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Research Paper

A New Approach Based on Centrality Value in Solving the Minimum Vertex Cover Problem: Malatya Centrality Algorithm

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Abstract — The graph is a data structures and models that used to describe many real-world problems. Many engineering problems, such as safety and transportation, have a graph-like structure and are based on a similar model. Therefore, these problems can be solved using similar methods to the graph data model. Vertex cover problem that is used in modeling many problems is one of the important NP-complete problems in graph theory. Vertex-cover realization by using minimum number of vertex is called Minimum Vertex Cover Problem (MVCP). Since MVCP is an optimization problem, many algorithms and approaches have been proposed to solve this problem. In this article, Malatya algorithm, which offers an effective solution for the vertex-cover problem, is proposed. Malatya algorithm offers a polynomial approach to the vertex cover problem. In the proposed approach, MVCP consists of two steps, calculating the Malatya centrality value and selecting the covering nodes. In the first step, Malatya centrality values are calculated for the nodes in the graph. These values are calculated using Malatya algorithm. Malatya centrality value of each node in the graph consists of the sum of the ratios of the degree of the node to the degrees of the adjacent nodes. The second step is a node selection problem for the vertex cover. The node with the maximum Malatya centrality value is selected from the nodes in the graph and added to the solution set. Then this node and its coincident edges are removed from the graph. Malatya centrality values are calculated again for the new graph, and the node with the maximum Malatya centrality value is selected from these values, and the coincident edges to this node are removed from the graph. This process is continued until all the edges in the graph are covered. It is shown on the sample graph that the proposed Malatya algorithm provides an effective solution for MVCP. Successful test results and analyzes show the effectiveness of Malatya algorithm.

Keywords: *Minimum vertex cover, Graph theory, Malatya centrality algorithm, Centrality algorithm.*

1. Introduction

A graph is a widely used basic mathematical model and a widely used data structure in computer science (Cormen, Leiserson, Rivest, & Clifford, 2001). Graph structure is used in modeling existing problems in many engineering fields such as transportation and security. Thus, algorithms and solutions developed for the graph are expected to provide solutions to these problems (Khattab, Mahafzah, & Sharieh, 2022). However, among the many problems defined on the graph, they are NP problems that cannot be solved in polynomial time. The Minimum Vertex-Cover Problem (MVCP) is one of the problems used in this field (Dinur & Safra, 2005).

Vertex-cover is one of the basic algorithms used on graphs (Thulasiraman & Swamy NS, 2011). The VC algorithm determines which nodes can be selected to cover all edges in a graph (Khattab et al., 2022). For this coverage process, different subsets of nodes in the graph can be determined. However, when all edges are covered by a minimum number of nodes, it is referred to as MVCP. MVCP is an NP-Complete optimization problem that is difficult to solve in polynomial time (Haider & Fayaz, 2020). Many algorithms have been proposed and approaches have been developed to solve this problem.

Many real-life problems can be modeled and solved based on MVCP (L. Wang, Du, Zhang, & Zhang, 2017). Current problems in many fields such as network, communication, engineering, and bioinformatics are modeled on the basis of MVCP (Yigit, Dagdeviren, Dagdeviren, & Challenger, 2021). Therefore, the algorithms developed for MVCP and the solutions found can also be used for real-life problems (Dagdeviren, 2021). Also, since MVCP is an NP-Complete optimization problem, research and development of possible solutions continue. Since MVCP is a problem that cannot be solved completely, different alternative approaches have been presented to solve this problem (Zhang et al., 2022). At the beginning of these are the heuristic and meta-heuristic methods.

MVCP is an NP-Complete optimization problem that cannot be solved in polynomial time. Therefore, there is not exact solution to this problem and it is difficult to find solutions close to the optimum solution set. MVCP is tried to be solved by approximate methods. In the literature, there are different approaches to solve MVCP. First, heuristics and metaheuristics approaches have been widely used to determine the minimum possible set of solutions. Kahattab et al. have proposed a new method to solve MVCP by using chemical reaction optimization and the best solution algorithm in their proposed method (Khattab et al., 2022). In this study, the initial population was formed by these methods and it was shown that the proposed method gave better results than the classical optimization methods. Quantum is another optimization method proposed by Link et al. to solve MVCP. The approximate optimization algorithm was used (Zhang et al., 2022). In this method, no linear time solution is developed for MVCP, but a probabilistic approach that can be performed on quantum computers is given. Dagdeviren proposed a new metaheuristic approach for MVCP by using the genetic algorithm and greedy approach together with his proposed approach (Dagdeviren, 2021). The proposed method is designed to monitor energy efficiency in wireless mobile networks. Three metaheuristic methods were used to demonstrate the effectiveness of the method. Another proposed approach for wireless mobile networks was developed by Yigit et al. (Yigit, Dagdeviren, & Challenger, 2022). In the study conducted in wireless networks and the field of the Internet of Things, two different approaches have been used for the orientation of sensors or objects. In the first approach, communication in the distributed disordered network is handled, while in the second method, information about the two dominant nodes in the graph structure is available. The analyzes showed that the system they proposed gave better results than their counterparts.

There are different approaches in the literature to solve MVCP. Jovanovic et al. developed the method for MVCP in weighted graphs (Jovanovic, Sanfilippo, & Voß, 2022). The proposed method consists of stages. In the first stage, the algorithm is prevented from getting stuck in the local minimum in total values. In the second stage, learning algorithms are added to problem-solving. In another approach, the highest-order nodes in the graph are used (Akram & Ugurlu, 2022). In this method, the minimum vertex-cover set is determined by knowing the value of the two highest-order nodes. A new method for MVCP based on Shapley – Shubik index estimation has been proposed by Gusev. In this method, graph theory and game theory are used together (Gusev, 2020). This study aims to use the Vertex-cover problem in transportation networks and to determine the optimum solutions for the networks. In another approach that solves MVCP effectively, a two-person-based master-apprentice evolution algorithm is executed to increase the diversity of solutions (Y. Wang, Lü, & Punnen, 2021). Next, a configuration control and solution-based approach are presented by combining the hybrid tabu search and local search procedure.

In this study, Malatya algorithm, which offers an effective solution for MVCP, which is an NP-complete problem, is proposed. MVCP consists of two steps, calculating the Malatya centrality value and determining the vertex to be selected for MVCP. While calculating the Malatya centrality value of the vertex, the node's degree and adjacent vertex degrees are used together. For each vertex in the graph, the Malatya Centrality value is produced by summing the ratios of its degree with the adjacent vertex degrees. Then, considering the calculated Malatya centrality values, the node with the highest value is selected and the relevant vertex and edges are removed from the graph. remaining graph The recalculation and selection processes are repeated for the structure. When the process is complete, the required set of nodes that make up the solution for MVCP is determined.

The rest of the article is as fallows. In the second part, the proposed method is mentioned. In the third part, the Malatya centrality value is calculated and the solution set for MVCP is determined. In the fourth part, evaluations of the proposed algorithm and applications on sample graphs are given. In the conclusion part, the results related to the proposed algorithm are given.

2. Material and Method

In this study, a new approach is proposed to solve MVCP effectively. The proposed approach consists of two steps. The first of these steps is the calculation of Malatya centrality values and the other is the vertex selections. It is the determination of the vertex to be selected to solve the vertex cover problem. Malatya centrality values for the nodes in the graph are calculated separately for each node. In the calculation of this value, the node's degree and the degrees of its adjacent nodes are used. Malatya centrality value of each node consists of the sum of the values obtained by dividing the node's degree by the degree of each adjacent node.

Malatya centrality values, the nodes in the graph are selected respectively for the edge coverage process, starting from the node with the maximum Malatya centrality value. The selected node and its adjacent edges are removed from the graph. This calculation and subtraction are continued until all edges are covered.

2.1. Centrality

Centrality is used in many fields, especially in graph theory and network analysis. Centrality is assigning numbers to the node depending on their position in the graph and ranking the node (Borgatti, 2005). In many applications, it is aimed to find the most efficient node in the graph or network. Numerous algorithms have been proposed to find the central node or nodes. However, the connections on the node are the same as in the given graph. It is at the beginning of the determining parameters to measure its centrality. This approach, called degree centrality, is included in the structure of many algorithms such as PageRank, which is widely used (Kumar, Duhan, & Sharma, 2011).

Proposed Malatya algorithm, the centrality value was expressed as the Malatya centrality value and node connections were used while calculating this value. For each node, the Malatya centrality value is calculated by using the node's degree and the degree of adjacent nodes. In determining the Malatya centrality value, the degrees of the adjacent nodes along with the node's value are also determinative.

2.2. Vertex Cover

Vertex-cover problem is to use the nodes in any graph to show whether all the edges in the graph are coverable. Many real-world problems can be expressed as VC problems. Placing cameras in corridors that see all rooms on the same floor or detecting repetitive DNA sequences are exemplary applications for this problem. Possible solutions for the VC problem also offer solutions for real-world problems (Hossain et al., 2020).

The vertex-cover problem can be achieved with minimum nodes expressed as MVCP. MVCP is an NP-complete optimization problem and the optimal solution is difficult to find. Various optimization methods have been proposed to find the solution to this problem and predict its results. MVCP in the literature Many approaches have been proposed to identify optimal or near-optimal solutions.

3. Proposed Malatya Algorithm

In this study, a new algorithm is proposed to solve MVCP. MVCP consists of two parts. First of all, Malatya centrality value is calculated for the nodes in the graph. Using these Malatya centrality values, the solution set for MVCP is calculated. The general outline of the proposed method is given in Figure 1. In the figure, firstly, the edge and node data of the graph are received. Then, Malatya centrality value is calculated for each node by using its degree and the degrees of its adjacent nodes. Starting from the highest of these values, the next node is selected each time until all the edges in the graph are covered. When all edges are selected, the node selection process is terminated. The resulting output will be the minimum vertex coverage set.

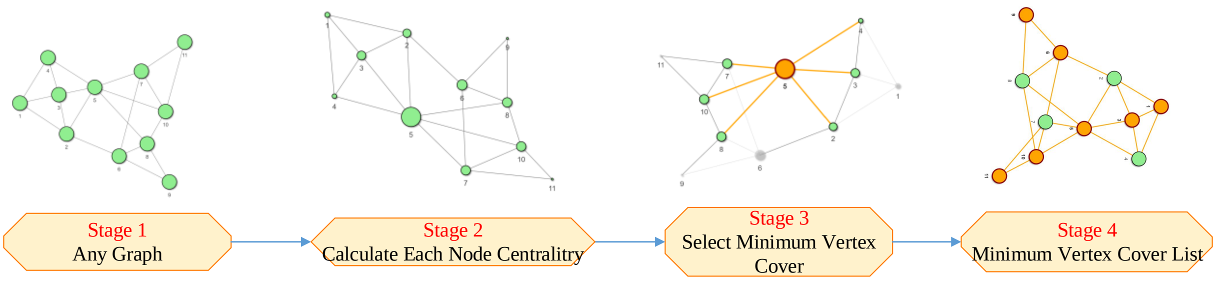

Figure 1. The general structure of the proposed algorithm

Proposed Malatya algorithm for the solution of MVCP consists of the following steps.

1 The degree of a node is obtained by adding the parts of its degree to the degrees of each of its adjacent nodes. The node distinguishing number to the resulting value and denoted by (v) for a node $\psi(v)$

2 The node with the maximum $\psi(v)$ value is selected.

3 The selected node and its coincident edges are removed from the graph.

4 Algorithm is completed when nodes covering all edges are selected; otherwise, it is returned to the first step with the new shape of the graph.

The algorithm used to calculate the Malatya Algorithm node (v) values are given in Equation 1. ψIn this equation, n represents all the nodes in the graph, respectively. For a node v_i, the set of adjacent nodes is represented as $N(v_i)$.

$$\psi(v_i) = \sum_{\forall v_j \in N(v_i)} \frac{d(v_i)}{d(v_j)} \qquad (1)$$

Algorithm 1, the Pseudocode of the proposed approach for MVCP is given. In the given codes, the operations of the proposed algorithm are listed in order. The codes of Malatya algorithm are given in lines 1-10. In lines 11-21, the vertex to be deleted is determined and its adjacent edges are deleted. This process is continued until all the edges are covered. Details about these codes are given in the explanations next to the relevant piece of code.

Algorithm 1. Proposed algorithm Pseudocode

Minimum Vertex Cover

```
1. G:(V,E)                                              // G graph
2. MalatyaCentralityMethod <- function (g){             //Malatya algorithm is defined
3.    VertexList <- c(V(g))                             // Throw vertex from graph to array
4.    for (i in VertexList )                            // Work as many vertexes in the array
5.         Vdegree <- degree ( g,v = V(g)[i])           // Calculate the node degree of the corresponding vertex
6.         AdjacentDegree <- degree ( g,v = neighbors ( g,v = V(g)[i]))     // Calculate the node degree of the neighbors of the relevant node
7.         Value <- Vdegree / AdjacentDegree            // Degree of related node / degree of the adjacent node
8.         MalatyaCentralityValue <- print ( paste (V(g)[i], sum (Value)), digits = 3)    // New centrality value results
9.     return ( MalatyaCentralityValue )                // Returns the maximum values of the graphs
10. }
11. FindMaxMalatyaCentralityValue <- function (g){      // method that returns the vertex name with maximum centrality
12.      maxVertex <- FindMaxVertex ( MalatyaCentralityMethod (g));      // calculates the maximum vertex degree
13.      V <- maxVertex ;
14.      DeleteEdges (MV);                              // The edges of the selected maximum vertex are deleted
15.      return ( maxVertex );
16. }
17. FindMinVertexCover <- function (g){                 // Minimum vertex Detects cover members
18.       while ( Edge.Count != 0)                      // Runs as long as there is an unreached edge in the graph
19.             FindMaxMalatyaCentralityValue (g);
20.                  print ( maxVertex );               // Minimum vertex print cover members
21. }
```

Calculation of Malatya centrality values of Malatya algorithm proposed in Algorithm 2 and the vertex operations of the cover problem are given in mathematical expressions. In this algorithm, $\psi(v_i)$Malatya centrality value, V_c solution set, $d(v_i)$ v_i. the degree of the node, |V| Indicates the number of nodes in the graph. With the given algorithm, the solution set is initially empty. Then Malatya centrality values are calculated and added to the relevant vertex solution set. When all edges are covered, the solution set is determined.

Algorithm 2. Mathematical representation of Malatya algorithm

Minimum Vertex-Cover Algorithm

Input: Adjacency matrix of G is A and G = (V, E) // G graph

Output: $V_c \subseteq V$, V_c is a set of nodes and it is a solution for vertex-cover problem

$V_c \leftarrow \emptyset$
While $E \neq \emptyset$ do
    $i \leftarrow 1, \ldots, |V|$
        $\psi(v_i) = \sum_{\forall v_j \in N(v_i)} \frac{d(v_i)}{d(v_j)}$
    $V_c = V_c \cup \{\max(\psi(v_i))\}$
    $V = V - \{v_i\}$, and $E = E - \forall(v_i, v_j) \in E$
Output=V_c

4. Experimental Results

In this study, Malatya algorithm, which offers an effective solution to MVCP, is proposed. MVCP consists of two steps, calculating Malatya centrality values and determining inclusive nodes. Details on these steps are given in section 3. The most important advantage of this method is that it provides a polynomial solution for MVCP, which is an NP-Complete problem. The efficiency of the proposed algorithm is shown in detail on sample graphs step by step. In this graph, the solution set containing the minimum number of vertex was determined by using Malatya algorithm for MVCP.

In figure 2, the details of the sample graph used to show the effectiveness of the Malatya algorithm are given. The nodes and edges of the graph are given in detail in the figure. This graph shows the edges of each node and the degrees of its adjacent nodes. In this graph, Malatya centrality value is calculated using Malatya algorithm and a suitable graph model is presented for determining inclusive nodes.

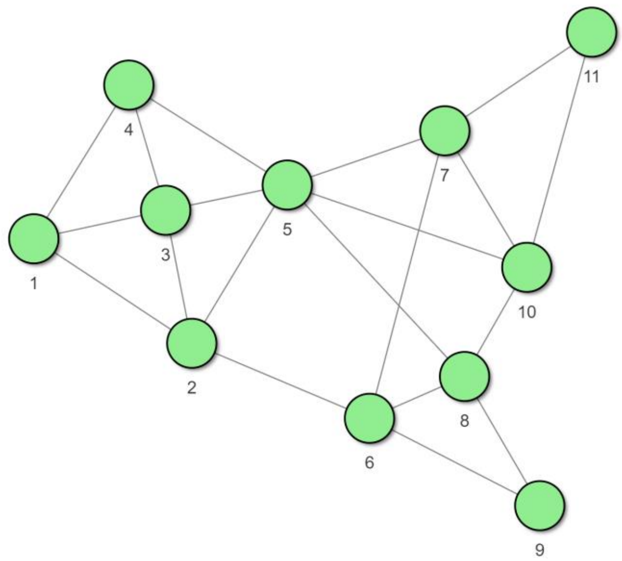

Figure 2. The graph model used in the calculation

Centrality values calculated using the proposed method in Figure 3 are given in a sample graph structure. In this graph, nodes with high Malatya centrality values are shown with a larger circle, while those with low centrality are shown with a small circle. As can be seen in the figure, the highest value node was calculated as node 5, while the lowest value was calculated as nodes 9 and 11.

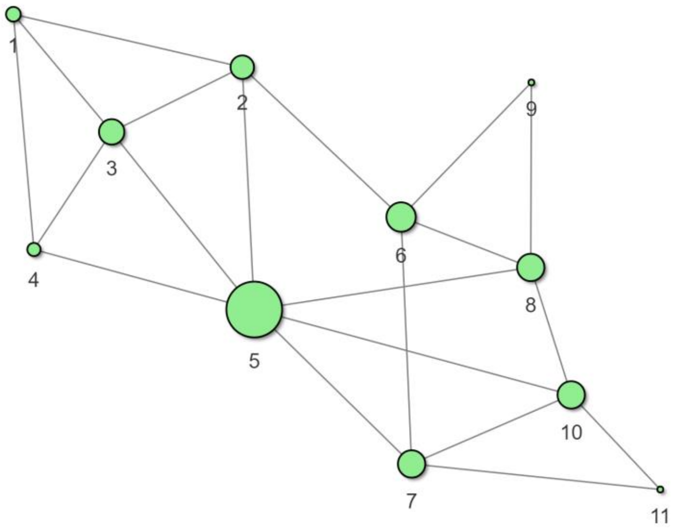

Figure 3. Graph structure showing Malatya centrality values

In Figure 4, the proposed algorithm is used to determine the vertex of cover problem, the first node is selected and the corresponding graph structure is given. The node with the highest centrality values calculated using this algorithm was selected. In this node selection, the central node for the sample graph structure was determined by considering both its own degree and the degree of adjacent nodes. Then the edges associated with this node are vertex selected for the cover problem. Node 5 and the edges related to this node are selected in the sample graph.

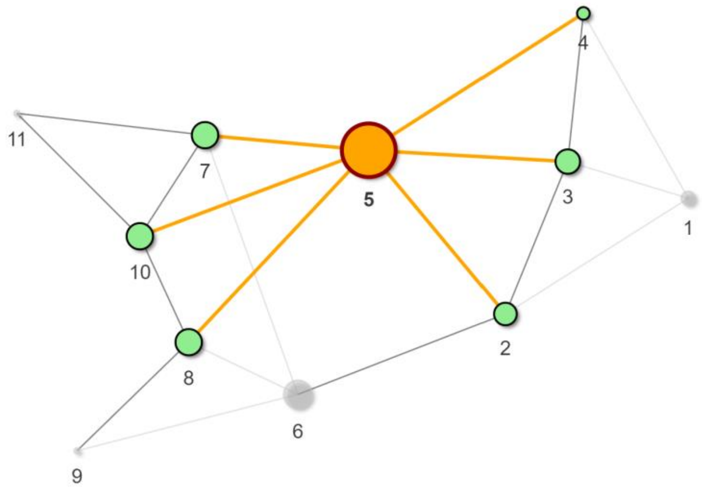

Figure 4. Example structure where the most effective node was determined

The set of nodes determined for the vertex cover problem by using the suggested method is given in Figure 5. Here, after node 5 with the highest Malatya centrality value was selected, the nodes with the highest value were selected respectively. vertex The solution set determined to solve the cover problem consists of nodes 1, 3, 5, 6, 9, 10, and 11. It is seen on the graph structure that this solution set is the minimum number for the sample graph.

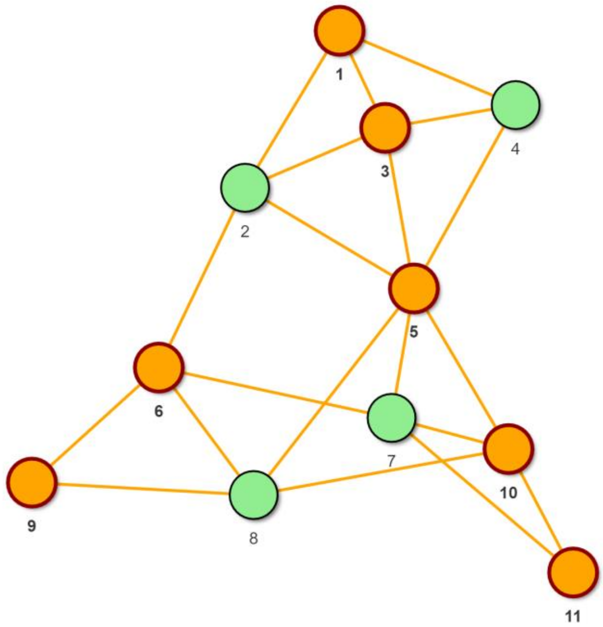

Figure 5. Minimum vertex nodeset used for the cover

5. Conclusion

A graph data model is used to model and illustrate many real-life problems. Many technical problems, such as security, and transportation, are based on the graph data model and its possible solutions. vertex cover is one of the important NP-Complete optimization problems in graph theory. Using a minimum number of nodes for vertex cover is referred to as MVCP. Many algorithms and approaches have been proposed to solve this problem. In this study, an approach has been developed to find an effective solution for MVCP. The proposed approach offers a polynomial approach, unlike the literature. The new approach consists of two steps. First, Malatya centrality value is calculated for each node using Malatya algorithm. In the calculation process, for each node, the sum of the ratios of the node's degree to the adjacent nodes' degrees is taken. In the second step, the nodes in the graph are sorted starting from the calculated maximum Malatya centrality values. Starting from the node with the maximum value, appropriate node selections are made for MVCP. The solution efficiency of Malatya algorithm proposed for MVCP has been tested with sample graph data. The effectiveness of the proposed Malatya algorithm was demonstrated by successful test and analysis results.

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