TOPSIS under a fuzzy SS environment.

Shabir et al. [29] proposed an algebraic approach to N-SS with application in DM via TOPSIS. Akram et al. [30] generalize the TOPSIS and ELECTRE-I methods in a bipolar fuzzy framework. Akram and Adeel [31] extended the TOPSIS for MCGDM via an interval-valued hesitant fuzzy N-SS context. Xu and Zhang [32] constructed a strategy based on maximizing deviation and the TOPSIS to explain multi-attribute DM problems. In 2014, Zhang and Xu [33] extended the TOPSIS in MCDM using Pythagorean FSs. Mousavi-Nasab and Sotoudeh-Anvari [34] gave an MCDM-based method using TOPSIS, COPRAS, and DEA for material selection problems. Mahmood et al. [35] pioneered a novel TOPSIS method based on lattice-ordered T-BSS with applications in DM.

Inspired by the previously mentioned earlier studies and the basic principle of MRBSs, we have observed that the BSS can manage the bipolarity of the data concerning specific alternatives with the assistance of two mappings. The positive side of the data is addressed by one mapping, whereas the other mapping measures the negative side. Keeping in mind the relationship between RSs and BSS, Karaaslan and Çağman [18] attempted to explore the roughness of BSS, which has certain shortcomings. To overcome these shortcomings, Shabir and Gul [19] pioneered the idea of MRBSs.

Moreover, to per best of our knowledge, there does not exist any investigation on the appropriate fusion of TOPSIS with MRBSs. This gap motivates the current research to propose a novel TOPSIS approach using MRBSs and discuss their application in DM.

In a nutshell, to expand the theory of MRBSs, the primary goal of this study is to establish a novel TOPSIS approach for MCGDM problems via the MRBSs environment. We introduce a DM algorithm that determines the best and worst decision among some alternatives, with implementation on selecting the optimal candidate for a particular post.

This article is structured as follows: Section 2 introduces basic notations related to RS, SS, BSS, BSRS, and MRBSs. These notions will assist us in discussing our work and suffice the paper for the reader. After this, we give the general procedure of the TOPSIS technique. Section 3 puts forward the new TOPSIS-based strategy for addressing MCGDM problems using MRBSs. Section 4 states our suggested algorithm for choosing the optimal alternative, which we validate through a fully developed case study in Section 5. Section 6 represents a comparative analysis between the proposed technique and the other existing methods in solving MCGDM problems. Finally, Section 7 ends with an outline of the current work and a few perspectives for the future.

2. Preliminaries

In this section, we recapitulate a few essential notions associated with the background of this study. Throughout this article, unless stated otherwise, we will use $\mho$ for an initial universe, $\mathfrak{A}$ for the set of all the parameters related to the objects in $\mho$, and $2^{\mho}$ for the power set of $\mho$.

Definition 2.1. [7] Let $\varnothing \neq \mho$ be a finite universe, and σ be an equivalence relation of $\mho \times \mho$. Then, $(\mho, \sigma)$ is stated to be an approximation space.

If $\varnothing \neq \mathcal{Q} \subseteq \mho$, then $\mathcal{Q}$ may or may not be expressed as a union of some equivalence classes of $\mho$. If $\mathcal{Q}$ is expressed as a union of some equivalence classes, then $\mathcal{Q}$ is said to be σ-definable; in any other case, it is referred to as σ-undefinable. If $\mathcal{Q}$ is σ-undefinable, then the lower and upper approximations of $\mathcal{Q}$ concerning σ are given as follows:

$$\underline{apr}_{\sigma}(\mathcal{Q}) = \{q \in \mho : [q]_{\sigma} \subseteq \mathcal{Q}\} \tag{1}$$

and

$$\overline{apr}_{\sigma}(\mathcal{Q}) = \{q \in \mho : [q]_{\sigma} \cap \mathcal{Q} \neq \emptyset\} \tag{2}$$

where

$$[q]_{\sigma} = \{r \in \mho : (q, r) \in \sigma\}$$

The boundary region of the RS is characterized as:

$$Bnd_\sigma(\mathcal{Q}) = \overline{apr}_\sigma(\mathcal{Q}) - \underline{apr}_\sigma(\mathcal{Q})$$

From Equations (1) and (2) we can see that

i. An element q belongs to the lower approximation $\underline{apr}_\sigma(\mathcal{Q})$ if all elements equivalent to q belong to $\mathcal{Q}$.

ii. An element q belongs to the upper approximation $\overline{apr}_\sigma(\mathcal{Q})$ if at least one element equivalent to q belongs to $\mathcal{Q}$.

Let $\mho$ be a non-void universe and $\mathfrak{A}$ be a set of parameters. Then, an SS is defined through a set-valued map, as described below.

Definition 2.2. [2] A pair $(\hat{f}, \mathfrak{A})$ is called an SS over $\mho$, where $\hat{f} : \mathfrak{A} \longrightarrow 2^{\mho}$ is a set-valued map.

In other words, an SS over $\mho$ gives a parameterized collection of subsets of $\mho$. An SS over $\mho$ may also be represented as:

$$(\hat{f}, \mathfrak{A}) = \{(\wp, \hat{f}(\wp)) : \wp \in \mathfrak{A}, \hat{f}(\wp) \in 2^{\mho}\}$$

A BSS is obtained through two set-valued maps by considering not only a set of parameters but also an associated set of parameters with an opposite meaning known as "not set of parameters".

Definition 2.3. [3] By a "NOT set of parameters" of $\mathfrak{A}$, we mean a set having the form $\widetilde{\mathfrak{A}} = \{\neg\wp : \wp \in \mathfrak{A}\}$ where $\neg\wp =$ not $\wp$, for all $\wp \in \mathfrak{A}$.

Definition 2.4. [13] A triplet $(\hat{f}, \hat{g} : \mathfrak{A})$ is termed as a BSS over $\mho$ where $\hat{f} : \mathfrak{A} \longrightarrow 2^{\mho}$ and $\hat{g} : \widetilde{\mathfrak{A}} \longrightarrow 2^{\mho}$ such that, for all $\wp \in \mathfrak{A}$, $\hat{f}(\wp) \cap \hat{g}(\neg\wp) = \varnothing$.

In other words, a BSS over $\mho$ offers a couple of parameterized families of subsets of $\mho$ and $\hat{f}(\wp) \cap \hat{g}(\neg\wp) = \varnothing$, for all $\wp \in \mathfrak{A}$, is used as a consistency constraint. A BSS might be characterized as:

$$(\hat{f}, \hat{g} : \mathfrak{A}) = \left\{(\wp, \hat{f}(\wp), \hat{g}(\neg\wp)) : \wp \in \mathfrak{A}, \neg\wp \in \widetilde{\mathfrak{A}} \text{ and } \hat{f}(\wp) \cap \hat{g}(\neg\wp) = \varnothing\right\}$$

After this, the collection of all the BSSs over $\mho$ will be denoted by $\mathcal{BSS}^{\mho}$.

Definition 2.5. [18] Let $(\hat{f}, \hat{g} : \mathfrak{A}) \in \mathcal{BSS}^{\mho}$. Then, $\beta = \langle \mho, (\hat{f}, \hat{g} : \mathfrak{A}) \rangle$ is termed as a $\mathcal{BSA}_s$ (bipolar soft approximation space). For any $\mathcal{Q} \subseteq \mho$, the bipolar soft rough approximations based on β are defined:

$$\underline{BS}_\beta(\mathcal{Q}) = \left(\underline{S}_{\beta^P}(\mathcal{Q}), \underline{S}_{\beta^N}(\mathcal{Q})\right)$$

and

$$\overline{BS}_\beta(\mathcal{Q}) = \left(\overline{S}_{\beta^P}(\mathcal{Q}), \overline{S}_{\beta^N}(\mathcal{Q})\right)$$

where

$$\underline{S}_{\beta^P}(\mathcal{Q}) = \left\{q \in \mho : \exists\ \wp \in \mathfrak{A}, \left[q \in \hat{f}(\wp) \subseteq \mathcal{Q}\right]\right\}$$

$$\underline{S}_{\beta^N}(\mathcal{Q}) = \left\{q \in \mho : \exists\ \neg\wp \in \widetilde{\mathfrak{A}}, \left[q \in \hat{g}(\neg\wp), \hat{g}(\neg\wp) \cap \mathcal{Q}^c \neq \varnothing\right]\right\}$$

$$\overline{S}_{\beta^P}(\mathcal{Q}) = \left\{q \in \mho : \exists\ \wp \in \mathfrak{A}, \left[q \in \hat{f}(\wp), \hat{f}(\wp) \cap \mathcal{Q} \neq \varnothing\right]\right\}$$

and

$$\overline{S}_{\beta^N}(\mathcal{Q}) = \left\{q \in \mho : \exists\ \neg\wp \in \widetilde{\mathfrak{A}}, \left[q \in \hat{g}(\neg\wp) \subseteq \mathcal{Q}^c\right]\right\}$$

Moreover, if $\underline{BS}_\beta(\mathcal{Q}) \neq \overline{BS}_\beta(\mathcal{Q})$, then $\mathcal{Q}$ is called a BSRS; else $\mathcal{Q}$ is called bipolar soft β-definable.

The boundary region of a BSRS is described as:

$$BND_{\beta}(\mathcal{Q}) = (\overline{S}_{\beta^P}(\mathcal{Q}) \setminus \underline{S}_{\beta^P}(\mathcal{Q}), \underline{S}_{\beta^N}(\mathcal{Q}) \setminus \overline{S}_{\beta^N}(\mathcal{Q}))$$

BSRSs were originally initiated by Karaaslan and Çağman [18] to manage the roughness of BSSs, which was subsequently altered and improved by Shabir and Gul [19] by launching the idea of MRBSs. MRBSs are characterized as follows:

Definition 2.6. [19] Let $(\hat{f}, \hat{g} : \mathfrak{A}) \in \mathcal{BSS}^{\mho}$ such that $\hat{f} : \mathfrak{A} \longrightarrow 2^{\mho}$ and $\hat{g} : \widetilde{\mathfrak{A}} \longrightarrow 2^{\mho}$. Construct two different maps as follows:

$$\begin{aligned} \Phi : \mho &\longrightarrow 2^{\mathfrak{A}} \\ q &\longmapsto \Phi(q) = \{\wp : q \in \hat{f}(\wp)\} \end{aligned}$$

and

$$\begin{aligned} \Psi : \mho &\longrightarrow 2^{\widetilde{\mathfrak{A}}} \\ q &\longmapsto \Psi(q) = \{\neg\wp : q \in \hat{g}(\neg\wp)\} \end{aligned}$$

Then, $\Omega = \langle \mho, (\Phi, \Psi) \rangle$ is called a modified rough bipolar soft approximation space (MRBS-AS).

For any $\varnothing \neq \mathcal{Q} \subseteq \mho$, the lower modified bipolar pair (LMBP) and the upper modified bipolar pair (UMBP) concerning Ω are defined in the following manner, respectively:

$$\underline{MBS}_{\Omega}(\mathcal{Q}) = (\underline{\mathcal{Q}}_{\Phi^+}, \underline{\mathcal{Q}}_{\Psi^-})$$

and

$$\overline{MBS}_{\Omega}(\mathcal{Q}) = (\overline{\mathcal{Q}}^{\Phi^+}, \overline{\mathcal{Q}}^{\Psi^-})$$

where

$$\begin{aligned} \underline{\mathcal{Q}}_{\Phi^+} &= \{p \in \mathcal{Q} : \Phi(p) \neq \Phi(r), \text{ for all } r \in \mathcal{Q}^c\} \\ \overline{\mathcal{Q}}^{\Phi^+} &= \{p \in \mho : \Phi(p) = \Phi(r), \text{ for some } r \in \mathcal{Q}\} \\ \underline{\mathcal{Q}}_{\Psi^-} &= \{p \in \mho : \Psi(p) = \Psi(r), \text{ for some } r \in \mathcal{Q}\} \end{aligned}$$

and

$$\overline{\mathcal{Q}}^{\Psi^-} = \{p \in \mathcal{Q} : \Psi(p) \neq \Psi(r), \text{ for all } r \in \mathcal{Q}^c\}$$

Here, $\mathcal{Q}^c = \mho - \mathcal{Q}$. Generally, $\underline{\mathcal{Q}}_{\Phi^+}$, $\overline{\mathcal{Q}}^{\Phi^+}$, $\underline{\mathcal{Q}}_{\Psi^-}$, and $\overline{\mathcal{Q}}^{\Psi^-}$ will be called Φ-lower positive, Φ-upper positive, Ψ-lower negative, and Ψ-upper negative MRBS-approximations of $\mathcal{Q} \subseteq \mho$, respectively. If $\underset{\sim}{MBS}_{\Omega}(\mathcal{Q}) \neq \widetilde{MBS}_{\Omega}(\mathcal{Q})$, then $\mathcal{Q}$ is said to be an MRBSs; otherwise, $\mathcal{Q}$ is said to be MRBS-definable.

The corresponding positive, boundary, and negative regions under MRBSs are listed as follows:

$$Pos_{\Omega}(\mathcal{Q}) = (\underline{\mathcal{Q}}_{\Phi^+}, \overline{\mathcal{Q}}^{\Psi^-})$$

$$Bnd_{\Omega}(\mathcal{Q}) = \left(\overline{\mathcal{Q}}^{\Phi^+} \setminus \underline{\mathcal{Q}}_{\Phi^+}, \underline{\mathcal{Q}}^{\Psi^-} \setminus \overline{\mathcal{Q}}_{\Psi^-}\right)$$

and

$$Neg_{\Omega}(\mathcal{Q}) = \left(\left(\overline{\mathcal{Q}}^{\Phi^+}\right)^c, \left(\underline{\mathcal{Q}}_{\Psi^-}\right)^c\right)$$

TOPSIS is one of the most frequently utilized techniques for MCDM because it ranks alternatives and chooses optimal alternatives in the concept evaluation procedure using Euclidean distances. Suppose that for any DM problem, there are n criteria and m alternatives. Then, a decision matrix is described as $\mathfrak{D} = [\delta_{ij}]_{m \times n}$ where $i \in \{1, 2, \cdots, m\}$, $j \in \{1, 2, \cdots, n\}$, and δ_{ij} demonstrates the preference value of an alternative for design criteria.

The procedure of TOPSIS described in [36] is as follows:

i. Construct the normalized decision matrix $\mathfrak{D}_{nor} = [r_{ij}]_{m\times n}$ where $r_{ij} = \dfrac{\delta_{ij}}{\sqrt{\sum_{i=1}^{m}\delta_{ij}^2}}$ and weighted normalized decision matrix. Here, $v_{ij} = \omega_j r_{ij}$ is a weighted normalized value where ω_j is the weight of a criteria.

ii. Evaluate the positive ideal solution (PIS) and negative ideal solution (NIS) as:

$$v_i^+ = \Big\{\Big(\bigvee_i (v_{ij}) \mid j \in \mathcal{I}\Big), \Big(\bigwedge_i (v_{ij}) \mid j \in \mathcal{J}\Big)\Big\}$$

and

$$v_i^- = \Big\{\Big(\bigwedge_i (v_{ij}) \mid j \in \mathcal{I}), (\bigvee_i (v_{ij}) \mid j \in \mathcal{J})\Big\}$$

where $\mathcal{I}$ and $\mathcal{J}$ are related to the benefit and cost criterion, respectively.

iii. Determine the separation measure of each alternative from PIS and NIS by using n-dimensional Euclidean distance:

$$\delta_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_i^+\right)^2}, \quad i \in \{1, 2, \cdots, m\}$$

and

$$\delta_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_i^-\right)^2}, \quad i \in \{1, 2, \cdots, m\}$$

iv. Evaluate the relative closeness coefficient of each alternative to the ideal solution, given as:

$$\mathcal{C}_i^* = \frac{\delta_i^-}{\delta_i^- + \delta_i^+}, \quad i \in \{1, 2, \cdots, m\}$$

v. Sort the alternative concerning the value of $\mathcal{C}_i^*$. The optimal alternative is the object with the highest value of $\mathcal{C}_i^*$. That alternative would have the least distance from the PIS and the largest distance from the NIS.

3. An Integrated Model of MCGDM using TOPSIS Technique and MRBSs

The MCGDM is one of the substantial components of modern decision theory. MCGDM aims to select the optimal from finite alternatives by incorporating the evaluation information of various alternatives acquired from a group of experts(decision-makers). It is instrumental in economic evaluation, clustering analysis, site selection, medical diagnosis, etc. In MCGDM, the primary step is to consider a finite number of alternatives in terms of multiple conflicting criteria based on the experts' opinions. Characterizing the evaluation information for several attributes is a significant problem in the MCGDM. In real-life MCGDM problems, uncertainty is inevitable because of imprecise judgment by decision-makers. TOPSIS is a practical and extensively used multi-criteria DM (MCDM) technique for sorting alternatives and determining the optimal alternative in the concept evaluation procedure. The aggregating function computed in TOPSIS indicates "closeness to ideal solution". To make criteria with the same units, TOPSIS employs vector normalization. The critical concept of TOPSIS is that the alternative that has been selected as the optimal should have the smallest distance from the PIS and the greatest from the NIS.

In this section, we utilize the TOPSIS technique for MCGDM based on the MRBSs. The systematic procedure of the TOPSIS under the MRBSs is explained as follows:

3.1. Description of Problem

In this subsection, we first give the essential explanation of the MCGDM problem under consideration. Suppose that $\mho = \{\mu_1, \mu_2, \ldots, \mu_n\}$ be the set consisting of n alternatives in which the best object is to be selected and $\mathfrak{A} = \{\wp_1, \wp_2, \ldots, \wp_m\}$ be the set of parameters (criterion) of objects. Assume that we have a group of independent experts $\mathcal{G} = \{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k\}$ consisting of k decision-makers to evaluate the objects in $\mho$. Each expert needs to review all the objects of $\mho$ and will be requested to only choose "the optimal alternatives" as their evaluation result. Hence each expert's primary evaluation result is a subset of $\mho$. For the sake of simplicity, we assume that the evaluations of these experts in $\mathcal{G}$ are of the same importance. Let $\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_k$ are non-void subsets of $\mho$, indicate primary evaluations of experts $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_k$, about n alternatives concerning m parameters, respectively, and $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_r \in \mathcal{BSS}^{\mho}$ are the real results previously captured for the same problems in various locations or various periods. Specifically, we can take the MRBS-approximations of the expert $\mathcal{P}_i$'s primary evaluation result $\mathcal{Q}_i$ concerning the MRBS-AS $\Omega = \langle \mho, (\Phi, \Psi) \rangle$. The Φ-lower positive approximation $\underline{\mathcal{Q}_i}_{\Phi^+}$ can be interpreted as the set consisting of the objects which are undoubtedly the optimum candidates according to the expert $\mathcal{P}_i$'s primary evaluation. Similarly, the Φ-upper positive approximation $\overline{\mathcal{Q}_i}^{\Phi^+}$ can be interpreted as the set consisting of the objects which are possibly the optimum candidates according to the expert $\mathcal{P}_i$'s primary evaluation. The Ψ-lower positive approximation $\underline{\mathcal{Q}_i}_{\Psi^-}$ can be interpreted as the set consisting of the objects which are possibly the worst candidates according to the expert $\mathcal{P}_i$'s primary evaluation. Likewise, the Ψ-upper negative approximation $\overline{\mathcal{Q}_i}^{\Psi^-}$ can be interpreted as the set consisting of the objects which are surely the worst candidates according to the expert $\mathcal{P}_i$'s primary evaluation. Then, the DM for this MCGDM problem is: "how to resolve differences of the evaluation conveyed by the individual experts to determine the object which is highly favorable by the entire group of experts".

3.2. Methodology

Here, we present the step-by-step mathematical formulation and process of the TOPSIS technique under the framework of MRBSs for the MCGDM problem.

Definition 3.1. Let $\underline{MBS}_{\mathcal{B}_q}(\mathcal{Q}_j) = (\underline{\mathcal{Q}_j}_{\Phi_q^+}, \underline{\mathcal{Q}_j}_{\Psi_q^-})$ be the LMBP and $\overline{MBS}_{\mathcal{B}_q}(\mathcal{Q}_j) = (\overline{\mathcal{Q}_j}^{\Phi_q^+}, \overline{\mathcal{Q}_j}^{\psi_q^-})$ be UMBP of $\mathcal{Q}_j$ such that $j \in \{1, 2, \cdots, k\}$ concerning $\mathcal{B}_q = (\hat{f}_q, \hat{g}_q : \mathfrak{A}) \in \mathcal{BSS}^{\mho}$, for $q \in \{1, 2, \ldots, r\}$. Then,

$$\underline{M} = \begin{pmatrix} \langle \underline{\mathcal{Q}_1}_{\Phi_1^+}, \underline{\mathcal{Q}_1}_{\Psi_1^-} \rangle & \langle \underline{\mathcal{Q}_2}_{\Phi_1^+}, \underline{\mathcal{Q}_2}_{\Psi_1^-} \rangle & \cdots & \langle \underline{\mathcal{Q}_k}_{\Phi_1^+}, \underline{\mathcal{Q}_k}_{\Psi_1^-} \rangle \\ \langle \underline{\mathcal{Q}_1}_{\Phi_2^+}, \underline{\mathcal{Q}_1}_{\Psi_2^-} \rangle & \langle \underline{\mathcal{Q}_2}_{\Phi_2^+}, \underline{\mathcal{Q}_2}_{\Psi_2^-} \rangle & \cdots & \langle \underline{\mathcal{Q}_k}_{\Phi_2^+}, \underline{\mathcal{Q}_k}_{\Psi_2^-} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \underline{\mathcal{Q}_1}_{\Phi_r^+}, \underline{\mathcal{Q}_1}_{\Psi_r^-} \rangle & \langle \underline{\mathcal{Q}_2}_{\Phi_r^+}, \underline{\mathcal{Q}_2}_{\Psi_r^-} \rangle & \cdots & \langle \underline{\mathcal{Q}_k}_{\Phi_r^+}, \underline{\mathcal{Q}_k}_{\Psi_r^-} \rangle \end{pmatrix}_{r \times k}$$

and

$$\overline{M} = \begin{pmatrix} \langle \overline{\mathcal{Q}_1}^{\Phi_1^+}, \overline{\mathcal{Q}_1}^{\Psi_1^-} \rangle & \langle \overline{\mathcal{Q}_2}^{\Phi_1^+}, \overline{\mathcal{Q}_2}^{\Psi_1^-} \rangle & \cdots & \langle \overline{\mathcal{Q}_k}^{\Phi_1^+}, \overline{\mathcal{Q}_k}^{\Psi_1^-} \rangle \\ \langle \overline{\mathcal{Q}_1}^{\Phi_2^+}, \overline{\mathcal{Q}_1}^{\Psi_2^-} \rangle & \langle \overline{\mathcal{Q}_2}^{\Phi_2^+}, \overline{\mathcal{Q}_2}^{\Psi_2^-} \rangle & \cdots & \langle \overline{\mathcal{Q}_k}^{\Phi_2^+}, \overline{\mathcal{Q}_k}^{\Psi_2^-} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \overline{\mathcal{Q}_1}^{\Phi_r^+}, \overline{\mathcal{Q}_1}^{\Psi_r^-} \rangle & \langle \overline{\mathcal{Q}_2}^{\Phi_r^+}, \overline{\mathcal{Q}_2}^{\Psi_r^-} \rangle & \cdots & \langle \overline{\mathcal{Q}_k}^{\Phi_r^+}, \overline{\mathcal{Q}_k}^{\Psi_r^-} \rangle \end{pmatrix}_{r \times k}$$

are stated to be modified bipolar soft lower and upper approximation matrices, respectively, where

$$\underline{\mathcal{Q}_j}_{\Phi_q^+} = (\underline{\mu_{1j}}_{\Phi_q^+}, \underline{\mu_{2j}}_{\Phi_q^+}, \ldots, \underline{\mu_{nj}}_{\Phi_q^+})$$

$$\underline{\mathcal{Q}_j}_{\Psi_q^-} = (\underline{\mu_{1j}}_{\Psi_q^-}, \underline{\mu_{2j}}_{\Psi_q^-}, \ldots, \underline{\mu_{nj}}_{\Psi_q^-})$$

$$\overline{\mathcal{Q}_j}^{\Phi_q^+} = (\overline{\mu_{1j}}^{\Phi_q^+}, \overline{\mu_{2j}}^{\Phi_q^+}, \ldots, \overline{\mu_{nj}}^{\Phi_q^+})$$

and

$$\overline{\mathcal{Q}_j}^{\Psi_q^-} = (\overline{\mu_{1j}}^{\Psi_q^-}, \overline{\mu_{2j}}^{\Psi_q^-}, \ldots, \overline{\mu_{nj}}^{\Psi_q^-})$$

Here,

$$\underline{\mu_{ij}}_{\Phi_q^+} = \begin{cases} 1, & \mu_i \in \underline{\mathcal{X}_j}_{\Phi_q^+} \\ 0, & \mu_i \notin \underline{\mathcal{X}_j}_{\Phi_q^+} \end{cases}$$

$$\underline{\mu_{ij}}_{\Psi_q^-} = \begin{cases} -\frac{1}{2}, & \mu_i \in \underline{\mathcal{X}_j}_{\Psi_q^-} \\ 0, & \mu_i \notin \underline{\mathcal{X}_j}_{\Psi_q^-} \end{cases}$$

$$\overline{\mu_{ij}}^{\Phi_q^+} = \begin{cases} \frac{1}{2}, & \mu_i \in \overline{\mathcal{X}_j}^{\Phi_q^+} \\ 0, & \mu_i \notin \overline{\mathcal{X}_j}^{\Phi_q^+} \end{cases}$$

and

$$\overline{\mu_{ij}}^{\Psi_q^-} = \begin{cases} -1, & \mu_i \in \overline{\mathcal{X}_j}^{\Psi_q^-} \\ 0, & \mu_i \notin \overline{\mathcal{X}_j}^{\Psi_q^-} \end{cases}$$

Remark 3.2. From Definition 3.1, we have

i. $\underline{\mathcal{Q}_j}_{\Phi_q^+}$ and $\overline{\mathcal{Q}_j}^{\Phi_q^+}$ show the Φ-lower and Φ-upper positive MRBS-approximation of the evaluation $\mathcal{Q}_j \subseteq \mho$ by the j^{th} expert related to q^{th} actual result represented by the BSS $\mathcal{B}_q = (\hat{f}_q, \hat{g}_q : \mathfrak{A})$.

ii. $\underline{\mathcal{Q}_j}_{\Psi_q^-}$ and $\overline{\mathcal{Q}_j}^{\Psi_q^-}$ show the Ψ-lower and Ψ-upper negative MRBS-approximation of the evaluation $\mathcal{Q}_j \subseteq \mho$ by the j^{th} expert related to q^{th} actual result represented by the BSS $\mathcal{B}_q = (\hat{f}_q, \hat{g}_q : \mathfrak{A})$.

Definition 3.3. Let $\underline{M}$ and $\overline{M}$ be modified bipolar soft lower and upper approximation matrices concerning $\underline{MBS}_{\mathcal{B}_q}(\mathcal{Q}_j)$ and $\overline{MBS}_{\mathcal{B}_q}(\mathcal{Q}_j)$. Then,

$$A = \underline{M} + \overline{M} = (\alpha_{ij})_{r\times k} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1k} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{r1} & \alpha_{r2} & \cdots & \alpha_{rk} \end{pmatrix}$$

is regarded as aggregated parameter matrix, where every element has the form:

$$\alpha_{ij} = \langle \alpha_{ij}^{\Phi^+}, \alpha_{ij}^{\Psi^-} \rangle = \left\langle \underline{\mathcal{X}_j}_{\Phi_q^+} \oplus \overline{\mathcal{X}_j}_{\Phi_q^+}, \underline{\mathcal{X}_j}_{\Psi_q^-} \oplus \overline{\mathcal{X}_j}_{\Psi_q^-} \right\rangle$$

such that $\alpha_{ij}^{\Phi^+} = \underline{\mathcal{X}_j}_{\Phi_q^+} \oplus \overline{\mathcal{X}_j}_{\Phi_q^+} = (\ldots, \underline{\mu_{mj}}_{\Phi_i^+} + \overline{\mu_{mj}}^{\Phi_i^+}, \ldots)$ and $\alpha_{ij}^{\Psi^-} = \underline{\mathcal{X}_j}_{\Psi_q^-} \oplus \overline{\mathcal{X}_j}_{\Psi_q^-} = (\ldots, \underline{\mu_{mj}}_{\Psi_i^-} + \overline{\mu_{mj}}^{\Psi_i^-}, \ldots)$. Here, the operation $\oplus$ stands for the vector addition.

Definition 3.4. Assume that A is an aggregated parameter matrix. Then,

$$S = \left(\langle s_{ij}^{\Phi^+}, s_{ij}^{\Psi^-} \rangle\right)_{r\times k}$$

is said to be a standardized decision matrix where $s_{ij}^{\Phi^+} = \left(\sum_{m=1}^{k} \alpha_{im}^{\Phi^+}\right)_j$ and $s_{ij}^{\Psi^-} = \left(\sum_{m=1}^{k} \alpha_{im}^{\Psi^-}\right)_j$ such that $i \in \{1, 2, \ldots, r\}$ and $j \in \{1, 2, \ldots, n\}$.

Remark 3.5. From Definition 3.4, we have noticed that $s_{ij}^{\Phi^+}$ is the positive information for the jth coordinate of the vector sum of the i^{th} row of the matrix A and $s_{ij}^{\Psi^-}$ is the negative information for the j^{th} coordinate of the vector sum of the i^{th} row of the matrix A. ın other words, each row in the matrix S is a vector obtained by taking the column sum of A. Thus, $\langle s_{ij}^{\Phi^+}, s_{ij}^{\Psi^-} \rangle$ represents the standardized MRBS-approximation of alternative u_i under the scenario of j^{th} real result previously acquired for the same problems in various locations or various periods.

Definition 3.6. Let S be a standardized decision matrix. Then,

$$\aleph = (n_{ij})_{r\times k} = \begin{pmatrix} n_{11} & n_{12} & \cdots & n_{1k} \\ n_{21} & n_{22} & \cdots & n_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ n_{r1} & n_{r2} & \cdots & n_{rk} \end{pmatrix}$$

is called a normalized decision matrix where each entry is of the form $n_{ij} = \langle \eta_{ij}^{\Phi^+}, \eta_{ij}^{\Psi^-} \rangle$ with the following conditions:

$$\eta_{ij}^{\Phi^+} = \frac{s_{ij}^{\Phi^+}}{\sqrt{\sum_{\ell=1}^{r} \left(s_{\ell j}^{\Phi^+}\right)^2}}$$

and

$$\eta_{ij}^{\Psi^-} = \frac{s_{ij}^{\Psi^-}}{\sqrt{\sum_{\ell=1}^{r} \left(s_{\ell j}^{\Psi^-}\right)^2}}$$

Definition 3.7. Let $\aleph$ be a normalized decision matrix. Then,

$$\mathfrak{D} = (\delta_{ij})_{r\times k} = \begin{pmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1k} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{r1} & \delta_{r2} & \cdots & \delta_{rk} \end{pmatrix}$$

is called an average weighted normalized decision matrix where each entry is of the form:

$$\delta_{ij} = \frac{\left|\eta_{ij}^{\Phi^+}\right| + \left|\eta_{ij}^{\Psi^-}\right|}{2}$$

Definition 3.8. Let $\mathfrak{D}$ be an average weighted normalized decision matrix. Then, the expressions:

$$MPIS = \{\delta_1^{\Phi^+}, \delta_2^{\Phi^+}, \cdots, \delta_k^{\Phi^+}\} = \{\max(\delta_{ij}) : i \in I_r\} \quad \text{such that} \quad I_r = \{1, 2, \cdots, r\}$$

and

$$MNIS = \{\delta_1^{\Psi^-}, \delta_2^{\Psi^-}, \cdots, \delta_k^{\Psi^-}\} = \{\min(\delta_{ij}) : i \in I_r\} \quad \text{such that} \quad I_r = \{1, 2, \cdots, r\}$$

are called modified PIS and modified NIS, respectively.

Definition 3.9. Let MPIS and MNIS be positive and negative ideal solutions. Then, the separation measurement of each alternative to MPIS is determined as follows:

$$\mathcal{S}_i^{\Phi^+} = \sqrt{\sum_{j=1}^{k} \left(\delta_{ij} - \delta_j^{\Phi^+}\right)^2}, \quad i \in \{1, 2, \cdots, r\}$$

Similarly, the separation measurement of each alternative to MNIS is evaluated as follows:

$$\mathcal{S}_i^{\Psi^-} = \sqrt{\sum_{j=1}^{k} \left(\delta_{ij} - \delta_j^{\Psi^-}\right)^2}, \quad i \in \{1, 2, \cdots, r\}$$

Definition 3.10. Let $\mathcal{S}_i^{\Phi^+}$ and $\mathcal{S}_i^{\Psi^-}$ be separation measurements of MPIS and MNIS, respectively. The relative closeness of alternatives to the ideal solution is defined as:

$$\mathfrak{C}_i^{(\Phi^+,\Psi^-)} = \frac{S_i^{\Psi^-}}{S_i^{\Psi^-} + S_i^{\Phi^+}}, \quad i \in \{1, 2, \cdots, r\}$$

Here, $0 \leq \mathfrak{C}_i^{(\Phi^+,\Psi^-)} \leq 1$, for all $i \in \{1, 2, \cdots, r\}$. The larger value of $\mathfrak{C}_i^{(\Phi^+,\Psi^-)}$ corresponds to the most desirable alternative. It has the least distance from the MPIS and the highest distance from the MNIS.

4. An Algorithm for the Proposed MCGDM Problem

In this section, we present an algorithm for the developed TOPSIS-based MCGDM problem considered in Section 3. The related steps are outlined as follows:

Step 1. Take primary evaluations $\mathcal{Q}_i$ of experts $\mathcal{P}_i$ such that $i \in \{1, 2, \cdots, k\}$.

Step 2. Construct $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_r$ using the real results.

Step 3. Determine $\underline{MBS}_{\mathcal{B}_q}(\mathcal{Q}_j)$ and $\overline{MBS}_{\mathcal{B}_q}(\mathcal{Q}_j)$, for $j \in \{1, 2, \cdots, k\}$ and $q \in \{1, 2, \cdots, r\}$, from Definition 2.6.

Step 4. Construct $\underline{M}$ and $\overline{M}$ according to Definition 3.1.

Step 5. Construct the aggregated parameter matrix from Definition 3.3.

Step 6. Compute the standardized decision matrix using Definition 3.4.

Step 7. Compute the normalized decision matrix according to Definition 3.6.

Step 8. Construct the average weighted normalized decision matrix using Definition 3.7.

Step 9. Determine the MPIS and the MNIS using Definition 3.8.

Step 10. According to Definition 3.9, calculate separation measurements of MPIS and MNIS for every alternative.

Step 11. Determine relative closeness of alternatives to ideal solutions using Definition 3.10.

Step 12. Ranking the preference order.

The flowchart of the above algorithm is displayed in Figure 1.

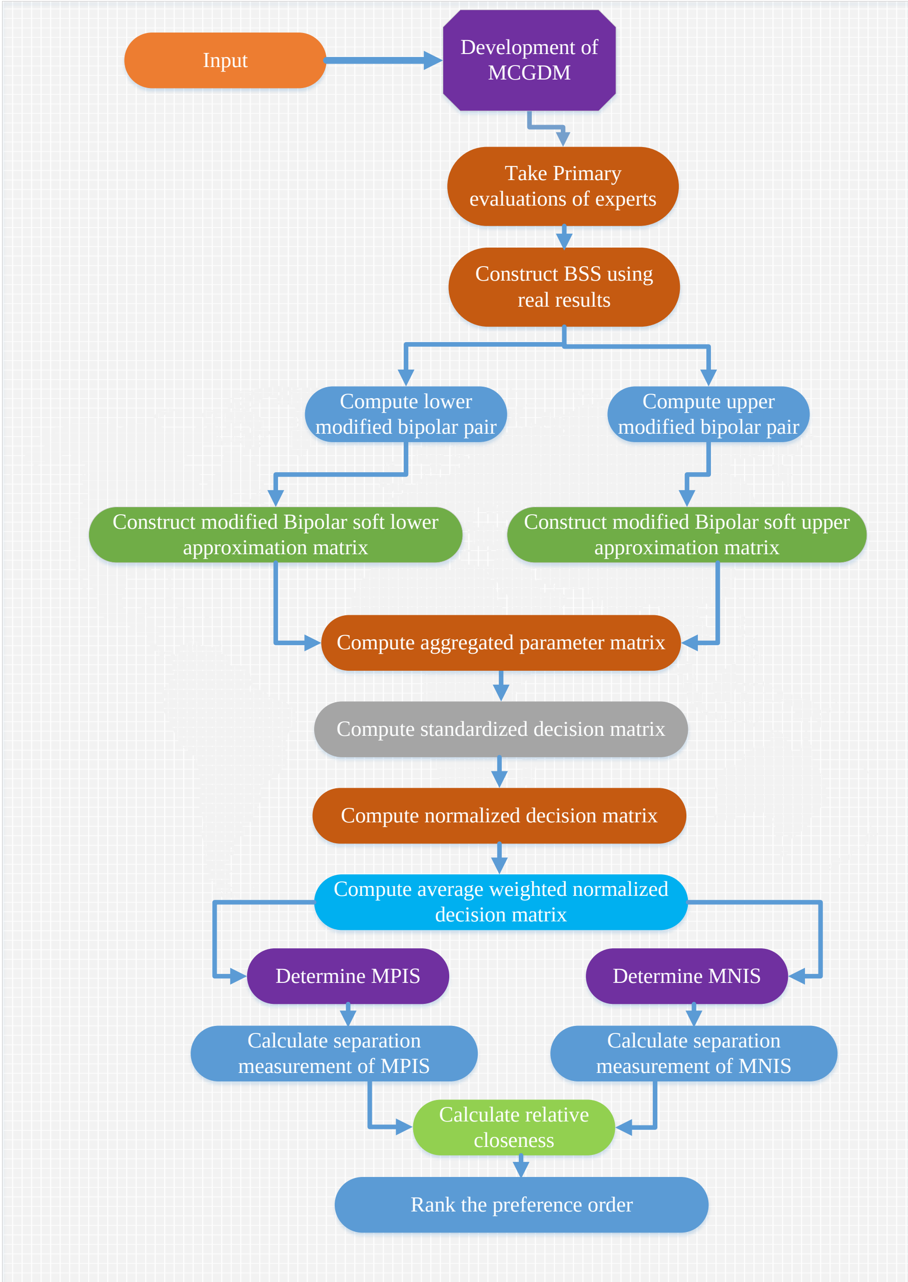

Figure 1. Flowchart of TOPSIS using MRBSs

5. Case Study

In this section, we discuss a design example of the MCGDM problem in MRBSs to illustrate the potential of the above-formulated TOPSIS method.

Example 5.1. Due to globalization's growing competition and mechanical upgrades, global markets are forcing companies to deliver top-quality things and services. This must be achieved through the participation of suitable employees. Employee selection is a procedure selection of people with the essential capabilities to perform a specific job at best. It chooses the information nature of employees and performs a crucial role in personnel management. Growing rivalry in worldwide markets encourages organizations to put greater emphasize on the recruitment process. Several companies determine the best job-hunter using rigorous and expensive identification methodologies. A candidate may be judged by various parameters such as managerial skills, ability to work under pressure, fluency in English, etc. It is wise to consult experts to accurately judge the candidates based on these parameters.

Assume that a production corporation wants to hire a marketing manager for a vacant post. Let $\mho = \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}$ be the set of five candidates who might fit the marketing manager position at the production company. A panel of experts $\mathcal{G} = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$ is set up to hire the most suitable candidate for this job. The panel will evaluate the candidates according to the set of parameters $\mathfrak{A} = \{\wp_1, \wp_2, \wp_3\}$ such that $\wp_1$ = managerial skills, $\wp_2$ = ability to work under pressure, and $\wp_3$ = fluency in English. The following calculations are performed to solve the MCGDM problem using the proposed methodology.

Step 1. The panel of experts $\mathcal{G} = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$ gives their primary evaluations for the candidates as:

$$\mathcal{Q}_1 = \{\mu_1, \mu_2, \mu_5\}, \quad \mathcal{Q}_2 = \{\mu_1, \mu_3, \mu_5\}, \quad \text{and} \quad \mathcal{Q}_3 = \{\mu_2, \mu_4, \mu_5\}$$

Step 2. Real results in three various times and places are displayed as BSSs $\mathcal{B}_1 = (\hat{f}_1, \hat{g}_1 : \mathfrak{A})$, $\mathcal{B}_2 = (\hat{f}_2, \hat{g}_2 : \mathfrak{A})$, and $\mathcal{B}_3 = (\hat{f}_3, \hat{g}_3 : \mathfrak{A})$ as follows:

$$\begin{aligned} \hat{f}_1 : \mathfrak{A} &\longrightarrow 2^{\mho} \\ \wp \longmapsto \hat{f}_1(\wp) &= \begin{cases} \{\mu_1\}, & \wp = \wp_1 \\ \{\mu_1, \mu_5\}, & \wp = \wp_2 \\ \{\mu_4, \mu_5\}, & \wp = \wp_3 \end{cases} \end{aligned} \qquad \begin{aligned} \hat{g}_1 : \widetilde{\mathfrak{A}} &\longrightarrow 2^{\mho} \\ \neg\wp \longmapsto \hat{g}_1(\neg\wp) &= \begin{cases} \{\mu_3, \mu_5\}, & \neg\wp = \neg\wp_1 \\ \{\mu_3\}, & \neg\wp = \neg\wp_2 \\ \{\mu_1, \mu_3\}, & \neg\wp = \neg\wp_3 \end{cases} \end{aligned}$$

$$\begin{aligned} \hat{f}_2 : \mathfrak{A} &\longrightarrow 2^{\mho} \\ \wp \longmapsto \hat{f}_2(\wp) &= \begin{cases} \{\mu_2\}, & \wp = \wp_1 \\ \{\mu_2, \mu_4\}, & \wp = \wp_2 \\ \{\mu_3, \mu_4\}, & \wp = \wp_3 \end{cases} \end{aligned} \qquad \begin{aligned} \hat{g}_2 : \widetilde{\mathfrak{A}} &\longrightarrow 2^{\mho} \\ \neg\wp \longmapsto \hat{g}_2(\neg\wp) &= \begin{cases} \{\mu_1, \mu_4\}, & \neg\wp = \neg\wp_1 \\ \{\mu_5\}, & \neg\wp = \neg\wp_2 \\ \{\mu_1, \mu_5\}, & \neg\wp = \neg\wp_3 \end{cases} \end{aligned}$$

and

$$\begin{aligned} \hat{f}_3 : \mathfrak{A} &\longrightarrow 2^{\mho} \\ \wp \longmapsto \hat{f}_3(\wp) &= \begin{cases} \{\mu_3, \mu_5\}, & \wp = \wp_1 \\ \{\mu_2\}, & \wp = \wp_2 \\ \{\mu_2, \mu_5\}, & \wp = \wp_3 \end{cases} \end{aligned} \qquad \begin{aligned} \hat{g}_3 : \widetilde{\mathfrak{A}} &\longrightarrow 2^{\mho} \\ \neg\wp \longmapsto \hat{g}_3(\wp) &= \begin{cases} \{\mu_1, \mu_2\}, & \neg\wp = \neg\wp_1 \\ \{\mu_4\}, & \neg\wp = \neg\wp_2 \\ \{\mu_1, \mu_3\}, & \neg\wp = \neg\wp_3 \end{cases} \end{aligned}$$

Step 3. Using Definition 2.6, the LMBP and the UMBP for $\mathcal{Q}_1$, $\mathcal{Q}_2$, and $\mathcal{Q}_3$ concerning $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{B}_3$ are as follows:

$$\begin{aligned} \underline{MBS}_{\mathcal{B}_1}(\mathcal{Q}_1) &= (\{\mu_1, \mu_5\}, \{\mu_1, \mu_2, \mu_4, \mu_5\}) \\ \underline{MBS}_{\mathcal{B}_1}(\mathcal{Q}_2) &= (\{\mu_1, \mu_5\}, \{\mu_1, \mu_3, \mu_5\}) \\ \underline{MBS}_{\mathcal{B}_1}(\mathcal{Q}_3) &= (\{\mu_4, \mu_5\}, \{\mu_2, \mu_4, \mu_5\}) \end{aligned} \qquad \begin{aligned} \overline{MBS}_{\mathcal{B}_1}(\mathcal{Q}_1) &= (\{\mu_1, \mu_2, \mu_3, \mu_5\}, \{\mu_1, \mu_5\}) \\ \overline{MBS}_{\mathcal{B}_1}(\mathcal{Q}_2) &= (\{\mu_1, \mu_2, \mu_3, \mu_5\}, \{\mu_1, \mu_3, \mu_5\}) \\ \overline{MBS}_{\mathcal{B}_1}(\mathcal{Q}_3) &= (\{\mu_2, \mu_3, \mu_4, \mu_5\}, \{\mu_2, \mu_4, \mu_5\}) \end{aligned}$$

$$\begin{aligned} \underline{MBS}_{\mathcal{B}_2}(\mathcal{Q}_1) &= (\{\mu_1, \mu_2, \mu_5\}, \{\mu_1, \mu_5\}) \\ \underline{MBS}_{\mathcal{B}_2}(\mathcal{Q}_2) &= (\{\mu_1, \mu_3, \mu_5\}, \{\mu_1, \mu_2, \mu_3, \mu_5\}) \\ \underline{MBS}_{\mathcal{B}_2}(\mathcal{Q}_3) &= (\{\mu_2, \mu_4\}, \{\mu_2, \mu_3, \mu_4, \mu_5\}) \end{aligned} \qquad \begin{aligned} \overline{MBS}_{\mathcal{B}_2}(\mathcal{Q}_1) &= (\{\mu_1, \mu_2, \mu_5\}, \{\mu_1, \mu_2, \mu_3, \mu_5\}) \\ \overline{MBS}_{\mathcal{B}_2}(\mathcal{Q}_2) &= (\{\mu_1, \mu_3, \mu_5\}, \{\mu_1, \mu_5\}) \\ \overline{MBS}_{\mathcal{B}_2}(\mathcal{Q}_3) &= (\{\mu_1, \mu_2, \mu_4, \mu_5\}, \{\mu_4, \mu_5\}) \end{aligned}$$

and

$$\begin{aligned} \underline{MBS}_{\mathcal{B}_3}(\mathcal{Q}_1) &= (\{\mu_2, \mu_5\}, \{\mu_2, \mu_4, \mu_5\}) \\ \underline{MBS}_{\mathcal{B}_3}(\mathcal{Q}_2) &= (\{\mu_3, \mu_5\}, \{\mu_1, \mu_3, \mu_5\}) \\ \underline{MBS}_{\mathcal{B}_3}(\mathcal{Q}_3) &= (\{\mu_2, \mu_5\}, \{\mu_2, \mu_4, \mu_5\}) \end{aligned} \qquad \begin{aligned} \overline{MBS}_{\mathcal{B}_3}(\mathcal{Q}_1) &= (\{\mu_1, \mu_2, \mu_4, \mu_5\}, \{\mu_2, \mu_4, \mu_5\}) \\ \overline{MBS}_{\mathcal{B}_3}(\mathcal{Q}_2) &= (\{\mu_1, \mu_3, \mu_4, \mu_5\}, \{\mu_1, \mu_3, \mu_5\}) \\ \overline{MBS}_{\mathcal{B}_3}(\mathcal{Q}_3) &= (\{\mu_1, \mu_2, \mu_4, \mu_5\}, \{\mu_2, \mu_4, \mu_5\}) \end{aligned}$$

Step 4. Using Definition 3.1, the modified bipolar soft lower upper approximation matrices are obtained as:

$$\underline{M} = \begin{pmatrix} \left\langle (1,0,0,0,1), (-\frac{1}{2},-\frac{1}{2},0,-\frac{1}{2},-\frac{1}{2}) \right\rangle & \left\langle (1,0,0,0,1), (-\frac{1}{2},0,-\frac{1}{2},0,-\frac{1}{2}) \right\rangle & \left\langle (0,0,0,1,1), (0,-\frac{1}{2},0,-\frac{1}{2},-\frac{1}{2}) \right\rangle \\ \left\langle (1,1,0,0,1), (-\frac{1}{2},0,0,0,-\frac{1}{2}) \right\rangle & \left\langle (1,0,1,0,1), (-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},0,-\frac{1}{2}) \right\rangle & \left\langle (0,1,0,1,0), (0,-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}) \right\rangle \\ \left\langle (0,1,0,0,1), (0,-\frac{1}{2},0,-\frac{1}{2},-\frac{1}{2}) \right\rangle & \left\langle (0,0,1,0,1), (-\frac{1}{2},0,-\frac{1}{2},0,-\frac{1}{2}) \right\rangle & \left\langle (0,1,0,0,1), (0,-\frac{1}{2},0,-\frac{1}{2},-\frac{1}{2}) \right\rangle \end{pmatrix}$$

and

$$\overline{M} = \begin{pmatrix} \left\langle (\frac{1}{2},\frac{1}{2},\frac{1}{2},0,\frac{1}{2}), (-1,0,0,0,-1) \right\rangle & \left\langle (\frac{1}{2},\frac{1}{2},\frac{1}{2},0,\frac{1}{2}), (-1,0,-1,0,-1) \right\rangle & \left\langle (0,\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}), (0,-1,0,-1,-1) \right\rangle \\ \left\langle (\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}), (-1,-1,-1,0,-1) \right\rangle & \left\langle (\frac{1}{2},0,\frac{1}{2},0,\frac{1}{2}), (-1,0,0,0,-1) \right\rangle & \left\langle (\frac{1}{2},\frac{1}{2},0,\frac{1}{2},\frac{1}{2}), (0,0,0,-1,-1) \right\rangle \\ \left\langle (\frac{1}{2},\frac{1}{2},0,\frac{1}{2},\frac{1}{2}), (0,-1,0,-1,-1) \right\rangle & \left\langle (\frac{1}{2},0,\frac{1}{2},\frac{1}{2},\frac{1}{2}), (-1,0,-1,0,-1) \right\rangle & \left\langle (\frac{1}{2},\frac{1}{2},0,\frac{1}{2},\frac{1}{2}), (0,-1,0,-1,-1) \right\rangle \end{pmatrix}$$

Step 5. According to Definition 3.3, the aggregated parameter matrix is constructed as:

$$A = \begin{pmatrix} \left\langle (1.5,0.5,0.5,0,1.5), (-1.5,-0.5,0,-0.5,-1.5) \right\rangle & \left\langle (1.5,0.5,0.5,0,1.5), (-1.5,0,-1.5,0,-1.5) \right\rangle & \left\langle (0,0.5,0.5,1.5,1.5), (0,-1.5,0,-1.5,-1.5) \right\rangle \\ \left\langle (1.5,1.5,0,0,1.5), (-1.5,-1,-1,0,-1.5) \right\rangle & \left\langle (1.5,0,1.5,0,1.5), (-1.5,-0.5,-0.5,0,-1.5) \right\rangle & \left\langle (0.5,1.5,0,1.5,0.5), (0,-0.5,-0.5,-1.5,-1.5) \right\rangle \\ \left\langle (0.5,1.5,0,0.5,1.5), (0,-1.5,0,-1.5,-1.5) \right\rangle & \left\langle (0.5,0,1.5,0.5,1.5), (-1.5,0,-1.5,0,-1.5) \right\rangle & \left\langle (0.5,1.5,0,0.5,1.5), (0,-1.5,0,-1.5,-1.5) \right\rangle \end{pmatrix}$$

Step 6. Compute standardized decision matrix using Definition 3.4, we have

$$S = \begin{pmatrix} \langle 3,-3 \rangle & \langle 1.5,-2 \rangle & \langle 1.5,-1.5 \rangle & \langle 1.5,-2 \rangle & \langle 4.5,-4.5 \rangle \\ \langle 3.5,-3 \rangle & \langle 3,-2 \rangle & \langle 1.5,-2 \rangle & \langle 1.5,-1.5 \rangle & \langle 3.5,-4.5 \rangle \\ \langle 1.5,-1.5 \rangle & \langle 3,-3 \rangle & \langle 1.5,-1.5 \rangle & \langle 1.5,-3 \rangle & \langle 4.5,-4.5 \rangle \end{pmatrix}$$

Step 7. According to Definition 3.6, the normalized decision matrix can be determined as:

$$\aleph = \begin{pmatrix} \langle 0.619,-0.666 \rangle & \langle 0.333,-0.485 \rangle & \langle 0.577,-0.514 \rangle & \langle 0.577,-0.512 \rangle & \langle 0.620,-0.577 \rangle \\ \langle 0.722,-0.666 \rangle & \langle 0.666,-0.485 \rangle & \langle 0.577,-0.686 \rangle & \langle 0.577,-0.384 \rangle & \langle 0.482,-0.577 \rangle \\ \langle 0.309,-0.333 \rangle & \langle 0.666,-0.728 \rangle & \langle 0.577,-0.514 \rangle & \langle 0.577,-0.768 \rangle & \langle 0.620,-0.577 \rangle \end{pmatrix}$$

Step 8. Using Definition 3.7, the weighted normalized decision matrix is obtained as follows:

$$\mathfrak{D} = \begin{pmatrix} 0.643 & 0.818 & 0.546 & 0.545 & 0.599 \\ 0.694 & 0.576 & 0.632 & 0.481 & 0.530 \\ 0.321 & 0.697 & 0.546 & 0.673 & 0.599 \end{pmatrix}$$

Step 9. According to Definition 3.8, MPIS and MNIS are obtained as follows:

$$MPIS = \{0.818, 0.694, 0.697\}$$

and

$$MNIS = \{0.545, 0.481, 0.321\}$$

Step 10. By Definition 3.9, the separation measurements of MPIS and MNIS for every parameter are calculated as:

$$\mathcal{S}_1^{\Phi^+} = 0.415 \qquad \mathcal{S}_1^{\Psi^-} = 0.234$$
$$\mathcal{S}_2^{\Phi^+} = 0.118 \qquad \mathcal{S}_2^{\Psi^-} = 0.474$$
$$\mathcal{S}_3^{\Phi^+} = 0.317 \qquad \mathcal{S}_3^{\Psi^-} = 0.271$$
$$\mathcal{S}_4^{\Phi^+} = 0.347 \qquad \mathcal{S}_4^{\Psi^-} = 0.352$$

and

$$\mathcal{S}_5^{\Phi^+} = 0.291 \qquad \mathcal{S}_5^{\Psi^-} = 0.287$$

Step 11. The relative closeness of each alternative to the ideal solution according to Definition 3.10 can be calculated as:

$$\mathfrak{C}_1^{(\Phi^+,\Psi^-)} = 0.361$$
$$\mathfrak{C}_2^{(\Phi^+,\Psi^-)} = 0.801$$
$$\mathfrak{C}_3^{(\Phi^+,\Psi^-)} = 0.461$$
$$\mathfrak{C}_4^{(\Phi^+,\Psi^-)} = 0.465$$

and

$$\mathfrak{C}_5^{(\Phi^+,\Psi^-)} = 0.497$$

Step 12. Ranking the preference order is given as:

$$\mu_2 \succeq \mu_5 \succeq \mu_4 \succeq \mu_3 \succeq \mu_1$$

This indicates that μ_2 is the optimal candidate for the marketing manager position. We also note that although the initial selection of three experts favored candidate μ_5 more, considering the previous three evaluations regarding BSS and the proposed TOPSIS method revealed a different ranking with more intelligence and insight into the given scenario. Note that "$\succeq$" is the symbol of the preference order of alternatives. The graphical display for the ranking of the candidates is also given in Figure 2.

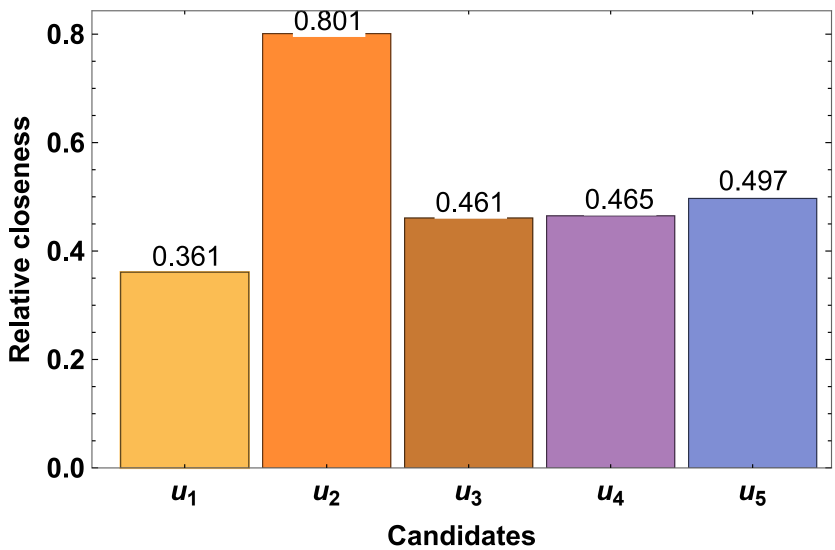

Figure 2. Graph for the ranking of candidates

6. Comparative Analysis and Discussion

In this section, we discuss the merits and drawbacks of the proposed technique and compare the suggested study with a few existing approaches.

6.1. Merits of the Proposed Model

Real-world MCGDM issues typically arise in a complex environment under ambiguous and imprecise data, which is tough to handle. The suggested model is highly appropriate for the considered problem when the information is complicated and uncertain, especially when the current information depends on the bipolar data by experts. Some advantages of the suggested approach are summarized as follows:

i. The suggested technique replicates each alternative's positive and negative characteristics as BSS. To manage aggressive DM, this integrated model is more comprehensive and suitable.

ii. This approach is also preferable because, in this method, the experts are free from any external constraints and requirements.

iii. There is no possibility of losing collective information throughout the process since aggregation is done in the final step.

iv. The established strategy not only takes experts' assessments but also integrates the previous experiences by the MRBS-approximations in real circumstances. Therefore, it is a more generalized approach for a better understanding available data and using artificial intelligence to make decisions.

6.2. Drawbacks of the Proposed Model

The suggested model has a few minor shortcomings, including its complicated structure and the massive information in the form of BSS. Such huge information is challenging to address because of enormous calculations, which are difficult to handle. However, one may establish MATLAB programming to ease these calculations simpler. Moreover, in the proposed model, parameters are independent of the environment. Therefore it cannot produce a ranking result when the parameters are dependent.

6.3. Comparison with Other Models

In this subsection, we compare the suggested strategy with TOPSIS approaches in fuzzy and bipolar fuzzy settings. Among the various MCDM approaches, the TOPSIS technique is the most favored one.

In the fuzzy TOPSIS technique, linguistic evaluations are used instead of numerical values. That is, the rating of the objects and the weights of criteria within the problem are evaluated utilizing fuzzy linguistic variables. Although the TOPSIS technique is the most effective approach in a fuzzy setting, it just gives us a mechanism to estimate the truth membership. On the other hand, the suggested TOPSIS technique offers a modified method for coping with MCGDM problems in which the subjective data is provided via a decision-maker in the form of BSS.

The researchers initiated and investigated bipolar fuzzy TOPSIS [30, 37] and extended the TOPSIS method based on IVHFNSSs [31]. It is generally known that the models can manage some DM problems to convey the idea of experts by using a crisp number. But, due to the uncertainty of the objective world and the complexity of the decision problems, they cannot address some group DM problems. For instance, some experts argue the membership degree of an object to a set and cannot compromise each other. One wants to assign 0.3, but the other prefers to choose 0.5. In this situation, MRBSs can be a perfect solution to this problem.

We explore the following points if we compare our proposed model with the TOPSIS techniques described in [27, 28, 38]. Firstly, these methods cannot address the bipolarity in the DM process, which is a critical feature of human cognition. Secondly, these techniques do not ensure harmony in decision-makers' opinions. Applying the most recent techniques presented in [18, 19] to Example 5.1 yields the following ranking results among the alternatives, displayed in Table 1.

Table 1. The ranking results of various methods to Example 5.1

Current Methods	Ranking Orders
Karaaslan and Çağman [18]	$\mu_5 \succeq \mu_2 \succeq \mu_3 \approx \mu_4 \succeq \mu_1$
Shabir and Gul [19]	$\mu_2 \approx \mu_1 \approx \mu_3 \succeq \mu_5 \succeq \mu_4$
Our proposed approach	$\mu_2 \succeq \mu_5 \succeq \mu_4 \succeq \mu_3 \succeq \mu_1$

A characteristics comparison of various approaches with suggested technique is given in Table 2. The comparison is evaluated with features: membership function (MF), non-membership function (NMF), parametrization, number of decision-makers, and ranking of alternatives.

Table 2. Characteristics comparison of different methods with proposed method

Methods	Characteristics				
	Handle MF	Handle NMF	Manage parametrization	Decision-makers	Ranking
Akram et al. [30]	Yes	Yes	No	One	Yes
Alghamdi et al. [37]	Yes	Yes	No	One	Yes
Eraslan and Karaaslan [28]	Yes	No	Yes	More than one	Yes
Feng [39]	Yes	No	Yes	More than one	Yes
Saeed et al. [40]	Yes	No	Yes	More than one	Yes
Sarwar [41]	Yes	No	No	More than one	Yes
Proposed Method	Yes	Yes	Yes	More than one	Yes

7. Conclusion

MRBSs are treated as practical tools for portraying the uncertainties and vagueness involved with the MCGDM problems. Thus decision-makers become more flexible in representing their judgment using MRBSs. In this work, we have presented a novel application of the MCGDM problem with the data having bipolarity and uncertainty. The framework is based on the TOPSIS method and MRBSs. We have defined a detailed mathematical procedure for the TOPSIS-based MRBSs method. The proposed approach integrates the strength of MRBSs theory in handling uncertainty and the advantage of the TOPSIS evaluation technique in MCGDM. An algorithm of DM is also established, which has two key benefits. Firstly, it evaluates the bipolarity of the data, containing uncertainty. Secondly, it considers the opinions of any (finite) number of experts about any (finite) number of objects. Additionally, we provide an application to demonstrate that the proposed strategy can effectively apply to specific issues, including uncertainty. At last, a comparative study of the suggested approach is conducted.

Numerous topics require further investigation. Bearing in mind the above, future perspectives will focus on the following:

i. The hybridization of the MRBS theory and more comprehensive selection models, such as VIKOR, ELECTRE, AHP, COPRAS, and PROMETHEE.

ii. The proposed method can be generalized to a fuzzy environment, and useful DM methods could be established.

Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

Conflicts of Interest

All the authors declare no conflict of interest.

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